# Price Variations in a Stock Market with Many Agents

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Large variations in stock prices happen with sufficient frequency to raise doubts about existing models, which all fail to account for non-Gaussian statistics. We construct simple models of a stock market, and argue that the large variations may be due to a crowd effect, where agents imitate each other's behavior. The variations over different time scales can be related to each other in a systematic way, similar to the Levy stable distribution proposed by Mandelbrot to describe real market indices. In the simplest, least realistic case, exact results for the statistics of the variations are derived by mapping onto a model of diffusing and annihilating particles, which has been solved by quantum field theory methods. When the agents imitate each other and respond to recent market volatility, different scaling behavior is obtained. In this case the statistics of price variations is consistent with empirical observations. The interplay between "rational" traders whose behavior is derived from fundamental analysis of the stock, including dividends, and "noise traders", whose behavior is governed solely by studying the market dynamics, is investigated. When the relative number of rational traders is small, "bubbles" often occur, where the market price moves outside the range justified by fundamental market analysis. When the number of rational traders is larger, the market price is generally locked within the price range they define.

## I. INTRODUCTION

The literature on competition among firms and the literature on stock market prices has been for the most part treated separately. Here we adhere to this approach, although eventually a model which makes explicit the feedbacks between the "real" or physical economy of firms and production must be linked to the paper economy of finance. Even the simplest of economic models tends to become enormously complicated when attempts at realistic modelling are made. Mathematical tractability as well as clarity can be lost in a welter of detail. The basic idea of our investigation is to select model segments of the estimated minimal structure needed to make an adequate description of the statistical properties of price variations in a stock market.

A motivation for this work is suggested by the early observations of Mandelbrot (1963, 1966,1967) on the nature of stock prices and the recent behavior of the derivatives markets where "10 sigma events" have been happening with sufficient frequency to raise doubts about existing models, which all fail to account for non-Gaussian variations in price. Mandelbrot observed that price variations of many market indices over different, but relatively short time intervals could be described by a stable Levy distribution, rather than being Gaussian. The Levy distribution has substantially more weight for large events than the Gaussian distribution where large events (beyond $\sim 4\sigma$) are prohibitively unlikely.

Mandelbrot's observation has since been augmented by fitting to a "truncated Levy distribution" where the scaling regime for price fluctuations is actually finite rather than infinite. Empirical observations suggest that the succession of daily, weekly, and monthly distributions progressively converge to a Gaussian (Akgiray and Booth, 1988). The presence of an intermediate scaling regime, where the price changes $\Delta p$ occur with a probability distribution $P(\Delta p) \sim (\Delta p)^{-\alpha-1}$, can mathematically account for slow convergence to a Gaussian distribution at long time scales. For example, the data for the S & P 500 index is reasonably fitted by a truncated Levy distribution with $\alpha \simeq 1.4$ over a time scale which ranges from a minute to a day, with convergence to a Gaussian at approximately one month

(Mantegna and Stanley, 1995). Also, Arneodo et al (1996) observed a $1/f^2$ power spectrum at long time scales consistent with Gaussian behavior; while at short time scales truncated Levy behavior was observed. They analyzed the DEM-USD exchange rate from October 1991 - November 1994. Although this Levy-type behavior is by now well documented empirically, there is as yet no mathematical model of a stock market which can explain the origin of large price variations with "fat tails", at least over a fairly broad range of time scales. Here such a model is introduced.

We construct an extremely simple, but completely defined, economic model of many agents trading stock. These agents form a market that exhibits large variations in prices resulting from differences in the agent's behavior. The agents are of two types: type (1) are "noise" traders whose current volatility may depend on recent changes in the market and whose choice of price to buy or sell may imitate choices of others; type (2) are "rational" agents each optimizing their own utility functions. There is only one type of stock and each agent can own at most one share. These simplifications are quite drastic for many reasons. First of all, in reality the price changes of different stocks are correlated to each other. Thus one cannot treat each stock as an independent market. In addition, we are ignoring price variations due to exogenous changes such as in interest rates, money supply, or wars breaking out. Finally, we have an extremely simple description of the individual agent's behavior.

In spite of these gross simplifications, some of our toy models exhibit a statistical pattern of price variations consistent with that observed empirically. This suggests that we may have captured a dynamical process that is sufficiently robust to describe price variations in real markets. One version of the model exhibits power law fluctuations at small time scales leading to a "Hurst" exponent $H \simeq 0.6$. Eventually at long time scales the fluctuations may converge to Gaussian with $H = 1/2$. The fat tails in the probability distribution for price variations in our model are a collective effect resulting from many different interacting agents. Our results suggest that large fluctuations in price may be endogenous to the dynamics of stock markets.