Game Theory, Complexity, and Simplicity Part II: Problems and Applications

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In Part I an expository overview of the basic concepts of the theory of games was presented. The different forms for representing a game were noted and the various solutions were discussed and illustrated. Here we examine the highly different approaches adopted by those who regard themselves as game theorists and by those others who utilize game theory.

We examine the four most important distinctions in the many approaches to the uses and development of the theory of games. We consider:

1. Game theory as mathematics;
2. Game theory as science; applied science and social engineering,
3. Game theory as philosophy;
4. Game theory as prescience and as advocacy.

**Game theory as mathematics**

Mathematics may be the handmaiden of both science and philosophy when utilized in conjunction with these activities. But it stands alone as an art form which can (but need not) be pursued with no immediate application in mind. The primitives accepted or the basic axioms postulated may often be based upon some individual’s model of some aspect of activities and institutions in the surrounding world. However, the mathematical development of the basic models postulated need not reflect a scientific concern with any particular set of observable phenomena. The mathematically oriented game theorist may devote his or her time and abilities to the development of analytical or computational tools to solve game theoretic problems *in abstracto* caring little about empirical relevance or even metaphor between the results obtained and the relevance of the results to a scientific study.

(1) **Preference and utility theory**

Several subfields have developed along with the analysis of games of strategy. In particular the study of preferences and utility is a virtually independent subject. Daniel Bernouilli (1738, 1954) first suggested that an individual’s subjective valuation of wealth increases at a diminishing rate. Over two hundred years later von Neumann and Morgenstern (1944) provided the precise axioms establishing the existence of a utility function given an individual with a complete preference ordering over a set of riskless options who is permitted to gamble over “lottery...
tickets” or risky combinations involving the options. Shapley (1975) produced a different set of axioms for measurable utility based on the comparison of intensity of preferences. A comprehensive survey by Fishburn (1994) provides an overview of utility theory including some of the debate on the psychological basis for different approaches. However the point stressed here is that the seminal work of von Neumann and Morgenstern heralded the widespread utilization of formal axiomatic models\(^1\) in the social sciences and led to the growth of a large formal literature on the study of a variety of preference structures.

Among the preference structures which have been investigated are fully ordered preferences (any order preserving transformation leaves them uninfluenced); preferences represented by a utility function (where only linear transformations do not influence the valuation); preferences anchored to an absolute zero; preferences which are partially ordered (this means that there are some pairs of outcomes which cannot be ranked with each other); lexicographic preferences where the valuation is made in several dimensions; for example speed of typing may be the most important desired talent for a secretary, but if there are two secretaries with equal talent, one goes to the next dimension, say his ability to take dictation. This secondary attribute is used as the tie breaker.

The formal definition of a game requires a way to describe “payoffs” or the worth of the outcomes of the game. Preference and utility theory provide the needed valuation. An enormous simplification of the mathematics of all cooperative solutions theories can be had by assuming that not only are preferences representable by a utility function defined up to a linear transformation, but that there exists a special commodity which we could call “manna” or “soma” or “transferable utility” or “ideal gold” or “money” such that the preferences of all players are linear for this commodity. If such a commodity exists in sufficient quantity and is appropriately distributed among all agents, then it can serve as a side payment mechanism among the players in any coalition\(^2\).

(2) The extensive and strategic forms of a game

Among the many seminal contributions of von Neumann and Morgenstern(1944) was the provision of a set theoretic description of the fine structure or extensive form of a game. Their notation and description was improved on by Kuhn (1953). The key observation is that an

\(^1\) Although there had been earlier axiomatizations of subjective probability (See Fishburn, 1994, p.1401)

\(^2\)It is an empirical question as to when this assumption serves as a good approximation for the payment structure in the game. In many economic problems which concentrate only on local optimization money appears to serve this purpose.
intuitively natural structure was provided to describe the choice problem faced by a player at every point in a game, together with a precise description of the information conditions prevailing at every point. Subtleties such as "information leaks" could be formally mathematized. The choice theoretic description leads immediately to a mathematically precise formal description of the concept of strategy. A strategy is a formal plan which can be viewed as a book of instructions which specifies the selection of moves by a player under every contingency. Unfortunately a contemplation of a chess game limited to around forty moves indicates of the order of $10^{1000}$ different strategies, with the numbers for actual chess being far larger. This arithmetic tells us that individuals (and machines) do not formulate or search through complete sets of strategies).

From the mathematician’s point of view the mere fact that it is humanly not feasible to specify all strategies in a game as simple as chess, does not hamper one from thinking about the precise concept of a strategy. Furthermore if all individuals in a game select a strategy for a game, given this set of strategies, there would be no need for the individuals to be present at the playing of the game. A referee or a machine could match all strategies and work out the full play of the game which would be manifested as a path starting at the first (or root) node of the tree and proceeding all the way to some terminal node.

The strategic form of an n-person game consists of a listing of the sets of strategies for all players and the payoffs associated with the selection of a strategy by each of the players. As indicated in Part I, much of the exposition of game theory has utilized the two player, two strategy, simple 2 x 2 matrix game, as is illustrated in Table 1 where the rows are the strategies for Player 1, the columns, the strategies for Player 2 and the first entry in each cell the payoff to Player 1 and the second entry the payoff to Player 2.

<table>
<thead>
<tr>
<th>Player 2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10, 10</td>
<td>-2, 15</td>
</tr>
<tr>
<td>2</td>
<td>15, -2</td>
<td>3, 3</td>
</tr>
</tbody>
</table>

Table 1

From the point of view of much of the mathematical analysis it does not matter if the set of strategies available to an individual is 2 or $2^{1000}$. The logical operations are the same. Thus it can be proved that chess is an “inessential game”, i.e. if one could do all the calculations there would be no reason to play chess as each side would have an optimal strategy (Zermelo, 1912)
(3) The cooperative or coalitional form of a game

The coalitional or cooperative form of a game is based on primitive concepts far different from the extensive or strategic forms. It is assumed, *a priori* that there is some largest amount or set of amounts that any coalition S, of players can achieve by cooperating. This amount is usually denoted by v(S) An underlying assumption is that two or more groups will collaborate if there is joint gain to be had by coordinating their behavior. A game in coalitional form consists of a set of players N and a characteristic function v(N) which is a *superadditive set function* which specifies the worth of all $2^n$ coalitions which can be formed from n players.

(4) Solutions to games

The game description involves a considerable effort in selecting the appropriate abstractions. All of the game descriptions tend to have their basis in empirical views of social, economic, political or other structures. Mathematical effort goes into selecting descriptions which are consistent and complete. The major requirement for mathematical analysis comes in a two fold manner at the next stage. This involves the formulation of what is meant by a solution to a game in extensive, strategic or coalitional form. Originally there were two major divisions in solution theory, into cooperative and noncooperative solutions. The former were associated primarily with the games in coalitional form and the latter with games in strategic form. However in the past twenty to thirty years there has been a growing concern with games in extensive form modified to include the infinite horizon. When games with an infinite horizon are considered, even for those with perfect information, difficult mathematical problems appear, as is indicated by Mycielski (1992) and Sorin (1992) The solutions suggested for games in extensive form with or without finite termination cannot avoid the deep problems posed in attempting to construct a view of process and a dynamics of the intermix of cooperative and competitive behavior. It is in the approach to dynamics that the connection between the development of game theory, behavioral approaches and complexity is at its closest. This theme is developed in Part III.

The primitive concepts that the mathematical game theorist need not worry about provide the description of players, preferences, the rules of the game and what constitutes a solution. Considerable mathematical ingenuity may be called for to derive a solution to a game based on one of the many solution concepts which may be proposed. The mathematician has his work cut out for him without any need to be concerned for either the relevance of the model or any mapping from the primitives to corresponding aspects of an economic, political, military or other phenomenon.
Cooperative solutions

The solution theories suggested both for games in coalitional form and games in strategic form avoid the need to consider dynamics. The cooperative theories explicitly assume that the set of all players agree on “making the pie as large as possible”, they then use their coalitional claims in various ways to establish claims for as large a splice as possible. The dynamics is implicit in the negotiations which are not described. As an n-person game involves $2^n$ coalitions virtually any of the cooperative game solutions for n larger than three or four require considerable combinatoric analysis. The core and the nucleolus require programming techniques to examine the $2^n$ conditions which must be satisfied. Programs such as that devised by Scarf (1973) have been utilized to calculate points in the core. The calculation of the value solution for games of size of more than ten players remains a formidable computational task, although simulation methods have been used for large voting games.

Stable sets

Von Neumann, was more than a great mathematican. He had a genuine interest in the development of a mathematics devoted to the analysis and illumination of the basic problems in the social sciences in general and economics, in particular. The von Neumann-Morgenstern solution concept of the stable set solution to an n-person cooperative game attacked a central problem perceived by both of them. They felt that it was premature to try to develop a dynamic theory of economic interaction. It appeared to be too complex. Yet they were not satisfied with the simplistic economic analogies to equilibrium in physical systems. They sought a more satisfactory concept of social equilibrium. In particular they did not believe that a static theory of economics would have the power to produce a unique prediction. At best all that one could hope for was a solution consisting of a set of equally plausible outcomes selected so that they were consistent with the economic and societal conditions imposed on the model. A solution was a selection of a set of imputations. The conditions imposed on a solution were economic optimization (i.e. a cooperative society would opt for economic efficiency) and a complex form of social stability.

Von Neumann was rarely wrong in his conjectures. He conjectured that the stable set was never empty. This turned out to be false, Lucas (1968) was able to construct a 10-person counterexample. The importance of the combinatoric aspects of game theory is highlighted by the

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3 An imputation is a vector with an entry for each agent indicating his payoff, where the sum of all payoffs equals the amount available to the society as a whole.

4 The details of the suggested stability are somewhat subtle and cannot be discussed in sufficient detail in this paper. The reader interested in this novel approach to social stability should consult either von Neumann and Morgenstern (1944) directly or Lucas (1992)
counterexamples to apparently reasonable conjectures, which can be established only in high dimensions.

The stable set solution generated considerable mathematical investigation (as is indicated by the Lucas 1992 survey). However, to some extent due to its lack of universal existence, it has not been utilized much in applied game theory.

Other cooperative solutions

The most utilized cooperative solutions in the development of cooperative game theory have been the core and the value. They have been described in Part I; lesser known, but with a substantial literature are the bargaining set, the kernel and the nucleolus of a game.

Intuitively the core reflects the imputations protected from the direct bargaining power of subgroups in a society (it reflects the limitations of countervailing power), the nucleolus is an imputation that has the property that the largest complaint against it by any group is minimized and the bargaining set and kernel reflect plausible requirements for an acceptable bargaining outcome. All of these solutions have lead to considerable mathematical investigation. A survey of the literatures on the bargaining set, kernel and nucleolus which covers, not only the mathematics, but indicates applications and gaming experiments has been provided by Maschler (1992)

Noncooperative solutions

The noncooperative equilibrium

The ruling solution concept for games in strategic form is the noncooperative equilibrium of Nash (1950). A fixed point argument was utilized to show that for a finite matrix game there was an outcome which satisfied mutually consistent expectations.

Unfortunately there may be many equilibria which satisfy the condition of mutually consistent expectations, thus one may ask if there are any criteria to select one over another. Harsanyi and Selten (1988) have followed a program to search for conditions which lead to the selection of a unique equilibrium point for any game. The strategic or matrix form of a game supresses the implicit dynamics and process aspects of a game of strategy. As soon as the extensive form is examined many new problems with the concept of the noncooperative equilibrium appear. These problems are so critical that it is argued in Part III that the context free, rationalistic, high computation, high information, common knowledge assumptions behind the noncooperative equilibrium solutions must be replaced, or at least reinforced by more behavioristic “mass particle” behavioristic models.

In their attempts to deal with the many paradoxes posed in the examination of noncooperative equilibria in extensive form games, a “subindustry” in game theory has come into being dealing with extra requirements or conditions to be satisfied by acceptable noncooperative
equilibria. For example Selten (1975) refines the set of equilibrium points by considering only
perfect equilibria. These have the additional properties that not only are they noncooperative
equilibria in the game being examined; they are also equilibria in every subgame which can be
formed from that game. In essence this means that for perfect equilibria the role of history and
threats do not matter. Each player looks at his position at any point in time and regards himself as
in a new game which starts at that point. A further refinement to these equilibria are trembling
hand perfect equilibria. These have the additional property that one can subject one’s strategies to
minor perturbations (a trembling hand causing the individual to press the wrong button. Van
Damme (1996) provides an exposition and analysis of many of the basic refinements of the Nash
equilibria.

Rubinstein (1986) is one of the few game theorists to have gone from a vague worry about
the degree of complexity of a strategy to an attempt to construct a formal model with a limitation on
the level of complexity of the strategies. He does this by introducing explicitly a finite machine to
play for each player. In a different departure Aumann (1976) formally introduced the concept of
common knowledge and started up a new branch of game theory concerned with situations in
which players do not know what the other players know about their level of knowledge of the rules
of the game. This is referred to further in the Section on game theory and philosophy.

(7) The minimax solution

From one point of view, the two-person zero sum game theory can be regarded as a special
case of general n-person non-cooperative games, where the payoff to one player is the negative of
the payoff to the other. However there is a far stronger justification for the minimax solution than
the noncooperative equilibrium. In particular the minimax solution yields a unique value for any
two-person zero sum game. Furthermore as von Neumann and Morgenstern (1944) argued, the
only requirement concerning expectations is that a player believes that his opponent is rational.
Given this belief, each player’s selection of his strategy is reduced to a one-person optimization
problem.

The recommendation to a player to employ the minimax strategy, may be regarded as
normative, but a strong case can be made for following this norm in a two-person zero-sum game.
In particular, the argument is sufficiently plausible that it is of use in military operations research in
weapons evaluation where weapons effectiveness can be studied in the context of twoperson zero
sum game.

Although the optimal strategies for matrix games of considerable size (say several hundred
strategies each) can be calculated by linear programming methods, the proliferation of strategies in
games as simple as checkers or chess rules out current computational methods. The optimal
strategy for chess remains unknown, even though it can be proved that it exists.
Game theory as science; Applied science and social engineering

The original topics addressed in the development of the theory of games were the solution to formal parlor games such as chess, checkers or Bridge and problems in economics where in each instance, as a first approximation, it was implicitly or explicitly assumed that the individual agents are conscious optimizing individuals with great computational sophistication. Since its inception game theory has been employed in the study of many aspects of economics, political science, sociology, psychology, biology, anthropology, military science and law. In the last thirty years a considerable interest has grown in experimental gaming, some of which has been directly concerned with the validation of game theoretic solutions. No attempt is made here to provide a full discussion of all applications, however several examples are given and references noted. We discuss voting theory and design, complex cost allocations, uses in economics and in socio-biology.

Voting theory and design

Lloyd Shapley (1953) proposed as a solution to an n-person game the value. It is a single point solution which awards all agents their combinatorial marginal contribution to every coalition that could be formed. It is a combinatoric generalization of the economic concept of marginal worth. Shapley and Shubik (1954) modeled strategic voting as a simple game, which is a game with only winning or losing coalitions, i.e. where either there is a sufficient number of votes to carry motion, or it fails. In such a game an additional player’s marginal worth is only larger than zero if he converts a losing coalition into a winning coalition. If we consider all ways in which an agent can join a coalition as equiprobable, then by calculating the frequencies for each player to be pivotal in a vote we obtain an intuitively reasonable measure of that individual’s power. For example, in a simple majority game, if an individual has more than fifty percent of the vote he has all of the power, thus we seek a nonlinear measure which associates the power each with the distribution of votes among all. Banzhaff (1965) argued against this index and offered an alternative in which he counts the number of coalitions in which a player is the swing voter (without weighting the coalitions for the frequency of the number of a priori ways a player could convert a losing coalition to a winning coalition). Dubey and Shapley (1979) provide a detailed mathematical analysis of the differences between the two indices. Both of the indices noted have been applied to single “win-lose” issues. Shapley (1977) has suggested away to extend the Shapley-Shubik index to situations involving an ideology “space” covering intensity of preference when voting between programs offered by different parties. Each individual occupies a point in an ideology space and distance between them gives some indication of the intensity of their differences. A perceptive survey of the problems with and applications of the power indices is given by Sraffin (1994). A different, but complementary approach to the study of voting involves
considering voting as a game in extensive form and concentrating on various voting schemes as mechanism design for desirable properties of voting systems. Brams (1994) provides a survey of the literature on voting procedures.

**Cost allocation**

Another domain of application based on the value and core is cost allocation. Shubik (1962) suggested that the Shapley value could be utilized as a means to assign joint costs and revenues. Since that publication an extensive literature on cooperative game theoretic applications to cost allocation has come into being, much of which has been covered in two publications of Young (1985, 1994). These include applications to telephone call pricing using the work of Aumann and Shapley (1974), municipal cost sharing, pricing aircraft landing fees and irrigation and water use systems. Well before the formalization of game theory the Tennessee Valley Authority utilized a procedure which yielded an outcome in the core of the associated game. (See Ransmeier, 1942).

**Economic Theory**

A suggested facetious description of the attitude of many economic theorists is that they view the functioning of many institutions as “good in practise but they do not work in theory”. The challenge of the applied game theorist is to refute this attitude by producing models which represent both practise and theory. Four sets of developments in game theory bear this out. They are: auctions, industrial organization, agency theory and the theory of money.

**Auctions**

Game theory as applied science has provided considerable insight in understanding auction procedures of many varieties. The specialized literatures on auctions is now voluminous, starting with Vickrey’s (1961) seminal paper. Engelbrecht-Wiggans, R., M. Shubik and R. Stark (1983) provide a an overview of the uses of game theory in auctions up to the early 1980s, and Wilson (1994) and Kagel (1995) provide detailed coverage of both the theory and experimentation and an earlier book by Cassady (1967) provides much historic and institutional detail.

There are many institutional forms to auctions; such as the dutch auction used in selling tulip bulbs. All buyers sit in front of a large price “clock” with a hand indicating price. As time passes the offering price descends, until the clock is stopped by an individual who presses a button indicating that he is willing to buy at that price.

The key observation concerning auctions is that they involve process analysis and call for the formulation of the extensive form of the game. The applied game theorist earns his keep in mechanism design and in facing up to the ad hoc aspects of lack of common knowledge and
assymmetric information. Kagel (1995) provides a thorough survey of experimental research on one-sided auctions where there is one buyer or seller confronting many sellers or buyers.

Auction design is now recognized as a useful application even though there is still considerable controversy in interpreting experimental studies and field data from actual auctions.

**Industrial organization**

There is a generation of economists characterized by Fudenberg and Tirole (1991) who confuse the general subject of game theory with variations of noncooperative equilibria applied to games in extensive form. For the purposes of studying problems in oligopolistic competition the use of the strategic form, the extensive form and modifications of the concept of a noncooperative equilibrium appears to be a reasonable approach to adopt. Shubik (1959) published the first book with the explicit application of noncooperative game theory to oligopolistic competition which integrated the previous work of Cournot, Edgeworth, Bertrand, Chamberlin and others into a noncooperative game framework where one could see the influence of numbers on the solutions. The contrast between behavior among few agents and behavior in mass economies raises deep questions concerning the economist’s “rational actor” approach and a highly different “mass particle approach.” This is considered further in Part III. Selten (1975,1978) raised several fundamental problems in the study of the concept of threat. Wilson (1992) provides a perceptive survey of much of the basic game theoretic literature developed in the 1980s on the oligopolistic aspects of entry, deterrence and struggle for market share stressing the problems of preemption, signaling and predation. Tirole (1988) presents an overall text on game theoretic models for the study of industrial organization.

There is a profusion of special models, many of which are highly sensitive to even minor changes information conditions and sequencing of moves. Three other topics of considerable application of extensive form game theory are agency theory, the study of moral hazard and contract theory. Dutta and Radner (1994) provide a detailed survey. All of these topics have been the source of considerable application in law and economics on an ad hoc basis. Their value depends on two features. The laying out of the details and “anatomy” of the structure being studied, and the appropriate portrayal of the information conditions and economic motivation of the agents.

**Theory of money**

One of the more unsatisfactory aspects of microeconomic theory has been the lack of a clear theory of money and financial institutions. This appears to be so because the emphasis in microeconomics has been on equilibrium analysis with most attempts to deal with dynamics handled by comparative statics which amounts to comparing two static time slices of the economy
with each other with no process mechanism specified. Shubik (1972), Shapley and Shubik (1976) and Dubey and Shubik (1978) developed and established the linkage between strategic market games and general equilibrium models of the economy. Karatzas, Sudderth and Shubik (1994, 1997) extended the models to the infinite horizon, essentially as a set of parallel stochastic dynamic programs with a continuum of agents. The important features to note are, as with applications to industrial organization, the models in extensive form are process models. The rules of the game virtually describe a reductionist form of economic and financial institutions. In contrast with the work on industrial organization, where the emphasis is on competition and cooperation among few agents; much of the theory of money is devoted to economies with a mass of agents. The role of money appears to be essentially manifested in disequilibrium dynamics. When the system is in equilibrium the need for the money disappears.

**Experimental and operational gaming**

In the past forty years there has been a considerable growth in the uses of gaming.\(^5\) Military operational gaming preceded game theory by well over a century (see Brewer and Shubik, 1979), but experimental gaming came into being after the publication of the theory of games. An up to date summary of experimental gaming as applied to economics has been provide by Kagel and Roth (1995) and a survey directed specifically at the relationship between game theory and gaming is given by Shubik (1998). It is clear as can be seen from the examination of the large experimental literature on the prisoner’s dilemma game repeated several times that the predictions of neither noncooperative nor cooperative theory are borne out. The results obtained appear to depend heavily on context, the briefing, the socio-psychological make up of the players, their unknown valuation of the physical payoffs and their knowledge of game theory. In short, it is extremely difficult to perform good experiments with human subjects whose life experience cannot be normalized to a tabula rasa by clearing memory and “rebooting their computer”. Even in that part of game theory subject to the least controversy, small zero sum matrix games, experimental difficulties abound. Possibly one of the most careful experiments in the literature of experimental game theory was performed by O’Neill (1987) utilizing a 4 x 4 matrix with only +(win) or - (lose) entries.

The experimental evidence does not support most game theoretic solution concepts, except in highly controlled and context relevant instances. The indications are that the dynamics of behavior among the few are far too rich to be captured universally by current game theory solutions. Other views of what constitutes a solution are called for.

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\(^5\) For a summary of the start of gaming up to the mid 1970s see Shubik (1975a,b) and Brewer and Shubik (1979).
Game theory and evolutionary biology

The paper of Maynard Smith and Price (1973) introducing the concept of an *evolutionarily stable strategy (ESS)* marked the start of the now active branch of game theory and biology known as *evolutionary game theory*. The key differentiation from conventional noncooperative game theory is that a mixed strategy, instead of being regarded as a probability distribution is interpreted as a frequency in which different agents from an otherwise homogeneous population utilize different pure strategies. An equilibrium is evolutionarily stable if it cannot be invaded. A simple example below illustrate an ESS.

We consider a simple so-called “Hawk-Dove” game. Suppose that the payoff to any individual is given by the following matrix:

<table>
<thead>
<tr>
<th></th>
<th>Attack</th>
<th>Retreat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attack</td>
<td>-1, -1</td>
<td>3, 0</td>
</tr>
<tr>
<td>Retreat</td>
<td>0, 3</td>
<td>3/2, 3/2</td>
</tr>
</tbody>
</table>

The population has some percentage of aggressive and docile animals. If two aggressive animals meet each other they attack and damage each other. If a “hawk” meets a “dove” who retreats against his attack, then the hawk takes over the territory and prospers with an expected 3 offspring. If two doves encounter each other they both retreat and end up undamaged and splitting the territory with and expected payoff of 3/2 each. Suppose that the distribution of the population is such that 3/5 consists of hawks and 2/5 consists of doves. The expected payoff to a hawk or dove in such a population is 3/5. Suppose however that there is a mutation which produces a few extra hawks. Their expected payoff from an encounter will be be less than 3/5. For example, suppose that the whole population except for one animal were hawks. Any hawk would obtain a payoff close to -1, but the solitary dove would obtain a positive payoff. This could not be stable and the population of doves would grow.

The payoffs in a biological game are interpreted as the expected number of surviving offspring.

The literature on evolutionary game theory has shown considerable development in the last two decades. There are two excellent surveys and a recent book which provide an overview of the developments. They are the surveys of van Damme (1996) and Hammerstein and Selten(1994) and the book of Weibull (1996).
**Game theory as philosophy**

The mathematical game theorist may operate on formal models, concerning himself with existence proofs, beautiful combinatorial problems and subtle computational methods. He does not need to be concerned with the origin of the models or why a given solution concept has been formalized. The social scientist other scientist or applied game theorist and consultant are deeply concerned with the relevance and appropriateness of the description of the game and the nature of the solution concept selected. Those with a more philosophical bent concentrate on several of the basic questions underlying the game theoretic formulation of human behavior.

One of the key divisions present in the early development of the theory of games was the splitting of the analysis of solutions to an n-person game into cooperative and noncooperative theories. Most of the cooperative theories have invariably been treated as normative; the major exception being the von Neumann and Morgenstern stable set solution. We noted in Part I that most of the cooperative solutions have been axiomatized, thus for example, the Shapley value is deduced from axioms concerning efficiency, symmetry and a form of additivity in awards to players when two completely independent games are considered as one. Game theorists with a philosophical bent seek to formalize concepts such as fair division, the nature and role of symmetry, efficiency, equity and power. These are the building blocks from which the various cooperative game theory solutions are formed.

The dominant solution theory for the noncooperative game in strategic form is the Nash noncooperative equilibrium. The justification for accepting it as the appropriate solution can be made along normative or behavioral lines. It can be argued that whatever outcome or set of outcomes which are accepted as a solution, any member of the set should have the property that it is stable against individual strategic defection, i.e. no single individual could improve his payoff by changing his strategy. A somewhat stronger and less plausible condition is that all individuals should share consistent expectations. In a one shot game this does not tell us how these expectations are formed. It is well known that even a simple game in strategic form may have many noncooperative equilibria. The Harsanyi and Selten (1988) program is devoted to providing the extra conditions to enable them to select a unique equilibrium point for any game.

Harsanyi’s (1994) view is that the study of ethics may be regarded as a branch of the general theory of rational behavior. The concept of rational behavior is normative, it indicates what human behavior would have to be like in order to satisfy consistency conditions in the pursuit of goals. Harsanyi(1994, p671) observes that “most philosophers also regard moral behavior as a special form of rational behavior. He suggests that the study of rational behavior in a social setting can be divided into game theory and ethics, where the study of ethics deals with two or more individuals having different interests, are concerned with promoting the common interests of their society in a rational manner. He proposes a utilitarian view of the social welfare function as a
linear function of all individual utilities with the same positive weight assigned to each. Harsanyi (1994, pp 692-694) offers a game-theoretic model for a rule utilitarian society as well as a critique of Rawls’ theory of justice.

The Shapley value may be considered as a solution representing “merit justice”; although it can be argued that the distinction between “need justice” and “merit justice” depends on the interpretation of the assumption of the veil of ignorance suggested by Rawls. A paradox in the interpretation of the Shapley value is that although one can argue that it provides a fair division measure, it simultaneously appears to provide a measure of power. The paradox is resolved when one observes that the initial specification of the characteristic function of a game slips in the power aspects into the initial conditions on which the fair division process is utilized. If the characteristic function were chosen behind a veil of ignorance it would be completely symmetric and need and merit justice would coincide.

Harsanyi and Selten, in their search for the unique appropriate noncooperative equilibrium point utilize as their primitive concept a game where all agents share as common knowledge complete information about all of the rules of the game. Even casual observation of human interactive decisionmaking suggests at least three basic aspects of reality of which the first two are ignored in the study of most game theoretic solutions. They are:

1. Individuals have limited computational and perceptual abilities.
2. In situations of even mild complexity the information requirements rule out any approach based on a fine-grained extensive form representation.
3. Individuals may not have common priors in many decisionmaking situations of any social, economic or political significance.

The first two conditions lead us in the direction of the construction of finite automata for playing games and the third leads towards a development of a game theory that is able to cope with the concept of common knowledge. Aumann’s (1976) initial contribution has already been noted. Geanakoplos’ (1994) careful survey, utilizing the concept of the overall “state of the world” and a “knowledge operator” presents a clear picture of the central paradoxes posed by an initial lack of common knowledge. A key illustration (which is left to the reader to solve) involves three children sitting, each wearing a red cap. Each child can see the cap of the other two. Each child knows only that her cap is either white or red. A teacher tells them all that at least one of them is wearing a red cap and then asks the first child if she knows the color of her cap. The child says no and this

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6 A “state of the world” specifies not only all states, but what every agent knows about all states and knows about what every other agent knows.
reply is heard by the other two. The teacher then asks the second child who also answers no. The third child when asked, says my cap is red.

This simple, essentially nonstrategic parlor game can be turned into a full fledged Bayesian game of strategy by attaching specific payoffs to the actions of each child. For example each child may have three strategies, to guess white, to guess red or to say I do not know. The payoff to the first might be -1,000,000, to the second, 10 and to the third, 0.

The basic theorem specified by Geanakoplos (1994, p.1454) “shows that it cannot be common knowledge that two or more players with common priors will want to bet with each other, even though they have different information”. We see speculation and panics around us; we here phrases such as “it is a difference of opinion that makes for the betting at horse races”. But these are not consistent with common knowledge. We must seek explanations in modifying our view of the rational superintelligent agents.

Game theory as prescience and as advocacy.

The two approaches to game theory, as mathematics and philosophy may be referred to as “highbrow game theory” where the emphasis is on mathematics and logic. The science, social science and applications can be viewed as “middlebrow” where the stress is more on detailed modeling and hand tailoring both the models and solution concepts to the problem at hand and devising tractable computations and approximations. The fourth type of game theory which, in many ways has been highly successful is its use as a preformal way in starting to explore aspects of the softer social sciences and for advocacy in the construction of metaphores and in the making of broad analogies using virtually no mathematics more than the formulation of a game as a 2 x 2 matrix game which is then discussed in essay form. This can best be described as “conversational game theory.” The term is not meant to be pejorative, but because much of the nature of the phenomena to be portrayed is difficult to portray precisely, the imprecision of essay form game theory is desirable as a start.

Schelling’s (1958) influential book The Strategy of Conflict was a prime example of this important genre. The formal game theory presented was elementary or wrong, but the ideas and the simple modeling called for a reorientation of the modeling of both the games and assumptions concerning behavior for those dealing with applications to international affairs. He dealt with bluffing, threats and counter threats and questioned knowledge about the rules of the game and the nature of human behavior. Anatol Rapoport (1960) in his book entitled Fights, games and debates and Boulding (1962) in his book, Conflict and defense produced essays far more devoted to context, to problems with mass behavior, such as psychological epidemics and to an appreciation of game theory as a great step in the understanding of conflict, but in many ways too rationalistic in its presentation of human behavior. Various game models have been utilized to study the Cuban
missile crisis, The United States and Soviet Union arms race, the Middle East wars, the Berlin and Formosa straights crises and many other situations. O’Neill (1994) provides an excellent overview of game theory models of peace and war and gives the references to many of these studies.

A considerable applied and conversational game theoretic literature exists, addressed to the threat structure of the nuclear standoff between the United States and the Soviet Union. This includes Ellsberg’s (1961) “The crude analysis of strategic choices” , Herman Kahn’s and Albert Wohlstetter’s many writings on thermonuclear warfare, doomsday machines and first launch policy.

The popular press ascribes to game theory, in general and to von Neumann in particular, the development of the calculus of deterrence, the doctrines of first strike and brinksmanship. Although undoubtedly von Neumann was a conservative and in some sense a hawk in his views of the Soviet Union; his development of game theory had nothing to do with defense considerations. In particular, virtually all of the defense analysis is based on noncooperative theory and von Neumann had considerable doubts about this theory; devoting much of his time to the development of a theory of cooperative games. He and Morgenstern felt strongly that the development of a game theoretic dynamics was premature when they were writing, and that an equilibrium analysis which focussed on a unique equilibrium would not provide a satisfactory model of human behavior.

The basic developments in the theory of games were not motivated by defense considerations. The later defense considerations were influenced by the development of game theory. The military funding for further developments at RAND and elsewhere came after von Neumann’s work on game theory. The work at RAND on duels, deterrence and operational gaming were only part of the broad development of game theory.

Many article in the press have appeared in the last ten years using phrases such as “a zero-sum society; a prisoner’s dilemma situation. The 2 x 2 games has made its way down to high school level. Thousands of papers have been written on variants of the Prisonner’s dilemma game’. There are 78 strategically different 2 x 2 games, if ties in payoff orderings are ruled out; there are many hundreds of strategically different 2 x 2 games if ties are permitted. The number of strategically different 3 x 3 games is astronomical. Yet the power of metaphor is such that much of the application of conversational game theory is based on analogies made between critical societal and political problems, such as nuclear war and 2 x 2 matrix games.

A simple experiment performed with my students over the years in a course taught on game theory in the social sciences was to give them three names and three matrices known to game

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7 See Rapoport, Guyer and Gordon. 1976 for an early listing and analysis of the 2 X 2 games or the Journal Conflict Resolution for many of the more recent references.
theorists as:” the prisoner’s dilemma”; “chicken”, and “the battle of the sexes”. The students were asked to associate the names with the matrices. There was little indication that students without previous knowledge of game theory were able to match the names and the matrices. Context appears to matter considerably and it is easy for the individual amateur to grasp many of the ideas and insights of game theory via a 2 x 2 matrix with a story attached to it. Unfortunately not all choices are binary and context appear to have considerable influence. The story provides the context. The game theorist, as a scientist should be delighted to see that interest in game theory analogies has spread, not merely to all of the social sciences and law, but to many policy problems and into the popular press. But the dangers of analogy and metaphor as well as their virtues must be considered. Furthermore the importance of context and the difficulties in matching context and the abstract model must be kept in mind especially when conversational game theory is utilized to influence policy.

The sociology of game theory

The mathematician, operations researchers, social psychologists, popular science writers, business consultants, strategic advisors and political commentators talking about “the nonzero sum society” have little in common. The experimental social psychologist matching oriental female undergraduates against Californian male undergraduates in a prisoner’s dilemma game has few interests in common with a pure mathematician studying the geometry of stable set solutions. Yet all of their activities may be broadly described as involving some aspect of game theory. There is employment for the pure mathematician and the writer of computational algorithms where neither of them have to be in the least concerned with any aspects of substantive science or advocacy. The subject has matured sufficiently that it can generate a rich supply of “nice” mathematical or computational problems. In contrast, those more concerned with utilizing the methods of game theory to illuminate problems in the behavioral sciences, law or even history are more concerned with how the models fit. In political science, public affairs, policy and administration the problems are “fuzzier” and are hardly amenable to full formalization. Many of the questions are imprecise and the answers are frequently more ephemeral inputs and aids to a political process than a search for understanding. The mere fact that the language of game theory has reached the level of general metaphor, however is a sign of the depth of its success. Thus play, zero-sum society, or non-zero sum game and other game theoretic phrases turn up today in the popular press.

The success of game theory has helped to illuminate its shortcomings. Although in the next twenty years there will probably be a continuation of the explosion in analysis and applications, the improvement in our understanding of the structure of interactive decision making has pointed (as von Neumann and Morgenstern suggested) to a radically different emphasis. The various fine-grained and coarse-grained formulations of the structure of a game, together with the different
game theoretic solutions suggested have been of considerable value in studying and furthering our understanding of a host of interesting and often, applied problems. But the very power and success of the methods of game theory have illuminated and helped to clarify the locus of the gaps in our understanding of human interactive decision making.

The relationship between analysis and evolution; between strategy and behavior; complexity and perception; inference and expectations; and the interactions of the few and the many pose many problems which have not yet been answered.

The methods of game theory have shown both the power and limitations of the study of equilibrium. The time is ripe for the first steps towards understanding dynamics. But this understanding appears to call for a radical departure from the solution concepts which have dominated game theoretic thought to date. This is the subject of Part III of this essay.
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