

Ricardo

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1 Introduction

This lecture looks at a model which really is a social system in which different agents with different skills and preferences take on different roles. While there was no need for markets on Robinson's island, it will be clear here. The questions asked about Robinson Crusoe's island are also relevant here: The existence of equilibrium, the optimality of equilibrium and the ability of the market to attain every possible optimum, and how does the equilibrium and optimum change as parameters of the model are varied. But here there are additional relevant questions pertaining to the description of equilibrium, such as describing the pattern of trade.

2 The Model

There are two countries, $i = 1, 2$, each with a single *primary factor of production* L_i . By *primary* we mean that it is not produced. Each country has a single consumer who owns the factor but does not consume it. We might think of the primary factor as that country's labor supply. There are two goods, A and B , each producible from the primary factor. Production is linear in the factor. Let a_i denote the output of good A per unit of L_i in country i , and let b_i denote the output of good B per unit of L_i in country i .

3 Autarchy

First suppose that the countries can engage in no trade at all. The *production possibility frontier* represents the trade off in production between goods A and B in country i . The ppf in country i is a straight line with slope $-a_i/b_i$. Assume that consumer preferences are such that for each good, the

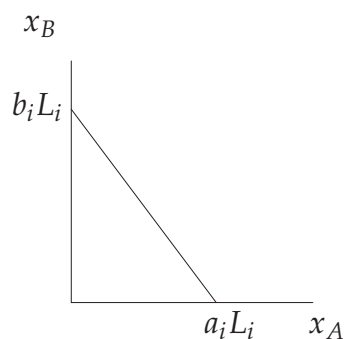


Figure 1: The Production Possibility Frontier

consumer's marginal utility is positive and becomes infinitely large as the quantity consumed of that good converges to 0.

3.1 Efficiency

Efficiency entails things: First, all of the factor must be used in production, so that the bundle of produced goods lies on the ppf. This is called *producer efficiency*. Second, the consumer's indifference curve at that bundle must be tangent to the ppf.

3.2 Competitive Equilibrium

Take labor as the numeraire good. We have not said whether production is specialized to particular firms for each good, or whether one firm produces everything. It does not matter; since there are

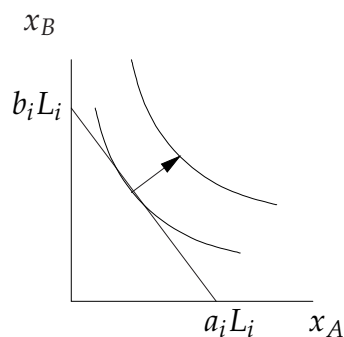


Figure 2: The Competitive Equilibrium

no production externalities, maximizing individual profit for each firm and maximizing aggregate profit are identical. Profits for good a are $p_A x_A - l_A = (p_A a_i - 1) l_A$, where l_A denotes the amount of primary factor devoted to the production of A . Thus profit maximization entails infinite supply when $p_A > 1/a_i$, any supply when $p_A = 1/a_i$, and 0 output when $p_A < 1/a_i$, and similarly for good b . Under our marginal utility assumption, both goods will be demanded at any positive price, and so $p_A = 1/a_i$ and $p_B = 1/b_i$; thus is the wage rate equal to the value of marginal product. This is all illustrated in figure 2. We can draw this picture in two dimensions because we know that all of the primary factor is going into production, that aggregate profits are 0, and so the consumer's income is just the value of his factor endowment, which equals the value of aggregate output.

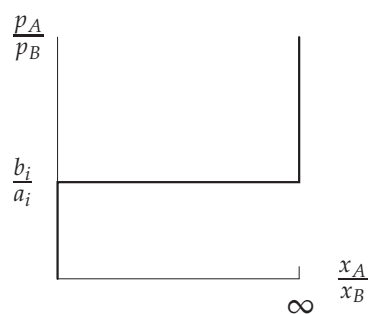


Figure 3: The Supply Curve

One way of understanding this market is to look at a version of the aggregate supply curve, which graphs the price ratio against the goods ratio. Any demand curve will cut the horizontal segment, and so the price ratio is the ratio of the inverse marginal products.

Equilibrium will be efficient since the budget line and the ppf are coincident.

4 Trade in Final Products

Now suppose that goods can be traded across the two countries. This exercise becomes interesting when the marginal products in the two countries are different. Suppose $a_1/b_1 > a_2/b_2$. Factors cannot be traded. If we think that both factors are labor, we are presuming that labor is immobile.

To derive the production possibility curve, suppose that all resources are going to the production of good B . Give up one unit of B to produce A . Where should that unit come from? The amount of A that can be produced by sacrificing one unit of B in country i is a_i/b_i , so the most is gained by producing the A in country 1. This remains the case as more A is produced, until country 1 is completely specialized in the production of good A . Only then do resources start shifting in country 2. The ppf is the vector sum of the ppf's of the two countries; denoting the ppf of country i by P_i , the

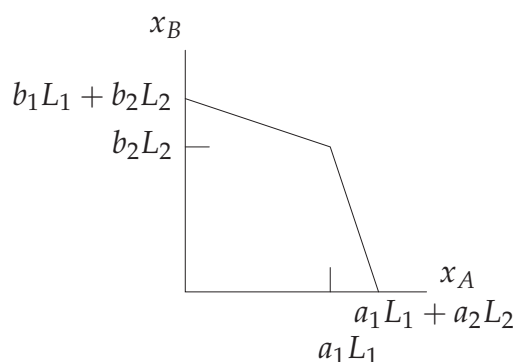


Figure 4: Production Possibilities with Trade

world ppf is $P_1 + P_2$. By this we mean that

$$P_1 + P_2 = \{x + y : x \in P_1 \text{ and } y \in P_2\}.$$

4.1 Competitive Equilibrium

What are the possibilities for equilibrium?

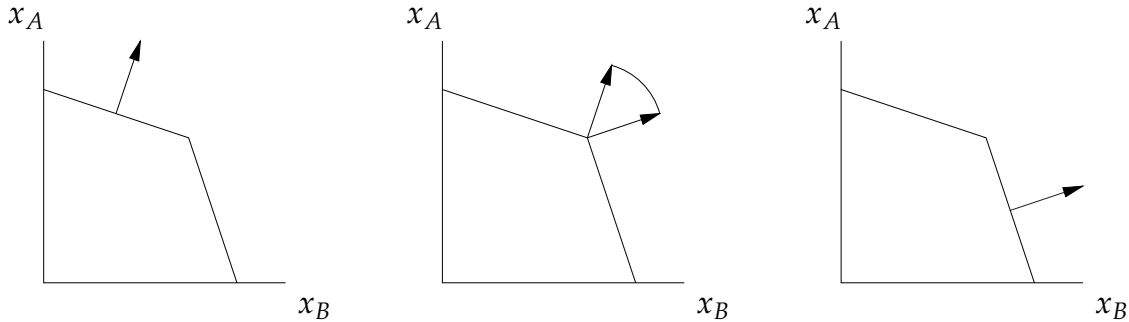


Figure 5: Profit Maximization

As figure 5 shows, there are three possibilities. In the left-hand ppf, country 2 is specialized in B while 1 is mixed. In the center, both countries are specialized, 1 in A and 2 in B , while in the right hand ppf, 1 is specialized in A and 2 is mixed. In any equilibrium, at least one country is specialized. If one country is mixed, then the p_A/p_B price ratio will be determined entirely by technology, while if both countries are specialized (as is the case in the center) the technology determines a cone within which equilibrium prices can occur.

To see that both can occur, suppose that both consumers have identical CES preferences, $u(x_A, x_B) = (\alpha x_A^{-\rho} + \beta x_B^{-\rho})^{-1/\rho}$ with ρ , α and β positive and $\alpha + \beta = 1$. This of course is the CES production function and the elasticity of substitution is $\sigma = 1/(1 + \rho)$. If the two consumers have wealth levels w_1 and w_2 respectively, then their demands will be

$$x_A^i = \frac{\alpha^\sigma p_A^{1-\sigma}}{\alpha^\sigma p_A^{1-\sigma} + \beta^\sigma p_B^{1-\sigma}} \frac{w_i}{p_A} \quad x_B^i = \frac{\beta^\sigma p_B^{1-\sigma}}{\alpha^\sigma p_A^{1-\sigma} + \beta^\sigma p_B^{1-\sigma}} \frac{w_i}{p_B}$$

and aggregate demand will be

$$x_A = \frac{\alpha^\sigma p_A^{1-\sigma}}{\alpha^\sigma p_A^{1-\sigma} + \beta^\sigma p_B^{1-\sigma}} \frac{w_1 + w_2}{p_A} \quad x_B = \frac{\beta^\sigma p_B^{1-\sigma}}{\alpha^\sigma p_A^{1-\sigma} + \beta^\sigma p_B^{1-\sigma}} \frac{w_1 + w_2}{p_B}.$$

An income redistribution has no effect. Aggregate demand and individual demand both lie on the ray coming from the origin described by the equation

$$\frac{x_B}{x_A} = \left(\frac{\beta}{\alpha}\right)^\sigma \left(\frac{p_A}{p_B}\right)^\sigma,$$

and how far out on that ray they lie is determined by income. In this case market demand can be derived from that of a representative consumer; that is, there is a *representative consumer* with the identical utility function such that market demand is the individual demand that would arise if the representative consumer had all the wealth.

The market 'supply curve' is plotted in figure 4.1. The demand curve can intersect the supply

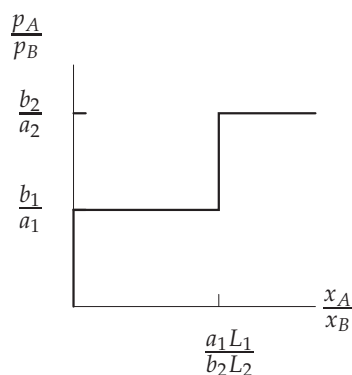


Figure 6: The World Supply Curve

curve at any of three places; the left horizontal segment, where firm 1 mixes and 2 produces only x_B ; the vertical segment wherein both firms are specialized — firm 1 in x_A and firm 2 in x_B ; and finally, the remaining horizontal segment wherein firm 2 specializes in B and firm 1 mixes.

4.2 Optimality

Optimality is more complicated here. Essentially, optimality divides into three questions: Are final products allocated efficiently between consumers (consumer efficiency)? Are goods being produced efficiently (producer efficiency)? And are the right mix of products being produced?

The answers are: (1) If \bar{x} is the output bundle, any division of the bundle $(\lambda\bar{x}, (10\lambda)\bar{x})$ with the first bundle going to country 1 and the second to country 2 is an optimal division of the bundle. (Why?) (2) Production is on the boundary of the production possibility frontier, so labor is being used efficiently. (3) The consumers' indifference curves are tangent to their budget lines and the isoprofit lines have the same slope. Production maximizes profit. There is no way of changing the output mix to increase profit, so there is no way of changing the output mix to increase utility.