Uncertainty, Difficulty, and Complexity.

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June 12, 1998

Abstract

In this paper, I formally define uncertainty, difficulty, and complexity both as measures of problems and environments and as analytic paradigms and discuss the importance of the difficulty and uncertainty paradigms in the study of institutions.

1 Introduction

Institutions, both formal and informal, align private incentives with the collective good. They enable coordination. They generate, assimilate, and provide information. They reduce uncertainty. They improve our ability to solve difficult problems. And, they channel and reduce the complexity of our daily interactions. Formal game theoretic models have helped us clarify our understanding of the first four of these roles. First, by describing how institutions structure incentives and induce equilibria, they link micromotives with macro-behavior (Hurwicz 1972, Alchian and Demsetz 1972, Reiter 1986, and Morrow 1995). We can then compare institutions by the equilibria they implement (Baron and Ferejohn 1989). When these models fail to make unique equilibrium predictions, they provides a second role for institutions: coordination, or more formally equilibrium selection. This can be done either focally or through refinements. Third, game theoretic approaches highlight informational aspects of problems. Typically, implementing a desired outcome becomes much easier when agents become more informed. Not surprisingly then, to the extent they can increase the amount and accuracy of information, institutions and organizations

*Department of Economics, University of Iowa, Iowa City, Iowa 52240. The ideas put forth in this paper have been improved through conversations with many people, most of all Stan Reiter. I would also like to thank Jenna Bednar, Lance Davis, Steve Durlauf, Lu Hong, Ken Kollman, John Ledyard, Doug North, Mike Ryall, Leigh Tesfatsion, and Simon Wilkie. Support from the MacArthur Foundation, the National Science Foundation, and the Santa Fe Institute is gratefully acknowledged.

improve outcomes, or at least reduce the operational costs of achieving those outcomes (Krehbiel 1992). Finally, game theory through formal incentive compatibility constraints translates privately held information into an incentival cost. Some uncertainty reduction can be bought. More generally and in a less mathematical structure, North (1990) demonstrates convincingly that institutions by reducing uncertainty allow economies to function.[1]

But what of the other two advantages? They undoubtedly exist. Institutions help us to find solutions to difficult problems. When searching for cures to diseases, building defense systems, or drafting welfare policy, we rely on institutional support and coordination. In addition, institutions reduce the complexity of our economic and social interactions. Stock markets allow us to diversify our wealth, and traffic lights, and more broadly traffic laws, simplify the task of driving. Yet, the standard game theoretic, rational optimizing agent approach does not adequately deal with either difficulty or complexity. That is not to say that there have not been efforts. Research on the size of message spaces explicitly addresses the tasks confronting agents (Jordan 1982, Mount and Reiter 1990). But on the whole, difficulty, and complexity do not occupy central locations in our current theoretic analyses of strategic and market interactions. They do not enter into discussions of the role of institutions. They even lack standard definitions.

The goals of this paper are to introduce the concepts of difficulty and complexity and to describe how they can improve our analyses of institutions. Given the widespread understanding of the notion of uncertainty and the frequent conflation of uncertainty with difficulty and complexity, I devote the first part of this paper to defining uncertainty, difficulty, and complexity both as measures of decision making environments and as analytic frameworks, or paradigms. I then address the role of institutions in helping to solve difficult problems and in reducing and channeling complexity.

I will first describe uncertainty, difficulty, and complexity as measures of problems or environments (collections of problems occurring simultaneously) and then discuss them as paradigms. Casual and journalistic usages of difficulty, uncertainty, and complexity mostly ring true despite the subtlety of the concepts. Investment decisions are often described as informed attempts to survive in an uncertain market. Engineering solutions are frequently referred to as a good approaches to difficult problems. And, successful new product lines are regularly called intelligent responses to a rapidly changing, complex world. The formal definitions reinforce these examples. Uncertainty refers to the lack of an additive objective probability distribution over outcomes (Knight 1921, Ellsberg 1987). And, when making investment decisions, we may not be able to construct a probability measure over all the possible states of the world, so we are correct in calling such a problem uncertain. The term difficulty applies to problems possessing large state spaces and nonlinear (epistatic) interactions

[1] Certainly, some institutions such as the Federal Deposit Insurance Corporation (FDIC) have uncertainty reduction as their primary purpose.

among variables (Page 1996). Few would deny that building a bridge requires solving nonlinear problems and that such problems are therefore difficult. Finally, complex environments arise from the accumulated interactions of multiple agents or particles (Gell-Mann, 1994, Anderson, Arrow, and Pines 1988). A company's new product line influences competitors' actions, which in turn influence the company's profits and set of responses. Again, the example fits the definition.[2]

Some problems are inherently uncertain, complex, or difficult: next January 8th's weather in Des Moines is uncertain; the floor of the commodities exchange in Chicago is complex; and protein folding problems are difficult. Yet, most problems do not fall neatly into one of the three categories. A problem or environment can be difficult but not complex, uncertain but not difficult, or both complex and difficult. Most problems that we think of as hard, such as deciding on an investment strategy, have degrees of uncertainty: market fundamentals depend on unknown random shocks, difficulty: minimizing risk depends on a multitude of price correlations, and complexity: future prices depend upon the actions of many other agents.

I next want to think of uncertainty, difficulty, and complexity as analytic paradigms, ways of formulating problems. When modeling a problem as uncertain, we assume the decisions makers fail to satisfy accepted axioms of rationality (Camerer and Weber 1992). Thus, they can make suboptimal choices, or at least choice that cannot be rationalized. The difficulty and complexity approaches extend our traditional analyses of decision and game theory models, the two primary modeling approaches in economics. [3] Decision problems that are too hard to solve because they have too many variables and are too nonlinear can be modeled as difficult. And models with strategic interdependence that do not settle quickly into equilibria can be modeled as complex.

The difficulty paradigm applies to decision problems, measuring how hard they are to solve optimally. The more difficult the problem, the less likely that agents locate the optimum and the greater benefit from increased search sophistication. Searching for a solution to a difficult problem can be thought of as crawling about on a rugged multi-dimensional landscape, where a location's altitude corresponds to its value. Loosely speaking, the ruggedness of a landscape corresponds to the problem's difficulty. Agents search until finding a local optimum, a peak on the landscape given their encoding and search heuristics. Final solutions may depend upon initial designs and the trajectory of search.

The complexity framework allows the modeling of dynamic environments with strategic interactions. Fixed point theorems guarantee the existence of equilibria for

[2]Lance Davis has pointed out that thinking of risk, uncertainty, and complexity as loosely forming a continuum can be helpful. If the probabilities of states of the world are known, the environment is risky. If the probabilities are not known, the environment is uncertain. Finally, if there is a sequence of states of the world and the transition function between those states cannot be uncovered by the agents, then the environment is complex.

[3]Simon Wilkie said the best advice to give graduate students in social science prior to taking their preliminary exams was "remember two things, people optimize and the world is in equilibrium."

closed systems but not for open environments, where new ideas, players, and technologies can enter the system, behavior need not settle into equilibria. Aggregate behavior may be perpetually novel, preventing individual agents from making optimal split second decisions. Agents may be forced to rely on simple rules of action (Lane and Maxfield 1995). Whereas a difficult problem can be thought of a searching for the highest point on a rugged landscape, in a complex environment, the landscape changes constantly. The actions of other agents, alter an agent's value function, her landscape. As in difficult environments, path dependence can occur. More interestingly, the potential for perpetual novelty in a complex adaptive system compromises comparisons across time, even discounted to a common monetary unit. The basket of goods available for consumption may depend upon the path of the economy (Matsuyama 1993). The sets of living agents may differ under two different paths of decisions, so even the notion of Pareto Optimality may not be well defined.

The existence of the three frameworks obliges a choice. In some cases, the most appropriate framework is obvious. Difficulty models should be used to capture the development of a cure for a disease, the search for a workable form of fusion, of the process of determining a welfare policy. Agents cannot enumerate all possible solutions. They must rely on encodings and heuristics to find good solutions. Other problems, such as the stock market, traffic, pricing by gas stations and should be modeled as complex Most problems though will have degrees of uncertainty, difficulty, and complexity, and the choice over analytic frameworks may hinge on training: economists tend to prefer uncertainty models; engineers and computer scientists construct difficult problems that approximate the real world and search for satisfactory, and, where possible optimal actions; and biologists explore complex adaptive models where the behavior of agents influences their environments, and the environment, in turn, influences the agents.

Many problems, such as automobile design, can be modeled under each of the three frameworks. Each generates a unique set of insights. First, under the uncertainty approach, a manufacturer might be considering several possible designs for a new mini-van. Key decisions may include whether to have folding or removable rear seats, sliding or hinged doors, and whether to offer an optional leather interiors. The profits from a design can be captured by a random variable that depends upon an uncertain state of the world. The source of the uncertainty may be about costs, design decisions of competitors, or other external factors, such as world oil prices, the weather, or the the whims of consumer preference. The presence of uncertainty may preclude optimal decision making.

The details of how attributes and nature influence profits under-pin the difficulty approach. They may search optimally given what they know about the problem, but they will become stuck on a local peak. As in decision theory, the difficulty approach takes the actions of competitors as fixed. In that a design consists of multiple attributes, it's value can be represented by a nonlinear function of those attributes. For example, car designers may know that consumers want safe cars offering good gas mileage. Designers may also want costs kept low. They may want a sleek aerodynamic design. These conflicting desires create nonlinear payoff functions. The potential number of designs precludes enumeration.[4]. The infeasibility of exhaustive search and the presence of nonlinearities makes choosing the optimal design unlikely. Decision makers who explore only a limited set of alternatives will locate a local peak, but they have no guarantees of global optimality.

The complexity approach recognizes that several companies design cars, manufacture parts, and compete for market share over time. A firm's decision to manufacture high mileage cars in one period influences payoffs and possibly strategy sets in future periods. Competitors' decisions, say to emphasize safety, also influence the value of the firm's own designs, and in some cases may even alter the preferences of the consumers. This ever changing decision making environment poses a problem. If a design team spends too much time collecting information on consumer preferences or perfecting a design, it runs the risk of solving a problem that existed in the past.[5]

In sum, the uncertainty, difficulty, and complex are attempts to model the flip side of bounded rationality. Empirically, we know that people rarely make mistakes on easy problems, though they do act inconsistently in face of great uncertainty. We know that individually, they are incapable of solving difficult problems optimally. And we know that tend to rely on rules of thumb when confronted with a complex environment. The study of bounded rationality, or more formally behavioral economics (see Conlisk 1996, Dawes 1988, Thayler 1995) should therefore benefit from formalization of the concepts of difficulty and complexity.

Before proceeding with the main part of the paper, several comments are in order as to the extant literature on uncertainty, difficulty, and complexity. The economics literature includes a plethora of uncertainty models (see Camerer and Weber 1992), a few that address difficulty (Tovey 1991, McCloud 1996, Gilboa 1992, Page 1996), and a growing literature on complexity (Arthur, Durlauf, Lane 1997, Durlauf 1995, Miller 1995). Political science also contains several papers dealing with complexity (Axelrod 1997). A skim through the complexity literature reveals an abundance of measures and conceptualizations. In many instances, the word complexity applies not to the environment but to the abilities of the problem solver. The extensive literature on the complexity of automata (Rubenstein 1988, Kalai and Stanford 1989, Gilboa 1993) describes the internal sophistication of agents, not the environmental complexity. The incomplete contracting literature describes environments as complex, but the term usually refers to a large state space that need not be complex in the sense I use here (Segal 1994). More often, a large state space correlates with greater difficulty, though it need not. The problem of selecting the largest integer between one and one billion has a large state space, but presents no difficulty. The lack of agreed upon definitions of difficulty and complexity leads to confusion at best, but at worst causes social

[4] An arithmetic argument shows that under reasonable assumptions, that the number of possible car designs far exceeds the number of atoms in the universe

[5] Kieron Meagher (1996) has a wonderful paper on the amount of information firms should gather given time considerations. Radner and Van Zandt (1992) also discuss information processing and returns to scale in firms.

scientists to overlook important aspects of the problems we study.

The remainder of this paper consists of five parts. The next three sections contain definitions of uncertainty, difficulty, and complexity. Each section also includes analyses of the problems of differentiated product design and welfare policy choice from the perspective of the approach being characterized. Both problems lend themselves to analysis with the three frameworks. One occurs in an economic context and the other on the boundary of economic and politics.[6] The welfare policy problem naturally admits a multidimensional model; it requires making decisions on who is eligible, who pays, and how the benefits are to be paid. Therefore, it can be usefully modeled as difficult. Policy values might also depend upon the actions of other government agents, on the actions of individuals, and on firms' policies. Therefore, welfare policy formulation occurs in a complex environment. Finally, macro–economic fluctuations remain uncertain to most of us, so welfare policy performance depends upon uncertain events as well. In the penultimate section, I discuss the role of institutions in difficult and complex environments. I describe how institutions might reduce or channel complexity and how they can help to solve difficult problems. In the discussion at the end of the paper, I provide additional examples and summarize.

2 Uncertainty

Knight (1921) advocated a distinction between *risk* (objective priors) and *uncertainty* (subjective priors that may be intervals). The probability that a flip of a fair coin lands heads up objectively equals one half. In contrast, the probability that an individual can carry an egg in his pocket for two weeks without it breaking might at best be subjectively placed in some interval, say $[0, 0.2]$.[7] This inability to assign a firm prior in the second case distinguishes uncertainty from risk. Subjective probability theory temporarily rendered this distinction unnecessary by reducing uncertainty to risk.[8] I say temporarily because the mathematical equivalence has not proven to be an accurate predictor of behavior.

The experimental literature finds significant levels of uncertainty aversion (Ellsberg 1961). People prefer a fifty–fifty proposition to either side of an unknown bet.

[6] A superficial economic spin on political versus market forces suggests that the latter are more effective: In competitive environments, the opportunity to accrue rents by improving an existing actions aligns individual incentives with social efficiency; whereas in political settings, people and organizations do not have access to all problems, so therefore, the government should prove less successful in locating good actions. Moreover, the incentives of individual agents to accrue power, maximize budgets, etc, may not naturally align with the greater goal of improving social welfare. While such arguments are not without merit, they lack realism. Government is not immune to competitive pressures, and many firms earn persistent rents (Whitman 1995).

[7] If the agent could perform the experiment many times, he could generate a probability distribution based upon statistical evidence. I am assuming the situation is novel and incapable of being repeated.

[8] An agent's preferences over outcomes implicitly define a unique set of probabilities over states of the world that can then be interpreted as the objective probabilities.

Take an urn containing one hundred balls all either blue or red. The exact number of each is unknown. A ball is randomly picked from the urn. A player gets to play two games. In the first game, he chooses between the following two bets:

Bet 1:

1. \$1.05 if a red ball is drawn. \$0 if a blue ball is drawn
2. \$1 if a flip of a fair coin lands heads up

and

Bet 2:

1. \$1.05 if a blue ball is drawn. \$0 if a red ball is drawn
2. \$1 if a flip of a fair coin lands heads up

Most people choose option 2 both times. If people satisfied the assumptions of rational decision making, this would imply that the total probability of either blue or red is less than one, a contradiction. These deviations from optimal decision making in the face of extreme uncertainty have led economists to construct models with agents who rely on subadditive probability measures.[9] I construct a skeletal version of these models below. My treatment is neither complete nor original.

2.1 Overview

In an uncertainty model, the value of an action is not known *a priori*. Let A denote the set of actions. The agent knows the set of all possible actions and the set of possible values of every action. Endowing agents with the ability to elaborate, evaluate, and compare all actions is a strong assumption. Agents may well be limited by their imagination or by computational constraints. An action's value depends upon the state of the world. Uncertainty exists when an agent does not know the true state of the world, when she does not understand how her action affects the state of the world, when the state of the world may depend upon the unknown actions of other agents, or when she does not understand the mapping from the other agents' actions to the state of the world.

In the uncertainty framework agents can elaborate, evaluate, and compare all actions. An agent facing uncertainty would never fail to recognize an obviously best choice. More precisely, if an action's value in every state exceeds the highest possible value of all other actions, then that action will be chosen. This idea can be formalized as follows:

[9] See Bewley (1986), Gilboa and Schmeidler (1994), Ghirardato (1996), and a survey paper by Camerer and Weber (1992).

Strong Dominance Preserving: *If there exists an $x \in A$ such that for all $y \in A$ with $y \neq x$, $v(x,s) > v(y,s)$, for all $s \in S$ then x is the action selected by the agent.*

Uncertainty has no bite for problems with strongly dominant actions. The dominant action is taken. The probabilities of the states of the world need not even be considered once it is realized that regardless of the state of the world one action performs best. Uncertainty matters when the value sets overlap. Consider an environment with three actions, x, y, and z and three possible states of the world (possibly actions of a competing agent) denoted by $1, 2$, and 3. Let the value for each of the three actions in each of the three state be as follows:

		State		
		1	**2**	**3**
	x	0	5	5
Action	y	5	5	0
	z	3	3	3

An agent uncertain about the probability of the three states of the world might decide to choose action z.[10] Action z offers the highest guaranteed minimum payoff and minimizes her regret. However, there does not exist a set of probabilities over the states of the world that makes z the optimal choice. Thus, uncertainty aversion may lead someone to choose an action that, although not be optimal for any probability distribution, seems safest.

2.2 A Formal Model of Uncertainty

I describe a formal model of decision making under uncertainty with a finite set of actions and a finite set of states. The method that I use here for computing utility mimics Choquet integration (Ghirardato 1996, Gilboa and Schmeidler 1994). The following definitions prove useful.

The set of *states* $S = \{1, 2, ..n\}$

The set of *actions* $A = \{1, 2, ..m\}$

The *payoff* from action a in state s, $u(a,s) \in [0,1]$

The *minimal payoff* given action a, $u^L(a) = min_s\, u(a,s)$

The agent's *distribution* over states $(p_1(a), p_2(a), \dots p_n(a))$, where $p_i(a)$ equals the probability that state i occurs given that the agent took action a.

[10] Being uncertain should not be confused with having a prior distribution over probability distributions. The latter collapses to a single probability distribution and a decision under risk.

Uncertainty models assume that the agent possesses a *subadditive* prior distribution; the sum of the p_i's is strictly less than one. Let $\sum_{i=1}^{n} p_i(a) = \theta < 1$. To capture uncertainty aversion, agents the excess probability $(1-\theta)$ accrues to the minimal payoff given the action taken. The expected payoff to action a, denoted by $E[u(a)]$ is written as follows

$$E[u(a)] = \sum_{i=1}^{n} p_i(a)u(s_i, a) \ + \ (1-\theta)u^L(a)$$

This formulation captures uncertainty aversion and provides an explanation for several empirical puzzles. We can use this formalization to explain how z could be chosen in the earlier numerical example. Let $p_i = \frac{1}{4}$ for $i = 1, 2, 3$. This implies that $(1-\theta) = \frac{1}{4}$. Straightforward calculations show that the expected utility from action z equals 3, and that the expected utility from each of the other two actions equals 2.5. This simple model helps to explain a possible, and plausible cause of uncertainty aversion. If people cannot assign full probability to all states of the world, they might assign the leftover to the worst possible state. They would then tend to make safer choices.

2.3 Uncertainty Applied to Differentiated Products

Consider an application of uncertainty to differentiated product design. A firm's action might consist of a price together with a set of product attributes. The profits resulting from a particular price, design choice pair depend upon potential buyers: their preferences, their incomes, and their expectations; characteristics of the economy: the prices of substitutes and complements, the rate of economic growth, and the rate of technological innovation; and the actions of competitors and suppliers: their product attributes and prices. The task of defining states of the world can be arduous. The decision maker may resort to simplifications. Her resulting beliefs over simplified states may no longer possess an additive probability distribution.

The uncertainty approach yields several insights to differentiated product design.[11] First, if a strongly dominant design exists, it is chosen. Second, decisions may not be consistent with any probability distribution. They may be biased in favor of more certain outcomes. Uncertainty aversion could steer agents away from novel choices. A radical new design may be left on the drawing board because of uncertainty over consumer reactions. In the example above, if x and y represent non traditional automobiles and z something similar to the status quo and if $1, 2$ and 3 represent the capricious preferences of consumers, then a risk averse firm may opt for option z. If the numbers in the table represent both profits to the firm and utility to the consumers, then either x or y is the optimal choice. In the uncertainty framework, firms need not make identical choices. Different attitudes towards uncertainty or different priors may lead to divergences in action.

[11] See Table at the end of the paper.

Third, the uncertainty approach provides an explanation for entry: the potential for suboptimal decisions opens the door for entrepreneurs who have different priors or less uncertainty aversion (Knight 1921). Return again to the three state, three action example. Consider an entrepreneur deciding whether to take an action that costs one dollar. If the entrepreneur believes state 1 to be unlikely, she may choose action x. If state 1 occurs, she loses money even though the expected value of action x exceeded that of action z. This is also true of risky environments; entrepreneurs may lose money ex post, though ex ante their expected profits given their information is positive.

Fourth, in uncertain environments, firms may benefit from hiring consultants to help resolve uncertainty. Consultants could provide better information to agents about the state of the world either by knowing the actions of others or by possessing a deeper understanding of the mapping from actions to states of the world. Under the uncertainty paradigm, consultants act solely as information providers. Finally, firms consisting of subunits or franchises may diversify their actions to reduce the uncertainty in the environment. Here multiple actors serve as a portfolio. They do not provide information about the value of other potential actions.

2.4 Uncertainty Applied to Welfare Policy

The problems of designing an automobile and of constructing a welfare policy share many features. Each consists of an enormous set of possible actions, and in each case the value of an action depends on many factors, some internal, some external. Therefore, many of the insights gathered from analyzing automobile design in an uncertain environment shall also apply to welfare policy — governments always choose a strongly dominant welfare policy if it exists, and governments tend to avoid novel policies owing to the uncertainty associated with them. The role of experts in the case of welfare policy also is similar to their role under differentiated products. Experts and consultants reduce uncertainty by providing information about the state of the world, actions of others, and the mapping from actions to states.

Important differences between policy and product environments do, however, exist. Their are no entrepreneurs per se. New governments cannot enter the market. Although in federal systems, constituent states may experiment with policies so politics does admit a form of entrepreneurialism. One role of states as laboratories would be to reduce uncertainty, to provide information about the state of the world, and to reduce the probability of making suboptimal decisions. Note though that even without uncertainty, ex post optimality may not be achieved. An expected utility maximizer confronting a risky decision can only guarantee ex ante efficiency. Yet, there is a crucial distinction. Under uncertainty, the decision can be suboptimal with respect to *any* prior distribution, leading to policy criticisms akin to “no matter what they were thinking, this was a stupid policy.” As a result, incumbent governments may be voted out of office on the basis of their policy decisions.

3 Difficulty

The term difficulty applies to self–contained problems that strain human and human–assisted information problem solvers. Difficult problems can be solved in isolation. Optimal solutions are independent of the actions of others. For example, proving a theorem can be a difficult problem as can designing an optimal keel for a sailboat or finding a solution to a discrete optimization problem with a large state space. The difficulty approach does not assume that the values of solutions cannot change with the actions of others, just that their ordinal ranking does not. A steam engine's efficiency does not vary when a breakthrough in combustion engines occurs. Of course, the value of the steam engine might plummet.

In the difficulty framework, agents know the value of each action they evaluate with certainty. The "difficulty" arises in locating and evaluating actions. The difficulty approach has its roots in computer science, where large, combinatorially exploding problems have been the subject of much study. Traveling salesperson problems and protein folding problems are difficult because the number of possible configurations precludes iterative examination.[12] In trying to solve a difficult problem, an agent may become stumped, unable to think of value–improving alternatives even though the space of actions is enormous. Professional problem solvers, from academics writing papers and constructing arguments to architects designing houses, often reach mental blocks within large state spaces. This inability to elaborate and evaluate all actions, much less to even think of a single new action, can also be attributed to search costs. Regardless of the explanation, when confronted with a difficult problem, agents consider only a subset of all possible actions.

The metaphor of adaptation on a rugged landscape, where agents crawl around on a geographic landscape searching for locations (actions) of high altitude (value), nicely characterizes the difficulty approach. The landscape does not have to remain fixed, it can shift up and down, but the ordinal rankings of actions must stay the same. Agents explore this landscape using heuristics, or rules of thumb. These heuristics may be serial—a single searcher iteratively climbing hills, or parallel—multiple searchers simultaneously roaming the space of actions. Eventually, agents become stuck on local optima. The distance from these local optima to the global optimum can be measured relative to values or to the space of actions. A heuristic may stop at a locally optimal action very near to the optimal action but with a much lower value, it may locate an action far from the global optimum with a relatively low value, or it may even locate an action far from the global optimum in action space but with a value close to the optimal value.

It is not easy to measure a problem's difficulty. Difficulty can be agent specific. It may depend upon how the agent encodes the problem and attempts to solve it (Hong

[12] Alternatively, a problem could be difficult because of lags in information: a guest at a hotel may take a long time to adjust the pressure and temperature of the shower, and, may often stick with a suboptimal combination (Sterman 1989). In this interpretation, if the number of setting is not large, then, in my characterization, the problem would not be difficult.

and Page 1997). Rather than undertake a complete treatment I shall mention just two complications. First, difficulty must be measured relative to an encoding and a heuristic. Encodings are crucial; given a heuristic any problem can be encoded so that it is easy to solve. Heuristics, or algorithms, also matter. The probability of locating the global optimum, the expected value of local optima, and the expected length of search all depend upon the heuristic employed to the encoding. The second complication, the domain over which to measure difficulty, may even be more problematic. A persuasive argument can be made that difficulty should be measured relative to upper contour sets (Page 1996). An upper contour set for a decision x consists of all decisions that have higher values. Imagine a problem that becomes easy once a solution above the median value has been located. Such a problem should not be considered difficult. Though it may be under standard measures of difficulty such as NP hard that measure worst case scenarios.[13]

The roles of encodings and search algorithms can be better understood by considering a simplified, difficult problem. According to legend, when creating a new brand of chocolate chip ice cream, Ben and Jerry (of the eponymous ice cream company) relied on a traditional encoding and a rather enjoyable heuristic. First, they constructed a two dimensional array of ice cream pints. Along one axis the chunks grew bigger, and along the other axis the number of chunks increased. The diagram below represents their encoding (the first coordinate represents the number of chunks, and the second coordinate measures the size of each chunk).

An Ice Cream Array

(# Chunks, Size Chunks)

4,1	4,2	4,3	4,4
3,1	3,2	3,3	3,4
2,1	2,2	2,3	2,4
1,1	1,2	1,3	1,4

In the lower left corner, the ice cream contains one small chunk and in the upper right, the ice cream contains four large chunks. In their problem, the array was much larger, making the task more difficult than the one presented here. To determine the optimal combination, they dug into a random pint and ate their way to a local

[13]A problem is classified as NP hard if the number of evaluations required to locate the optimal solution increase exponentially in the size of the problem.

optimum.[14] This was their heuristic. They eventually reached a local optimum, whether it was global depends upon nonlinearities between the size and number of chunks.

3.1 A Formal Difficulty Model

In a formal difficulty framework, agents rely on heuristics to search a large set of actions hoping to locate an action with a high value.

The set of *actions* $\Omega = \{1, 2, ..m\}$

The value function $V : \Omega \rightarrow \Re$

In the Hong and Page (1997) formulation, each problem solver has an internal language. The problem solver encodes the actions in this internal language. The internal language may be interpreted either at the neurological level—human brains perceive and store information, and these perceptions differ across individuals—or the metaphorical level—a person interprets problems based upon her training as an economist, a lawyer, etc.

An *internal language* for problem solver i, $,_i$

Problems solvers apply names to actions using this internal language using a *perspective*

A *perspective* $M : S(\Omega) \rightarrow ,_i$, where $S(\Omega)$ is a subset of Ω.

A heuristic, A_i is a mapping from elements in the internal language of problem solver i to sets in the internal language. Given a $\gamma \in ,_i$, $A_i(\gamma) \subseteq ,_i$ equals a set of neighboring actions in the internal representation of problem solver i. Here I consider a class of heuristics that consists of a set of functions defined on $,_i$. As a special case, let $f_{ij} : ,_i \rightarrow ,_i$, for $j = 1$ to m. A heuristic $A_i = \{f_{i1}, ..f_{im}\}$, where $A_i(\gamma) = \{f_{i1}(\gamma), ..f_{im}(\gamma)\}$.

A *heuristic* $A_i = \{f_{i1}, ..f_{im}\} : ,_i \rightarrow ,_i^m$ where $f_{ij} : ,_i \rightarrow ,_i$

An agent (M, A) induces a neighborhood structure on the set of actions. An action is locally optimal if it has the highest value in its induced neighborhood. By definition difficult problems have multiple local optima relative to an agent's perspective and heuristic; otherwise, they would not be difficult. The multiplicity of local optima makes possible sensitivity to starting points and search paths. If agents rely on distinct perspectives and heuristics, they can have different sets of local optimal: a local optimum for one agent may be improved upon by another agent, not because

[14]Initially, this problem might appear to have negative search costs, but presumably at some point the marginal return from consuming ice cream must become negative.

the other agent is smarter, but because she is different.[15]

3.2 Difficulty Applied to Differentiated Products

The differentiated products problem can be modeled within the difficulty approach rather nicely. Agents begin from a status quo solution and experiment nearby, searching for improving variants. In the case of a particular product, say, boxed macaroni and cheese, agents may vary product attributes—testing tangier cheeses or pastas of various shapes and sizes, or they may vary processes— stamping rather than cutting the pasta shapes. Evaluating each variation entails costs, but once evaluated, the market demand would be known with near certainty.

Under the difficulty approach, final actions need not be optimal, although they must be locally optimal relative to the search heuristic and perspective employed by the agent. Recall that in the uncertainty framework, a strongly dominant choice is always located. Under the difficulty approach this need not be true; dominant need not imply obvious. In that agents differ in how they solve problems, they may locate different actions. Here differences arise not from having different priors or information but from the diversity of perspectives and heuristics. No two people or firms think exactly alike. The difficulty of the problem may also influence the heuristic chosen; as the problem becomes more difficult, the benefits to more sophisticated heuristics may increase.

In difficult environments, entrepreneurs enter only when they see a better action. No uncertainty exists, so an entering entrepreneur always makes money. This contrasts with the uncertainty framework where entrepreneurs could enter and lose money depending upon the state of the world realized. Like entrepreneurs, consultants also improve upon actions in difficult environments. Unlike in the uncertainty case, where the consultants provided information, here they point the agent toward a better action, an action the agent was incapable of elaborating prior to hiring the consultant. This distinction can be clarified by reconsidering Ben and Jerry's search for the perfect pint. Suppose that Ben and Jerry have found a locally optimal pint, at (4,2). Suppose further that the consultant encodes the problem differently. She arrays the pints according to caloric rank. A portion of her encoding surrounding the local optimum is shown below:

3,2	2,3	**4,2**	3,3	2,4	4,3

If the consultant's heuristic calls for searching the two adjacent pints in each direction given her ordering, then starting at the pint labeled (4,2) and search all of the pints between (3,2) and (2,4). Suppose that the pint labelled (2,4), containing

[15] Page and Hong (1997b) show how diversity in perspectives and heuristics can be beneficial in the absence of communication problems.

slightly more calories and much larger, though fewer chunks than (4,2) tastes better than (4,2). The consultant, by relying on a different encoding—linear rank by calories, locates a better pint. In order to find this improvement, *the consultant need not be smarter*. She need only interpret the problem differently or apply a different heuristic. Ben and Jerry may well be better able to solve the problem then the consultant, their expected value from search might be higher than hers. Yet, this does not prevent the consultant being able to improve upon their action. She need only be different.

Finally, firms with subunits that face a difficult problem, for example, designing a new chicken sandwich or pizza, can use those subunits or franchises as laboratories to search the space of actions. The franchises can perform parallel search. As these separate searches proceed, information from one search might prove valuable to another. For example, if the local optima discovered by the individual searches reveal a pattern, then the location of the global optima might be attainable through introspection. In addition to providing information about the distribution of values, as occurs in search from a random distribution, here the searches provide information about the structure of the underlying problem.

3.3 Difficulty Applied to Welfare Policy

The difficulty approach applied to welfare policy can explain how decisions may be far from optimal both in value and in its attributes. The difficulty approach suggests a need for policy advisors who have different perspectives on the problem. These policy advisors supply ideas for better policies but not information that clarifies the value of known policies. Policy advisors can suggest alternative encodings, new ways to see old problems. They can advocate new search heuristics. They can propose new actions that they have arrived at by applying their human capital to an existing problem. Or they can reveal information about the underlying problem as a result of studying the time path of policies and their values.

If the government employs states as policy laboratories, then these states may provide information about the values of policies. Further, because the states have different institutional arrangements and leaderships, the approaches they use to solve the problem may, and probably would, differ. Moreover, assuming that states find local optima, the attributes of these local optima may themselves be important. Kollman, Miller, and Page (1997) describe the benefits from searching between the local optima of the states. The local optima contain information about the values of other policies. This information can be exploited to find even better policies. States then should not mimic the winner, so much as they should learn from the winner how to do even better.

4 Complexity

In a complex environment the value of a decision depends upon choices made by other agents, upon nature, or upon both (Lane and Maxfield 1994). Complexity differs from difficulty in that the ordinal ranking of an action's values can changes base upon actions of others. Returning to the landscape metaphor, in a complex environment the actions of others can cause the landscape to undulate. A complex environment also differs from an uncertain environment. In the latter, the multiple outcomes associated with an action are just assumed. In the former, the multiplicity results from interdependence of actions. In addition, actions occur in real time. At each instant, agents can take actions that in turn may influence the set of available future actions, their payoffs, and the actions of other agents.[16] As stated in the introduction, a complex environment can be thought of as a series of uncertain problems where levels of uncertainty in previous periods may depend on current actions. Because agents respond in real time, the sophistication, or internal complexity, of a search algorithm now has attendant costs in delay. Early responses based on simple strategies may exploit opportunities while more sophisticated heuristics are still processing information. A complex environment exhibits perpetual novelty, decentralized interactions, continual adaption, and lack global control (Holland and Miller 1988, Arthur, Durlauf, Lane 1997).

The children's game of tag is a good example of a complex environment. The value of a particular action: stand and rest, run, trot, be alert, depends upon the actions of the other players. In tag, players rely on simple actions, and yet the resulting dynamic patterns of behavior may be complex enough to make optimal responses unlikely. Apparently optimal actions, such as a quick evasive movement may avoid an immediate tag, but feed back into the system resulting in a path of the game where the agent is 'it' for a long time.[17]

A complex environment consists of a set of agents each with both exogenous and endogenous characteristics, an internal state, and a set of actions. The set of actions may be a function of characteristics and the internal state of the agent. The aggregate actions of the agents form a configuration at each point in time. Agents purposively respond to the configuration, thereby creating a new configuration. Agents respond

[16] Not making a decision and as a result retaining the status quo can be interpreted as choosing the same action as in the previous period. In light of the interdependencies, timing of action updating plays an important role. Unfortunately, the order of action updating influences outcomes in subtle ways. Huberman and Glance (1993) have shown that asynchronous updating leads to distinct dynamics and distributions over steady states from synchronous updating. Under synchronous updating agents change actions lock-step in military fashion. Huberman and Glance, along with Bersini and Detours (1994) find that much of the interesting behavior found in cellular automata models are artifacts of synchronous updating. Page (1995) demonstrates that when the sequencing of updating is asynchronous and incentive based - those with the greatest benefit from updating update first - entirely different dynamics occur.

[17] Complexity need not only be the aggregate of simple behaviors and rules. Competing firms may rely on elaborate strategies implemented by sophisticated heuristics.

to both global and local features of the configuration (Kirman 1997). Agents cannot necessarily respond optimally to their environment given its complexity and the speed at which events occur. Instead, they adapt rules of behavior—possibly derived from internal models (Tesfatsion 1997). The resulting patterns of behavior need not attain an equilibrium. They may create a perpetually novel sequence of configurations.

Complex environments often exhibit clustered volatility in both the actions and in the internal models employed by agents (Arthur, Durlauf, Lane 1997). The clustered volatility arises from correlated shifts in landscapes. Suppose that several agents simultaneously alter their behavior. Their actions may shift the landscapes of other agents, creating opportunities for improvement. As these other agents search for better actions, they in turn shift the landscape of still more agents. A clustering of action changes occurs. Such dynamics would be mathematically unlikely to result from mixed strategies. In a mixed strategy, the probability that one player alters her strategy is independent of the actions of other players.

The complexity approach complements the prevailing equilibrium paradigm (Judd 1994). Mathematical models in the social sciences typically characterize situations where interacting forces balance – worlds settle into equilibria. In the complexity approach, the purposive behavior of agents leads to either patterns (which could be interpreted as equilibrium cycles) or perpetual novelty. The latter cases are called complex. A blurry line distinguishes systems tending towards equilibrium and perpetually novel systems. In finite systems—systems with a finite number of configurations—any deterministic transition rule results in a cycle of some, possibly enormous, length. This cycle could be construed as a dynamic equilibrium. Yet, a billion period long cycle might be more accurately described as perpetually novel, for agents would have neither the ability or any incentive to calculate and predict the entire cycle. In economies with noisy, or random, transition rules, Markov theory or random field theory often applies, and again, a hard line equilibrium theorist could consider the limiting distribution of configurations to be an equilibrium distribution. The support of this equilibrium distribution may contain billions of configurations. If so, the equilibrium distribution need not be especially informative.

Assumptions about agent level ability distinguish complexity models of strategic interdependence from game theoretic models. In complexity models, agents do not possess the informational or computational abilities to learn or predict the equilibrium cycle. This begs not one, but two questions: First, under even mild rates of learning would the system not still eventually settle down into equilibrium? and second, does this not preclude any predictions from complexity theory, as any predicted pattern could be exploited, and therefore no longer exist? The second question can be dealt with more quickly, so I address it first. Complexity theory might predict clustered volatility or convergence to a region with unpredictability within that region. Case in point, Kollman, Miller and Page (1993) find that competing political parties tend to converge to a region near the median of voters preferences. Within that region, they find an elaborate dance of the parties. Here complexity theory yields a prediction (a qualitatively accurate one I might add), yet its utterance in no way implies its irrelevance. The same can be said about some, but not all, predictions of volatility, cascades, or cycles.

The first question has been and will continue to be the subject of much debate in social sciences. I will merely dip my toe into the water here. First, if centuries pass before attaining the equilibrium, then perhaps the path deserves more attention than the destination. The destination may be meaningful in predicting the direction of the path, but it also may not be. Second, the concept of equilibrium applies for closed systems but it need not apply for open systems. If new agents, information, products, and ideas enter the system according to some flow, then the system need not settle into an equilibrium at all. Ironically, one of the arguments often made for equilibrium attainment, learning, may in fact cause the system to be open. Nachbar (1997), in a recent paper on learning in repeated games, mathematically describes the tendency for strategic interactions to become complex. He shows that under a technical condition on beliefs (*neutrality*) that each player chooses "a strategy that his opponent was certain would not be played."[18] This result says that at some point the best response strategy may not belong to the initial set of possible strategies. So, a rational agent would add new strategies, but this could continue forever, and the strategy set would explode. In time, the neutrality condition may be shown to be too strong. However, the intuition is powerful: the process of learning often results in the use of new strategies. New strategies represent one route to perpetual novelty.

To better see Nachbar's insight, imagine two agents playing the game of matching pennies who continually develop more and more sophisticated strategies. Matching pennies should be thought of as an extreme simplification of the real world. Think of competition between Burger King and McDonald's for market share as the real world counterpart to matching pennies. McDonald's wants to keep differentiating itself, and Burger King want to mimic the industry leader. In matching pennies, the action distribution may converge to an equal number of tails and heads, but the updating and refining of strategies may continue unabated. Equilibrium theory may be helpful in predicting final distributions, but it might tell us little about the periods of clustered volatility, the lengths of winning streaks, and the computational costs expended in attempting to gain an edge.

I should comment briefly on another open system that creates complexity. In a recent book *How Nature Works*, Per Bak (Bak 1996) discusses the concept of self organized criticality (SOC). The canonical example of a critical self organization is a sand pile. Sand dropped onto a table from a single source forms a conical pile. As sand drops, avalanches occur. The size of the avalanches satisfy a power law distribution, leading scientists to refer to the sand pile as *critical*. Bak argues that critical states can form though a process of self organization. He mentions as phenomena that demonstrate self organized criticality earthquakes, traffic jams, and stock market crashes.[19] SOC and complexity are not the same thing. However, a subsystem that

[18] The point of his paper is to demonstrate a shortcoming of Kalai and Lehrer's (1993) learning model, not to demonstrate the trend to complexity.

[19] Bak's argument, though correct for physical systems, may overstate the amount of complexity in

organizes itself into a critical state may cause a larger system to be complex. Huge avalanches in the subsystem may echo across the larger system, maintaining novelty.

4.1 A Formal Model of a Complex Environment

I next describe a model of a complex environment. I do not intend for it to represent the general case. Instead, it should be viewed as a class of examples. I provide one interpretation of the model below, but the reader can easily imagine more.

The set of *agents* $M = \{1, 2, ..m\}$

The set of *locations* $L = \{1, 2, ..n\}$

The *information at location L*, $I_\ell \in R^p$

The set of *actions* for agent i, $A_i = \{1, 2, ..k_i\}$

The *internal states* of agent i, $S_i = \{1, 2, ..r_i\}$

A *configuration* $C = \{(l_1, a_1, s_1), (l_2, a_2, s_2), ..(l_m, a_m, s_m)\}$

The *time periods* $T = \{1, 2, 3, \ldots\}$

If all agents have the same set of actions and internal states then at each moment in time the set of possible configurations has cardinality $(rnk)^m$. For even moderately sized r, m, k and n, this number is unsuitably large for elaboration. And we have not even considered the strategies of the agents or the information they possess.

This general framework could be used to model many environments. I will describe a model of gasoline buying and selling. Agents can either be sellers or buyers. The buyers have three possible actions at a location: ignore, look at the price, or purchase. A buyer's information might include gas reserves and knowledge of some prices. A seller's actions consists of a price. The internal states of a seller might include thresholds for profits or sales. The seller's local information might include the cost of gas (a random variable), last week's sales, and the number of buyers who drove by and looked at the gas price. If gathering this information has attendant costs, then this system is more likely to be complex than to result in an equilibrium. Why? Because there is a tension between the number and extent of the buyers' looking and sellers' setting prices. If buyers check many prices before buying, then prices

socio-economic systems. It errs in equating human behavior with that of particles. Though humans may be bounded, we do not all follow the same rules. Self organized criticality may be less likely to occur with idiosyncratic behavior on the part of the agents. More importantly,if an environment would be susceptible to self organized criticality *and* if self organized criticality is bad, then human actors would intervene to prevent critical states from forming.

fall toward marginal costs. Once the price has fallen, buyers have no incentive to look. Once buyers cease comparing prices, then the sellers have incentives to raise them. Once a few agents start to look, it makes sense for some firms to lower their prices. The expected result of this sort of model would be unpredictable patterns of prices, reserves, search strategies, and pricing strategies. In other words, complexity.

Alternatively, a game theoretic analysis would posit a stochastic equilibrium as the outcome of this environment. Agents would play mixed strategies generating an equilibrium. While this would also create fluctuations, the model would not explicitly include learning nor would there be clustered volatility of price changes. Agents would play the same mixed strategy each period. A richer stochastic equilibrium model might allow for agents to update their beliefs and strategies over time. This would create an interesting dynamic world, but at each moment in time, agents would be making optimal actions given the actions and strategies of the other agents. In a complex model, no such restriction is placed upon the actions of agents. The agents may just be following rules of thumb that they decide to abandon once the rules no longer perform well. The distinction boils down to whether agents optimize, or whether they rely on rules of thumb. Failure to optimize need not result in opportunities to exploit rents. Rent appropriation may require knowledge of the strategies of others.

4.2 Complexity Applied to Differentiated Products

The complexity approach applied to the differentiated products problem focuses on the interplay between firms' strategies. Rather than attempting to locate and optimal design holding the other firms as having fixed designs, a firm makes choices knowing that its design influences future choices by competitors. If one firm drastically changes its action, it alters the adaptive environments of its competitors. This in turn leads to further adaptations.

The question of entry can also be addressed in the complexity framework. A firm may enter if it sees an opportunity for profit, taking into account the likely responses of competitors. However, depending on the amount of complexity, an entering firm may or may not be able to anticipate these reactions. Remember that in a complex environment, future states of the world may be, and typically are, unpredictable. Any one one of three scenarios is likely to occur. Either an entering firm makes money initially, though loses later as other firms make adaptive responses. Of, a potential entrant may notice a profit opportunity but choose not to enter based upon its expectations of the adaptive responses of existing firms. Or finally, a firm may enter and initially lose money, but anticipate earning profits following the adaptive responses of the firms in the market.

Consultants can play several roles in complex environments. They can provide information about the actions of others, strategies of others, and likely responses of others. They can help firms construct new strategies. Taking the strategies (not the actions) of the other players as fixed, constructing an optimal strategy may be a difficult problem. Consultants might help firms by providing diverse approaches to solving problems in the form of new perspectives and heuristics.

In complex environments, franchise firms can treat subunits as separate players. They can test strategies and learn how the system responds. Learning from a successful adaptation has pitfalls in a complex environment. The local niche of the subunit may not share many features with the niches of the other subunits. Also, the fact that a strategy works better for one player does not imply that it will work for multiple players.

4.3 Complexity Applied to Welfare Policy

Using the complexity framework, the problem of deciding upon a welfare policy can be interpreted in a wider context, consisting of individuals, firms, and organizations adapting in response to the behaviors of others. A change in welfare policy alters agents' payoffs. Agents may adaptively respond – for example fathers may move into or out of the household – in turn may change the incentives for other agents in the economy. The resulting economy–wide effects may in turn require further changes in the welfare policies. This feedback from policy, to actions, back to policy may be sufficiently intricate to force reliance on rules of thumb. Agents need not be repeatedly fooled. They learn. But, unlike in a rational expectations model, or a game theoretic model, the agents continue to mistakes forever. This occurs because of the potential for perpetual novelty. In new circumstances, old heuristics may be inadequate.

The complexity approach assumes that welfare policy decisions interact with other layers and divisions of government. The effectiveness of welfare policy depends upon the health of the macro-economy that in turn depends, at least partially, on actions of the government such as the money supply, military spending, the size of the deficit, etc. In addition, less macro–significant policies like educational spending may influence the successes and failures of a welfare policy. But with welfare policy, defense spending, and regulation of the money supply, the interactions may be improved if welfare policy and money supply decisions are made with an awareness of other government policies.

A complexity model of government policy may exhibit clustered volatility in policy changes. If the government drastically changes its policy on education, it alters the environments for monetary policy, defense and welfare policy. Whether or not the environmental shift results in policy changes depends partially upon the government's incentive structure. Nevertheless, a policy change in one area creates the possibility of better policies in another area as a result of the landscape shifts. One role of experts would be to point out these interdependencies between layers and levels of government and the importance of shifting policies in one domain in response to policies in another.

Adapted policies need not function harmoniously; they may conflict. Currently, the United States government pursues a monetary policy designed to limit inflation. This policy puts breaks on the economy and some would argue guarantees moderate levels of unemployment. At the same time, the government pursues a welfare policy that calls for no pay for no work. While not entirely inconsistent, at a minimum such policies create tensions. Ideally, government subunits would cooperatively coadapt policies. A second role of experts would be to point out the potentials for greater cooperation.

As we have already mentioned, in a federal system, the central government can use states as policy laboratories. Although in complex environments, the interdependence of actions mitigates benefits of experimentation. A particular welfare reform may be wildly successful in Wisconsin, but if Wisconsin's portfolio of governmental policies and private support networks differ substantially from Texas', the reform may fare poorly in Texas. Worse yet, a policy may be successful in one state by imposing negative externalities on other states (Bednar 1998). Policy mimicry could then lead to dissolution or at least the threat of it. These arguments should not be interpreted as saying that experimentation cannot beget useful knowledge in complex environments, but instead, that the extent of learning is limited and that imitation of the successful should be undertaken with forethought. In game theoretic models with replication dynamics, all agents play the same game, so no such dangers exist.

5 Institutions, Difficulty, and Complexity

I now turn to the role of institutions, the original purpose of this inquiry. Historically, there has been a distinction in the literature as to what constitutes an institution and what is an organization. The former is a set of rules, either formal or informal, that govern behavior and the latter is coordinated activity taking place within these rules (Menard 1995, North 1991). I rely on this distinction for reasons of clarity. When I speak of an organization, I mean a firm or a bureaucracy, and when I refer to institutions I mean either codified rules or equilibrium actions.[20] I begin by elaborating briefly on how organizations coordinate activities to solve difficult problems and on how they help people to cope with complexity. I then discuss institutions. Building from North's (1990) thesis that institutions reduce uncertainty by providing a stable structure for human interaction, I argue that institutions can reduce complexity and that they also can channel complexity from one decision making domain to another more capable domain.

5.1 Organizations

Organizations coordinate activities. This coordination reduces transactions costs (Coase 1952, Williamson 1975). Transactions costs may be informational (Alchian and Demsetz 1972, Milgrom and Roberts 1991), incentival[21](Spence and Zeckhaser

[20]Davis and North 1971 distinguish among institutional environments and institutional arrangements. Here, by institution, I mean something close to their institutional environment.

[21]The costs of aligning the incentives of the agent with the goals of the organization.

1971, Reiter 1986, Hurwicz 1972), or related to the uncertainty of interactions—their timing, their outcomes, and the set of allowable actions.

Organizations also facilitate difficult problem solving by coordinating activities and creating a common value function. This idea has long existed in the literature though under different terminology (March and Olsen 1984, Williamson 1991). Building the first atomic bombs required solving difficult problems in both physics and engineering, as did putting people (men so far) on the moon. These immensely difficult problems probably would not have been solved so quickly had not organizations been formed with the solutions to these problems as their primary purposes. We attack the difficult problems of the day (developing a workable form of fusion, curing cancer, and cleaning up the environment) with organizations. Sometimes we rely on private organizations to solve our difficult problems, sometimes public organizations, and sometimes hybrids between public and private. Presumably, a belief that organizations often outperform uncoordinated market arrangements in solving difficult problems motivates these endeavors.

Organizations can also assist agents in dealing with complex environments. The stock market, undoubtedly a complex environment, overwhelms most individuals. Organizations can both simplify the system for individual agents and arm them with better tools. To perform the first task, organizations exploit informational returns to scale. They identify trends and spot arbitrage opportunities by closely monitoring the entire market. Even simple heuristics such as sell if the price goes above x, require real time price awareness, until recently a practice far too costly for the individual investor.

Some problems such as purchasing a house are both complex and difficult, and organizations help consumers with both aspects of the problem. First, consider just one difficult aspect of buying a house: calculating its market price. The information needed to answer these questions includes square footage, number of bedrooms, age of the roof, etc. Access to multiple listing services and knowledge of neighborhoods and prices enables real estate agents to steer purchasers towards appropriate homes. Home buying also has inherent complexities. The set of houses currently for sale depends upon the number of buyers relative to the number of sellers. A home's price depends upon its neighborhood, the length of time the house has been on the market, the number of potential buyers, and on the patience of the seller. These all depend on the actions of others. If in addition, a potential buyer must sell her existing home, then a domino effect, where one sale begets another, can occur. Real estate agents, through amassing information and recognizing pricing patterns have an incentive to create sales. Therefore, real estate agents may facilitate small groups of sales that otherwise may not have occurred.[22]

[22]Real estate agents may also create sales sales cascades by eroding neighborhood values. Herein lies a danger. Organizations may also manipulate a complex environment to their own ends. The task of preventing this from occurring falls on institutions. Of course, this raises the possibility of an infinite regress. Who protects people from the institutions?

5.2 Institutions

Institutions, the informal and formal rules that govern behavior, organize interactions, so that people, with our limited ability to gather and process information, can thrive. Undoubtedly, institutions help to reduce uncertainty. Here I want to focus on other roles: institutions as creator, channeler, and reducer of complexity and institutions as preserver and generator of the diversity necessary for solving difficult problems.[23] To see how institutions perform these tasks, let's consider first a specific institution, American political parties. They reduce voter uncertainty about potential candidates. According to Aldrich (1995), "the reason to enter a party is to win more, and here that means reducing uncertainty over future outcomes." (pg 35). In that parties have incentives to differentiate themselves, they also guarantee a diversity of thought, something crucial in solving difficult problems. The differences between Democrats and Republicans may allow them to collectively find better solutions than they could independently even if they find reaching consensus frustrating.[24] Political parties also reduce complexity. In plurality rule elections, the presence of more than two credible candidates drives a wedge between sincere and strategic voting. Parties, by keeping the number of candidates at two, simplify both the strategic environment for candidates and the decision making environment for voters.

Other prominent institutions also reduce complexity and make problems less difficult. Consider money. Money makes some problems, such as saving, lending and borrowing less difficult. Money also makes the economy less complex. Bilateral exchanges require less sophisticated strategies then multilateral exchanges. Although, money also creates new complexities by spawning a plethora of financial instruments. Similarly, the stock market reduces the complexity of raising funds but also spawns a giant financial industry that generates its own complexities. On balance, these two institutions probably rid us of far more complexity than they create.

Simple examples best demonstrate the interplay between institutions and the difficulty of problems people face and the complexity of interactions. For example, deciding what type and level of human capital to acquire is both difficult and complex. It is difficult because of the enormous choice set and the externalities between skills. It is complex, because the payoffs to a particular skill set depend upon the skill acquisitions of others. Let's concentrate only the difficult aspects. Schools develop curricula that offer collections of skills, taught in a sequence that leads to increased comprehension alleviating parents of the need to micro–manage their children's education. The institution solves what would be a difficult problem.

Institutions can also structure incentives so that people find better solutions to difficult problems. Congressional committees can reduce informational costs (Krehbiel

[23]By channeling complexity, I mean simply that an institution can influence the location of the interdependencies. This contrasts with the case where institutions create (resp. reduce) complexity by making the decision making environment more (less) interdependent.

[24]Organizational theorists generally accept that diverse groups locate better solutions but feel less satisfied. The former result agrees with the model developed in the section of this paper describing difficulty.

1992), but they also bring people together with different beliefs, perspectives and heuristics to work on common problems. Sometimes, compromises among people of differing views result in good solutions to difficult problems.[25]

To describe how institutions can reduce and channel complexity, I consider some examples dealing with traffic. A friend who recently returned from Lima, Peru remarked on the total absence, or at least disobeyance, of traffic laws. Drivers' respond to this complex environment through informal institutions, honking before entering an intersection, waving to signal priority. These emergent institutions generate inefficient outcomes and enormous complexity. More elaborate institutions such as traffic lights would reduce complexity and increase efficiency. Yet, greater institutional sophistication does not always imply less complexity. The opposite may also occur. Imagine allocating access to intersections through a market mechanism. Vehicles could be equipped with bidding devices loaded onto their front bumpers. The winner of the intersection, the highest bidder, could be flashed a "go" signal on her dashboard while all others could be told to stop. Such an institutional arrangement might lead to a more efficient allocation of intersections, but it would definitely create a more complexity. Agents would be burdened with the tasks of formulating and improving bidding strategies based upon the strategies of others.

Some traffic institutions channel complexity from one location to another. Car pool lanes exist to encourage ride sharing and increase traffic flows. Although they reduce the complexity of the traffic patterns, they increase the complexity of people's decisions of departure times and passengers. Now people must coordinate with others. This increase in complexity becomes more obvious in the extreme case. If a law required all vehicles to contain at least three passengers, then even California freeways would be mostly vacant. Choosing a route to work would be easy. However, locating passengers and settling on departure times would not be. Millions of people would be searching for passengers and rides.

5.3 Institutional Design

As mentioned above, the predominant paradigm in formal institutional analysis is game theory. Whether studying congressional committees (Krehbiel 1992), firms (Milgrom and Roberts 1991), governments (Austen-Smith and Riker 1990), or auctions (Myerson 1991), most models compute and compare the institutionally generated equilibria. The discussion above hints at two complementary research agendas also worth pursuing: (1) analyzing the ability of organizations to structure interactions so that better solutions can be found to difficult problems and (2) comparing institutions by how they channel, reduce or create complexity and by the difficulty of the problems that they create for people. Both of these agendas would complement the predominant paradigm of comparing equilibria, and not surprisingly, both are well underway. A growing organizational design literature explores problem solving

[25] Jenna Bednar has pointed out to me that the compromise among large and small states that led to two houses of government is a good example of this phenomenon.

by bounded agents. In a series of papers, Radner (1993) and Radner and VanZandt (1992) have explored optimal information processing.[26] In these models the task and the skills of the agents are exogenous, and the authors derive optimal organizational forms.

At some level, these research agendas do not represent a significant departure from early mechanism design models. Mechanism design analyzes how to implement social choice functions and correspondences, and from the outset, the literature has paid attention to the tasks confronting agents and how the agents might attain equilibria (Hurwicz 1972). Early mechanism design papers talked about adjustment processes with agents following simple rules. The agents did not instantaneously compute equilibrium strategies. They were boundedly rational. Initially, mechanism design also indirectly addressed difficulty by including minimizing the dimensionality of the message space needed to implement a particular outcomes as a goal (Reiter 1986). Unfortunately, later work found that message space dimension varies directly with difficulty.[27] Social scientists are now attempting to measure difficulty by other means (Whitman and Friedman 1996 and Page 1996). Complexity, as I discuss next, has entered discussions only tangentially.

With the rise in simulation models, complexity considerations will become more prominent. Even when mathematical proofs exist that show that learning leads to an equilibrium, computational experiments enable us to analyze the the complexity of the process leading to that equilibrium, which is often more interesting. Construction institutions so that learning is easier, that the environment is less complex is not a new concern. In the mechanism design literature, implementation in strongly dominant strategies is always preferred to Nash implementation. Why? Because in the former, ordinal rankings of actions are independent of the actions of others: *the environment is not complex*. In Nash implementation, the environment can be complex and attaining an equilibrium may take longer. Complexity theory may enable us to refine game theoretic concepts further, to say one game creates a less complex environment than another, rather than to merely distinguish between complex and not complex. Jackson (1993) partially address this point in his paper on implementation in undominated strategies. He argues against mechanisms that rely on "name the largest integer" games to get rid of equilibria. Agents could get caught in the subgame, never getting to the equilibrium. Richards (1997) more directly discusses the complexity of games. She suggests that more equilibria, hence smaller basins of attraction, should lead to slower convergence. She focuses on coordination games, but her mode of analysis seems extendable.

Ideally, the complexity framework would adopt wholesale the formalism of the mechanism design approach. This is unlikely because the former focuses on implementing a particular social choice rule and not on comparing time paths of the interaction. Instead of computing equilibria, agents are constantly adapting to their

[26]See also the aforementioned papers by Meagher (1996) and McCloud (1996).

[27]Mount and Reiter (1990) in researching the computational requirements of locating equilibria discovered a tradeoff between message space size and difficulty.

environment. Given agents' time constraints and limited computational abilities, the relevant question becomes how to construct institutions so that agents can rely on behavioral rules and still perform well. For example, in current policy debates about school voucher programs, complexity considerations often enter implicitly. School choice may lead to better sorting and more efficient equilibria (it also may not), but it creates more initial complexity as students sort into schools. Parents care about their children's' classmates. So payoffs are interdependent. Multiple equilibria probably exist. Equilibrium analysis can only compare the final outcomes. Complexity theory offers the possibility of describing the length and costs of the paths to those outcomes. The path, its length, and the volatility along it have real economic significance.

6 Discussion

In this paper, I have considered uncertainty, difficulty, and complexity both as measures of environments and as analytic paradigms. Considering specific examples helps us to understand the distinctions between these three measures of environments. Participating in multinational arms control negotiations, choosing a route to work in a busy city, and making pricing decisions for a grocery store with many competitors are complex environments. They should be modeled using the complexity framework. In these dynamic problems, the actions of agents are interdependent. The state space sizes, the frequency of decisions, and the rate the environmental parameters change make attaining an equilibrium unlikely. In contrast, developing good tasting, fat free ice cream, deciding how to route a company's twelve thousand salespeople, and constructing a computer automated inventory control system are difficult problems and should be modeled as such. Each problem can be solved in isolation and is sufficiently hard that solutions would likely not be optimal. Moreover, any individual, firm, or organizational decision would depend upon perspectives and encodings and probably would be path dependent. Finally, a decision among technologies, choosing the date and time for a ski tournament, and selecting from among a set of potential employees to hire are uncertain problems. They should be modeled as uncertain. Here, the states of the world determining payoffs are not likely to be known, nor are firm priors a reasonable assumption. Agents may avoid uncertainty and choose actions that are not consistent with additive priors.

Though I have considered two specific problems, welfare policy and differentiated product design, many other problems and classes of problems can be interpreted within the complexity, uncertainty, and difficulty frameworks. Comparing across environments often clarifies insights. For example, complete information games may be either difficult or complex. Locating an equilibrium can be considered a difficult problem because of the rationality assumption—an equilibrium or the set of equilibria can be computed by an algorithm. Computing the equilibrium may be difficult or easy depending on the number of players, strategies, and payoffs. Other games such as chess and Go cannot be solved explicitly. They can be considered complex because they are dynamic and the actions of agents are interdependent.

Some situations, such as playing a game of poker against shady characters, may be uncertain, difficult, and complex. A player faces uncertainty owing to subjective priors over the honesty and strategies of her opponents. Within the context of individual hands, the player confronts difficult combinatoric problems. And finally, in that the actions of others change the player's payoffs and that she learns about the other players' strategies over time, the environment is complex. Applying the three frameworks to this problems leads to three distinct intuitions. An uncertainty approach might predict that she would play a safe strategy, avoiding testing the honesty of her opponents. A difficulty model might predict that her ability to choose the optimal action varies directly with the difficulty of the combinatorial task presented to her within each game. In addition, her mistakes may be systematically related to her encodings and heuristics. Owing to a bias, she may, for example, never try for a straight or a flush, even when it's the optimal strategy. A complexity model might say that she adapts a strategy, say of bluffing or raising, over time that fills a niche relative to the strategies of her opponents. Each framework makes a distinct contribution to our understanding.

Eventually, the concepts of difficulty and complexity may become as common to our analyses as are the notions of optimality and equilibria. When analyzing behavior in institutions, we will not only ask "what is the optimal strategy for an agent?" but also we will wonder "how difficult is it for the agent to formulate this strategy?" and "how can agents be given incentives to collectively accumulate the skills necessary to make an optimal decision?" When analyzing the outcomes of institutions, we will not just compute and compare equilibria. We will also calculate the complexity of the dynamics leading to the equilibria, if it even exists. The resulting analysis will provide deeper insights and, hopefully, better predictions.

Differentiated Products

Framework	Features	Heterogeneity	Entrants	Consultants	Franchises
Uncertainty	Recognize dominant, avoid novel	Hetero. priors	Different priors can lose $	Provide info re states	Provide info re states
Difficulty	locally optimal action	Hetero perspective heuristics	Perspectives heuristics differ make $	Provide new perspectives heuristics	Allow parallel search
Complexity	Periods of clustered volatility	Caused by path dependency	Initially make $ losses later	Formulate dynamic strategies	Limited use due to novelty

Welfare Policy

Framework	Features	Policy Experts	States as Labs
Uncertainty	Choose dominant policy Avoid uncertain policies	Provide infor about world and probabilities	Enables Gov't to diversify its risk
Difficulty	Locally optimal Path dependent Heuristic matters	Different not smarter Give heuristics perspectives	Parallel search local optima informative
Complexity	Opportunities for improvements clustered	Reveal policy interdependencies promote cooperation	Require states to take similar policies on other issues

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