FUNDAMENTAL SOURCES OF UNPREDICTABILITY

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In discussing the fundamental sources of unpredictability, I shall concentrate on those indeterminacies that are definitely required by theory. Let me therefore begin by eliminating from consideration supposed indeterminacies stemming from doubts that some people may entertain about basic principles or from certain kinds of ignorance that I believe to be temporary and likely to be remedied in the relatively near future. I assume the following:

1) Quantum mechanics is correct. The formulation and interpretation of quantum mechanics are still undergoing some necessary evolution, especially in order to accommodate quantum cosmology in a comfortable way, but the basic character of quantum mechanics has always been the same and we may suppose it will remain unchanged.

2) The elementary particles and their interactions obey a definite dynamical law, discoverable by inquiring complex adaptive systems such as the human scientific enterprise. Although the process of discovery involves a sequence of approximate schemata, there is an endpoint to the process after a finite amount of research. (Of course, it can never be possible to prove that the resulting unified theory is perfect; one can only verify it in the usual way by comparing predictions to available observations.)

Humans may already have found this unified quantum field theory in the form of superstring theory, which has, to begin with, no arbitrary parameters. (Of course, spontaneous symmetry breaking may give rise to some parameters and even to a choice of solutions, with probabilities for the various alternatives. I shall deal with that possibility further on.)

This second assumption is equivalent to stating that there is no necessary fundamental unpredictability stemming from ignorance of the universal dynamical law.

3) The density matrix (in the Schrsˇdinger picture) of the universe near the beginning of its expansion is also knowable. It must in any case be comparatively simple and very far from equilibrium. The second law of thermodynamics and the other associated arrows of time are explained by these properties of the initial density matrix along with the fact that the universe is still very young -- the interval of ten billion years is extremely short compared with the relaxation time from the special initial condition.

Hartle and Hawking have proposed an ingenious pure state candidate for the density matrix, such that the wave function is in principle calculable from the action functional of the fundamental dynamical theory -- the two basic laws of physics would then become one. Another possibility is indicated by the work of Linde and others, namely that the universe emerged and become isolated from a larger system (which I call a multiverse) as a kind of bubble, one among very many. The density matrix would then be impure, with probabilities corresponding to the statistics of such bubbles, but it would still have a simple character. Here ÒsimpleÓ means expressible by means of a concise formula [1,2,3] in terms of the fundamental fields at the time.

Let us return to the promising candidate for the role of fundamental dynamical law for all matter. Superstring theory was thought for a number of years to exhibit several different mutually exclusive forms, of which the "heterotic" one was the most likely to agree with nature. However, it now appears that all the different forms are related.

Also, the heterotic form, studied in a simple approximation, exhibits many different solutions, corresponding to different numbers of spatial dimensions, different sets of elementary particles, and/or different interactions among particles. Many of those solutions may prove to be spurious when the approximation is improved, but what if multiple solutions remain? It looks as if they will all turn out to be related, just as the different forms of the theory are related, in a way that is reminiscent of the relationship of phases in condensed matter theory.

If there really are multiple solutions, as well as multiple forms of the theory, all manifestations of the same basic law, what determines the ÒphaseÓ that characterizes our universe? One might imagine a principle that determines the choice, say a principle of least S, where S indicates the quantum-corrected, Euclideanized action. Another possibility is a probabilistic situation, in which the likelihood of a particular solution is proportional to exp (- 2S). The one with the lowest value of S would then be the most likely, but not the only possible choice [1]. These likelihoods, or a priori probabilities, would be converted into statistical probabilities if our universe is really a multiverse containing an enormous number of largely independent universes. If not, we could still imagine such a multiverse as a mathematical abstraction, an aid to thinking about probability in connection with the universe.

Suppose we know the two basic laws that govern the behavior of all matter. What then? Can we predict, in principle, the history of the universe? Of course not. Because the laws of physics are quantum-mechanical, prediction is limited to probabilities for alternative histories of the universe. The limitations go far beyond the famous, but rather trivial uncertainty principle of Heisenberg. In order to give an account of those limitations, we must sketch the decoherent histories approach to quantum mechanics, as developed by James Hartle and me [4,5,6]. Our approach can be regarded as part of an ongoing effort by a number of theoretical physicists to construct a modern interpretation of quantum mechanics, one that is compatible with quantum cosmology and that explains in a convincing way how the quasiclassical world of familiar experience emerges from the underlying quantum universe. Some of the other authors who are part of this movement are R. Omn?s, R. Griffiths, D. Zeh, W. Zurek, J. P. Paz, C. Isham, and N. Linden.

Viewed in the most general way, from the standpoint of the whole universe (necessary in order to assure compatibility with quantum cosmology), the application of quantum mechanics is always to histories, since predictions of the probabilities of future events are always made subject to the explicit or implicit assumption that certain things have already happened, things that have meaning only if certain other things happened earlier, and so forth.

A very important advantage of the decoherent histories method is that it permits quantum mechanics to be formulated in a straightforward manner while meeting the requirements of general relativity. The correct unified quantum field theory must, of course, include quantized Einsteinian gravitation. Superstring theory does so. Indeed, it predicts, in a suitable approximation, the general-relativistic theory of gravitation within the framework of quantum mechanics. Moreover, the preposterous infinite corrections in perturbation theory that plagued previous attempts to incorporate gravitation into quantum field theory are absent.

Of course, there is still the apparent difficulty that quantum mechanics is usually formulated in terms of a temporal succession of spacelike surfaces, hard to define when the metric of spacetime is quantized. The problem with the usual formulation is even more serious when the topology of Euclideanized spacetime undergoes changes in the course of quantum fluctuations. Fortunately, it has been shown by Hartle [7] that our approach can be used to create a slight generalization of quantum mechanics in which the quantized metric poses no difficulties. In what follows, let us, for the sake of simplicity, ignore the complications that arise from general relativity and deal with the approximation in which we have a conventional Hilbert space, an ordinary time variable, and a Hamiltonian.

Furthermore, in outlining the decoherent histories interpretation of quantum mechanics, let us make the simplifying assumption of a pure density matrix r of the universe in the Heisenberg picture (equal to the initial density matrix in the Schrsˇdinger picture), so that

r = |Y><Y|, (1)

as in the Hartle-Hawking situation.

Probabilities enter only because |Y> is being compared to states representing alternative histories a.. The absolute square of the scalar product between |Y> and each of those (normalized) states is proportional to the probability of the corresponding history.

Without a great loss of generality, we can construct the states representing histories by means of sequences of projection operators at a succession of times t1, t2, etc. At each time tk, we have a set of mutually exclusive and exhaustive alternatives ak, which may depend on the previous alternatives, in accordance with causality. Thus the projection operators at time tk are labeled P(ak; ak-1...a2a1). A state corresponding to a history a = a1a2a3a4.........an is then proportional to Ca|Y>, where

Ca = P(an; an-1...a2a1) P(an-1; an-2...a2a1)...........P(a2;a1) P(a1). (2)

Since the alternatives at each time tk are mutually exclusive and exhaustive, we have

P(ak; ak-1...a2a1) P(a'k; ak-1...a2a1) = d(ak,a'k) P(ak; ak-1...a2a1) (3)

and

S P(ak ; ak-1...a2a1) = I, (4)
ak

where I is the unit operator. As a result, we obtain

S Ca = I, (5)
a

so that

Y> = S Ca|Y>. (6)
a

For probabilities to be assignable to the histories a, there must not be any interference terms between them, since the probability of history a or history b has to equal the sum of the probabilities of the two histories a and b. The set of histories without interference terms is called a decoherent set. There are various degrees of decoherence, but for simplicity let us deal mainly with what we call medium decoherence, which means that the various states Ca|Y> are all orthogonal to one another. The norm of each of those (unnormalized) states is its probability.

Except for trivial cases, sets of fine-grained histories are not decoherent, and the histories a in a decoherent set must be coarse-grained. By definition, a set of fine-grained histories would specify the values of a complete set of variables at every instant of time. For example, in nonrelativistic quantum mechanics without spin, the positions or momenta of all particles might be given at all times. (The Heisenberg uncertainty principle makes it impossible, of course, to specify exactly both momentum and position for the same particle at the same time.) The coarse-grained histories may be regarded as bundles of fine-grained histories, in which, for example, all times except a discrete set are eliminated by summing over projections onto all values of all variables at all the times that are not in the discrete set. At the discrete times that remain, projections onto all values of many of the variables are summed over. The surviving projection operators at the discrete times would project onto ranges of values of the variables (at those times) that do not have their projections summed over. (In a recent article [6] , Hartle and I have shown that in place of medium decoherence a much stronger form of decoherence is actually needed, implying a much coarser graining of histories.)

We define a realm to be an exhaustive set of mutually exclusive decoherent coarse-grained histories. We call it a quasiclassical realm if the projection operators tend, over long stretches of time, to be onto similar ranges of values of similar operators (i.e., roughly related by time displacement) obeying, with high probability, deterministic laws of time development modified by frequent small fluctuations and occasional major branchings.

The word ÒbranchingÓ refers to the metaphor of a tree of possible decoherent coarse- grained histories, branching as time goes forward, with probabilities for the different alternatives at each branching. Of course, the events actually experienced in this universe select only one outcome at each branching, unpredictable in advance except for probabilities. Once the particular outcome has occurred, if the result is ascertained, then those probabilities ÒcollapseÓ to one and zero. When we deal with decoherent histories, there is no mysterious Òcollapse of the wave function,Ó only the familiar collapse of probabilities that occurs at the race track when we see which horse actually wins a race.

A maximal quasiclassical realm is maximally fine-grained subject to decoherence and quasiclassicality. The quasiclassical realm of familiar experience is a coarse graining of a particular maximal quasiclassical realm, which we can describe in terms of projections onto ranges of values of so-called hydrodynamic variables. Those variables are defined as follows. At each of a set of discrete times, we consider a set of conserved quantities, such as momentum density, energy density, and electric charge density, and nearly conserved quantities, such as nuclear species densities, dislocation densities, and so forth. These quantities are integrated over small volumes, large enough to give sufficient inertia to offer resistance to most fluctuations, but small enough to have rough internal equilibrium. The ranges of values of these quantities and the time intervals are adjusted for decoherence, quasiclassicality, and maximality.

There are obviously many possible variations in detail in the description of this usual maximal quasiclassical realm, but presumably the details do not matter much, so that we are describing essentially a single set of coarse-grained histories. Every complex adaptive system we know has evolved to utilize some coarse-graining of this realm. It is fascinating to speculate about whether other, entirely different quasiclassical realms are exhibited by the theory and, if so, whether complex adaptive systems could evolve to utilize them, but that is a subject to be treated elsewhere. In any case, probabilities and statements about events happening have meaning only within a given realm.

What is crucial in the study of the fundamental sources of unpredictability is that any complex adaptive system [1,2], (including a composite complex adaptive system such as the human scientific enterprise), makes use of an extreme coarse graining of the usual maximal quasiclassical realm. We use the term IGUS (information gathering and utilizing system) to describe a complex adaptive system as observer. Most of the variables in the universe are inaccessible to an IGUS, referring as they do to remote or hidden places, such as the interiors of distant stars, or to small scales at which measurements are unlikely to be made. Thus, as the maximal set of histories unfolds, in an unimaginably long sequence of accidents with probabilistic outcomes, most of the information about outcomes that have actually occurred (or, in the language of special relativity, occurred in the past light cone) is unknown to the IGUS and unavailable for helping to predict the future. Such information must be summed over, with the result that the actual realm utilized by the IGUS is very coarse-grained indeed as regards the past, and of course similar considerations apply to the future. (The use of statistical mechanics is an example of such extra coarse graining.)

Ignorance as a source of unpredictability has been understood for hundreds of years, if not longer. In this discussion, we are merely putting unavoidable ignorance in the context of quantum mechanics.

The indeterminacies we have discussed are all exacerbated by amplification mechanisms. Such mechanisms are responsible for connecting events at the quantum level with quasiclassical histories. Take a measurement situation such as the Stern-Gerlach experiment, where a sodium atom with valence electron spin up develops a certain photographic grain, while it would develop a different photographic grain if its spin were down. Here a quantum variable is correlated directly with a branching in quasiclassical history. This is true whether a human being (or a chinchilla or cockroach) actually looks at the result or not. There are, of course, many natural situations, not set up by human beings, in which events at the atomic or subatomic scale cause branchings in the quasiclassical realm. For instance, the decay of a radioactive nucleus in a sheet of mica can produce a long-lasting track, recording the direction of the decay as well as the fact that it took place. That phenomenon is the basis of fission-track dating.

An important source of amplification is chaos, widespread in nonlinear systems. Classically, the term refers to cases in which the outcome of a process is sensitive to the tiniest details of the input, through divergence of classical orbits from one another, a divergence that is exponential in time. Chaos in quantum mechanics is more subtle, but it can lead to amplification of quantum fluctuations so that they materially affect quasiclassical history. The most obvious effect of chaos, however, is to enhance enormously the effect of ignorance of prior outcomes (including the effects of measurement errors).

Now, in addition to everything we have discussed so far, we must deal with the issue of computation. Difficulty of calculation can be a source of unpredictability. Suppose we are given the theory and the maximal realm coarse-grained -- for past and future -- in accordance with information that can be available to an IGUS (with uncertainties of measurement taken into account). There is then at any time tk a definite set of probabilities for the fully coarse-grained future histories. But is the calculation of those probabilities possible? Obviously it is not practical at present unless a gigantic amount of further coarse graining is introduced. It is probable that, as time goes on and calculational techniques improve, the further coarse graining could be progressively reduced. But does that make the whole problem approach tractability in principle? Even if we look just at the cases of degrees of freedom with high inertia, as in orbits of heavy objects, where classical physics applies to an excellent approximation, it is not clear that accurate prediction over very long periods of time is a tractable problem in principle, because of classical chaos. It is possible therefore that questions of calculability in principle will always have to be discussed with reference to requirements for additional coarse graining and to limited accuracy for calculated probabilities.

To recapitulate, we assume that the fundamental theory of matter and the initial condition of the universe are simple and knowable and that quantum mechanics is correct, apart from slight generalization. The fundamental sources of unpredictability are then the following:

1) Possible indeterminacy from the initial condition of the universe if it is impure.

2) Possible indeterminacy from the choice of solutions of the fundamental theory (for this universe) if the choice is probabilistic.

3) Coarse graining required to achieve decoherence of histories in a maximal realm, say the usual maximal quasiclassical realm. (The decoherence should really be strong decoherence, with the extra coarse graining that implies.) The uncertainty principle is automatically included.

4) The probabilistic character of all the branchings in this realm in the future.

5) The huge amount of additional coarse graining resulting from unavoidable ignorance on the part of any given IGUS about the results of many of the branchings in the past.

6) Still more coarse graining to make calculations practical with available computational tools.

7) Limitations on accuracy of calculation with available computational tools.

We have also discussed the enhancement of indeterminacies by amplification mechanisms, including chaos.

Now it is possible to regard 1) and 2), if they are present, as representing initial accidents for this universe on a par with all the subsequent accidents. Let us take that point of view. Then we can say that the history of the universe is co-determined by the basic laws (the dynamical theory of all matter and the initial condition of the universe) and the outcomes of an inconceivably long sequence of accidents, and we can describe the fundamental sources of unpredictability as:

a) The coarse graining necessary for a maximal realm, say the usual maximal quasiclassical realm, with all its accidents.

b) The probabilistic character of all the accidents (branchings) of that realm in the future.

c) Ignorance on the part of a given IGUS of the outcomes of most of the accidents that have already occurred, together with the exacerbation of the resulting unpredictability by amplification mechanisms.

d) Approximations and limitations on accuracy imposed by computational tools available.

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