Equilibrium Selection in the Stochastic Iterated Prisoner's Dilemma

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I. Introduction

> Many of the benefits sought by living things are disproportionately available to cooperating groups. While there are considerable differences in what is meant by the terms "benefits" and "sought", this statement, insofar as it is true, lays down a fundamental basis for all social life..." (Axelrod, 1984)

The Prisoner's Dilemma has received a great deal of attention in the literature because it captures the problems associated with fostering cooperation in social life. The game itself can be summarized by the following payoff matrix;

		Player B	
		Cooperate	Defect
Player A	Cooperate	c_1, c_2	s_1, t_2
	Defect	t_1, s_2	d_1, d_2

where $t_i > c_i > d_i > s_i$ and $(c_i + c_i) > (t_i + s_i)$. The temptation payoff, t_i, is greater than the payoff to agents if everyone cooperates, c_i. This cooperative payoff in turn is greater than d_i, the punishment for mutual defection, which is greater still than the sucker's payoff s_i. The second inequality simply restricts the payoffs to exclude the possibility that players may do better to alternate between being the "sucker" than to always cooperate.

For each player, Defect (D) dominates Cooperate (C). No matter what his opponent does, Defect is the player's unique best response. Thus, the iterated deletion of dominated strategies gives a unique answer to the one-shot PD game, namely {D, D}. This constitutes a Nash equilibrium, since no player, taking the strategy of his opponent as given, wishes to change his own strategy choice. Notice that while individually rational play prescribes the {D,D} outcome, this equilibrium is Pareto dominated by {C,C}, the collectively optimal outcome. It is this [tension] between individually rational behavior and the collectively optimal outcome that makes the PD game so interesting to economists.

Of course, many social interactions are not of the one-shot variety, but are rather played repeatedly. Thus, there has been great interest in studying repeated games, where players meet over an infinite (indefinite) horizon[1] to play the one-shot (stage) game. These games are of particular interest as they allow for reputation building, retaliation, and, in some instances, cooperation. There exists an extensive literature on these IPD game[2], focused largely, however,

[1] There is a substantial literature on the finitely repeated IPD, which will not be discussed in this paper. See for instance Selten, R. (1978) "The Chain-Store Paradox," *Theory and Decision* 9:127-159; Selten, R. and Stoecker, R. (1986), "End Behavior in Sequences of Finite Prisoner's Dilemma Supergames," *Journal of Economic Behavior and Organization* 7:47-61; or Nachbar (1992) "Evolution in the Finitely Repeated Prisoner's Dilemma," *Journal of Economic Behavior and Organization* 19:307-326.
[2] See for instance Friedman (1971) for theoretical work, and Axelrod (1984) for overview of theory and description of evolutionary approach.

on a fixed payoff game. That is, while the history of play may change over time, the payoffs do not.

Is this a useful way to represent social dilemmas? In some cases perhaps. I argue, however, that the nature of most repeated interactions is better captured in games where payoffs change with each meeting. Such a game is referred to as a Stochastic Iterated Prisoner's Dilemma (SIPD). Little is known about such games[3]. This paper attempts to shed light on the nature of these games, in particular on their equilibrium properties.

II. The Stochastic Iterated Prisoner's Dilemma (SIPD)

A. Motivation

It is not difficult to look out the window and find examples of games where stochastic payoffs - play an important role. One interesting example comes from recent work by Eaton and Eswaran (1996), who discuss know-how sharing within the context of an SIPD. In their model, producers discover better ways of producing their goods each period, with incremental innovations being drawn randomly from an exogenously specified distribution. The important thing to note is that producers never discover the same thing twice. Further, the value of their discoveries (to themselves and to rival producers) changes from period to period. The players problem under these circumstances is considerably more complex than under a fixed payoff scenario. Specifically, their decision over whether or not to share their discoveries with rival producers must take into account not only the possibility of future interactions[4], but also the stochastic nature of innovation.

Stochasticity also plays an important role in many social interactions of interest. Academics regularly exchange favors with colleagues; we read drafts of papers, help with computing difficulties, etc. The payoffs associated with these exchanges of favors, however, are not constant, but change from day to day. The same is true of the (anonymous) interactions we have in driving to work. How late am I in leaving the house? How important is the meeting I'm going to be late for? The game's payoffs - the cooperative payoff, and the payoff associated with defecting - will depend on the answer to these questions, which will change from day to day.

Are these considerations important? Do the equilibrium properties of the these stochastic games tell us anything about social dilemma's that we haven't already learned from study of the fixed-payoff IPD game? This paper answers 'yes' to both these questions.

B. Modeling

There are a number of approaches one can take toward modeling this game. One logical starting point is a standard game theoretical approach. Here, agents are modeled as fully rational, able to compute the subgame perfect Nash equilibrium of the game. However, in their recent paper, Eaton and Eswaran look at the SIPD game from this game theoretic point of view, and find that,

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[3]I am aware of only one piece of research on this topic - see Eaton and Eswaran (1996).

[4]It is this possibility of meeting again which makes cooperation possible. Axelrod (1984) writes that "The future can therefore cast a shadow back upon the present, and thereby affect the current strategic situation." (p.12)