IMPERFECT COLLECTIVE SECURITY

and

ARMS RACE DYNAMICS:

WHY A LITTLE COOPERATION CAN MAKE A BIG DIFFERENCE

Joshua M. Epstein

This paper is based on lectures given at the 1992 Santa Fe Institute Complex Systems Summer School and will appear in the author's book *Nonlinear Dynamics, Mathematical Biology, and Social Science: A Lecture Notes Volume in the Santa Fe Institute Studies in the Sciences of Complexity*, Addison-Wesley, 1997.

IMPERFECT COLLECTIVE SECURITY and ARMS RACE DYNAMICS: WHY A LITTLE COOPERATION CAN MAKE A BIG DIFFERENCE

Joshua M. Epstein‡

May 1, 1996

This paper uses simple mathematical models to explore the relationship between security regimes and arms race dynamics.[1] The main focus is a regime known as collective security, which is receiving wide attention. Little of the attention is mathematical, however, and, to my knowledge, none of it involves dynamical systems. One aim of this paper, then, is to formalize collective security in a dynamical systems context, which will allow us to extract some unexpected results. This formalization, of course, requires a rigorous definition of collective security. To wit: Imagine three countries x, y, and z. *Perfect* collective security would then operate as follows: If x attacks y, z allocates all force to y; if y attacks z, x allocates all force to z; and so on. The general rule is simply that *the odd man out instantly allocates all force to the attacked party.* In more biological--or sociobiological--terms, perfect collective security is a form of reciprocal altruism.[2]

Now, a heated debate surrounds this idea.[3] The debate turns on the question, "How much

---

‡. The Brookings Institution and Santa Fe Institute.

1. A regime is a "set of implicit or explicit principles, norms, rules, and decision-making procedures around which actors' expectations converge in a given area of international relations." See Stephen Krasner, *International Regimes,* Cornell University Press, Ithaca, N.Y., 1983, p. 2. On the emergence of norms generally, see Robert Axelrod, "An Evolutionary Approach to Norms," *The American Political Science Review* 80, December 1986.

2. See Edward O. Wilson, *Sociobiology*, Harvard University Press, 1975; Edward O. Wilson, *On Human Nature*, Harvard University Press, 1978, chap. 7; James L. Gould and Carol Grant Gould, *Sexual Selection*, Scientific American Library, New York, N.Y., 1989, pp. 244-246; and John Maynard Smith, *Evolutionary Genetics*, Oxford University Press, 1989, pp. 167-169.

3 A vigorous debate on the general topic has appeared in Harvard's journal, *International Security*. In particular, see John J. Mearsheimer, "The False Promise of International Institutions," *International Security*, Vol. 19, No. 3, Winter 1994/95, and Charles A. Kupchan and Clifford A. Kupchan, "The Promise of Collective Security," *International Security*, Vol. 20, No. 1, Summer 1995. See also Charles A. Kupchan and Clifford A. Kupchan, "Concerts, Collective Security, and the Future of Europe," *International Security*, Vol. 16, No. 1, Summer 1991. For a collection of thoughtful analyses, see George W. Downs (ed.), *Collective Security Beyond the Cold War*, The University of Michigan Press, Ann Arbor, 1994.

cooperation[4] is *possible*?" Skeptics argue that substantial levels of cooperation are not possible, and therefore that collective security can be dismissed. Proponents counter that a substantial degree of cooperation is possible and, therefore, that collective security is worth pursuing. Notice, however, that both positions *assume* substantial levels of cooperation to be *necessary* for collective security to be worthwhile. What about this central assumption? What about *a little bit* of collective security; is there merit in a highly diluted form? As nonlinear dynamicists--accutely aware that small perturbations can have huge effects--we are intrigued by the question.

And in fact, a central conclusion of this analysis is precisely that collective security regimes--even in highly diluted forms--can exert remarkably powerful stabilizing effects; in arms race models sufficiently nonlinear to produce really volatile dynamics, highly imperfect collective security regimes can damp the explosive oscillations and induce convergence to stable equilibria below initial armament levels. Put differently, the injection of *tiny* degrees of altruism can profoundly calm the otherwise volatile dynamics. The benefits of participating in the system are very great and, because of the nonlinearity, the required level of commitment from individual participants is very low.

In addition, one might assume that the more volatile a system is, the less value there will be in a given, low, level of collective security. But, counterintuitively, within the class of models examined here, precisely the reverse is true! These results would appear to invite a reorientation--or, at least, an extension--of research on altruism in a variety of fields.

Altruism: How Little Is Enough?

Specifically, the present analysis demonstrates that, in some dynamical systems, exceedingly low levels of *individual* altruism produce high levels of *collective* harmony. Everything depends on the intervening *dynamics*. For instance, in a memorable phrase, Edward Wilson writes, "the genes hold *culture* on a leash"[5] (emphasis added). But, what he actually argues is that the genes hold *individuals* on a leash. The present analysis suggests that, even if the individuals' leash is very short, the culture's leash might be very long. Indeed, the analysis seems to open an emotionally appealing niche: perhaps we can be optimistic about the prospect of *social* harmony while retaining a certain degree of skepticism toward the prospect of *individual* altruism. In short, the issue is not simply how much individual altruism is *possible*; but, for social harmony, how little is *enough*?

---

[4] Throughout, I will use the term "cooperation" and "reciprocal altruism," as just defined, interchangeably.

[5]. Wilson, *On Human Nature*, p. 167.

Organization and Methodology

The discussion is organized as follows. First, by way of introduction, the simplest two-country form of Lewis Frye Richardson's classic purely competitive arms race model is presented.[6] That model is then generalized slightly and expanded to encompass three competitors, the minimum number necessary to examine collective security.[7] In addition to Richardsonian competition and collective security, I will examine a regime characterized by the presence of a world policeman, or "globocop."[8] In this model, a force, C, located outside the competitive three-actor system is held at the ready to swoop in to support any attacked party. The object is to compare these three regimes formally. I do so first assuming that the underlying arms race dynamics are linear, as in Richardson's model.[9] Then I assume nonlinear arms race dynamics of a specific sort. I would hope that the full nonlinear model introduced here will contribute to the theoretical arms race literature in its own right.

Now, I make no attempt to test any model. Nor do I claim that any of these models is "right." Rather, the models are coarse lenses under which we compare the regimes. To be specific, the general procedure would be as follows. Take some model of Hobbesian--or, if you prefer, anarchic--arms race competition. Call that model M_1 (I use the linear Richardson model).[10] Then construct (see below) that model's collective security variant, M_1^C. And, for expository purposes, suppose collective security damps the competition in the sense that if $M_1^C(0) = M_1(0)$, then $M_1^C(t) < M_1(t)$ for all positive t.[11] Now, take a second Hobbesian model (I use a nonlinear Richardson model), M_2, construct its collective security variant, M_2^C, and compare the dynamics. Again, suppose that collective security damps the competition. Continue in this way. If this *comparative* result recurs without exception over a huge set of model pairs, $\left\{\left(M_i, M_i^C\right)\right\}$, then one may be justified in concluding that collective security exercises a systematically depressive effect on competitive dynamics. This discussion suggests that may be the case, though more of this "structural sensitivity analysis" would be needed before confidence is obtained.[12]

---

6. Lewis F. Richardson, *Arms and Insecurity*, Boxwood, 1960.
7. I do not treat the n-actor, or globally inclusive, case here.
8. In principle, globocop could be a consortium of powers. I thank Brian Pollins for this name.
9. Assuming linearity, globocop is not an interesting variation on Richardson. In fact, it *is* linear Richardson with grievance terms translated by a fixed amount. Hence, little is said about globocop in Part I. Indeed, since its effects are, from a qualitative mathematical standpoint, pretty straightforward even in nonlinear cases, globocop is included for completeness but will receive relatively little attention.
10 Technically, the model is affine.
11 To spell this out completely, we are positing $M_1(t) \equiv \left(M_{11}(t), M_{12}(t), \ldots, M_{1n}(t)\right)$ and $M_1^C(t) \equiv \left(M_{11}^C(t), M_{12}^C(t), \ldots, M_{1n}^C(t)\right)$. Then $M_1^C(t) < M_1(t)$ iff $M_{1k}^C(t) < M_{1k}(t)$ for every k.
12 Ultimately, an elegant way to proceed would be to characterize mathematically the entire class of formal arms race models under which collective security (or globocop) would show a depressive effect, and then examine whether models falling outside that class are at all plausible. If not, we

From a methodological standpoint, it is also important to distinguish this analysis from other treatments of the issue. In particular, I am *not* examining the stability of collective security from a game theoretic standpoint.[13] No claim is made as to the likelihood of compliance with, or defection from, the system. Rather, I am trying to contribute a dynamical systems perspective to the theoretical literature, asking, with all else fixed, what is the dynamic effect of purely "institutional"--or, rule regime--change? The results bear on the game theoretic literature inasmuch as compliance depends on payoffs.[14] The payoffs associated with compliance--this analysis suggests--may be surprisingly high, and individual behavior may change as a result. Or, to couch it more prosaically, maybe if leaders appreciated the potential payoff of even limited collective security, they would be more interested in broad compliance. Indeed, the analysis would appear to raise starkly the question where to draw the line between altruism and self-interest in the international system.

Finally, I make no attempt to evaluate the practicality of implementing collective security in Europe, the Middle East, or any other particular region. However, if collective security in practice would behave at all like the idealized regime examined here, then its institution--even in highly diluted forms--might well be worth substantial effort. To begin at the beginning, let us revisit Richardson's original model.

Part I. Linear Models

The Classic Richardson Model

The following differential equations constitute Richardson's basic model, with x and y the actors:

---

might wish to conclude that, by virtue of its membership in that class, the "right" model--whatever it is--will indicate the same depressive effect for collective security (or globocop), and *ipso facto*, that the effect is quite real. The basic idea, again, would be to say something reasonable about *comparative* dynamics, *without* claiming to know the "right" arms race model. Relatedly, it is worth stating explicitly that no sensitivity analysis on the parameter values given in the Appendix is conducted here. It is of considerable interest that there *exist* parameter settings at which a sharp sensitivity to rule regime is evident. A separate study would examine the robustness of this result under a wide range of parameter settings.

13. See Emerson M. S. Niou and Peter C. Ordeshook, "Realism versus Neoliberalism: A Formulation," *American Journal of Political Science*, vol. 35, no. 2, May 1991, pp. 481-511. Also relevant are Robert Axelrod, "The Evolution of Strategies in the Iterated Prisoner's Dilemma" in L. Davis (ed.), *Genetic Algorithms and Simulated Annealing*, Pitman Publishing, London, 1987; and Robert Axelrod, *The Evolution of Cooperation*, Basic Books, New York, 1984.

14. The location of mixed strategy equilibria--and of evolutionarily stable strategies in bimatrix games--depend on payoff magnitudes, not just orderings. The speed of convergence to any stable equilibria also depends on payoff magnitudes.

$$\dot{x} = a_2 y - a_1 x + g_x, \tag{1}$$
$$\dot{y} = b_1 x - b_2 y + g_y. \tag{2}$$

The basic idea is that a state's arms race behavior depends on three overriding factors: the perceived external threat, the economic burden of military competition, and the magnitude of grievances against the other party. Each merits a brief discussion.

"Grievances"

The constants, g_x and g_y, are usually interpreted simply as grievances. And, on this reading, a core message of Richardson's model is that there can be no permanent disarmament without the resolution of underlying political grievances. Even if disarmament is total (i.e., $x(t) = y(t) = 0$), arms racing will reemerge if g_x or g_y is greater than zero. For instance, take Eq. (1), and assume $x(t)$ and $y(t)$ are both zero. If $g_x > 0$, then, since the growth *rate* $\dot{x}(t)$ equals g_x, that rate, too, must exceed zero. Hence $x(t)$ begins to grow, which, via Eq. (2), stimulates a growth reaction in $y(t)$, which feeds back to further stimulate $x(t)$, and the race is on. Or, as Richardson himself put it, "mutual disarmament without satisfaction is not permanent."[15]

It seems to me that g_x and g_y can be interpreted somewhat more broadly. States compete militarily not only because there are grievances, but also because they lack confidence that grievances can be resolved without resort to arms. The emergence of institutions offering high confidence that differences could be resolved nonviolently might permit disarmament despite outstanding grievances. So, I think of the terms, g_x and g_y, as capturing both the underlying grievances and the level of confidence that grievances can be resolved nonmilitarily. If confidence ranges from zero to one, then each g could be interpreted as: (grievance)•(1-confidence), for example. It is far from clear how one would measure g_x or g_y under either interpretation. Luckily, their measurement is not necessary for our purposes.

The "Economic Fatigue" Term

Leaving the grievance terms aside, the rate at which x grows, $\dot{x}$, is proportional to the perceived external threat y, and vice versa for the growth rate $\dot{y}$. Without some damping term, this is a pure positive feedback system that simply blows up. Richardson posited economic fatigue terms ($-a_1 x$ and $-b_2 y$) that damp the process. Clearly, so long as the fatigue coefficients a_1 and b_2 are greater than zero, the process is damped. Importantly, if a_1 and b_2 are negative, then $-a_1$ and $-b_2$ are positive, meaning that military growth rates increase the larger is the military establishment. In such cases, x and y may both grow even in the absence of underlying

15. Richardson, *Arms and Insecurity*, p. 17.

grievances or any perceived threat. This would be autocatalytic growth in the "military-industrial complex." The "fatigue" coefficients might be thought of as embodying the net civil-military, or "guns versus butter" balance in society. If the terms are negative, then there is auto-catalytic military expansion, or "militarism" for short.

The External Threat Terms

Finally, the terms $b_1 x$ and $a_2 y$ incorporate perceived external threats, obvious components of any plausible model of arms race behavior.[16] I will generalize this model below, adding a third party and introducing certain nonlinearities, among other things. But clearly, Richardson's linear model has some basic appeal, parsimoniously relating arms race behavior to grievances, perceived external threats, and internal economic fatigue.[17]

Simple Analytics

The simple analytics of the famous model deserve a concise review. Defining

$$A = \begin{pmatrix} -a_1 & a_2 \\ b_1 & -b_2 \end{pmatrix},$$

the basic Richardson model is simply

$$\dot{x} = Ax + g \;;\; x, g \in \Re^2. \tag{3}$$

A positive equilibrium $\bar{x}$, if it exists, is given by

$$\bar{x} = -A^{-1} g \,.$$

Stability is independent of g; that is, $\bar{x}$ is a stable equilibrium of Eq. (3) if and only if the origin is a stable equilibrium of $\dot{z} = Az$, where $z = x - \bar{x}$. That requirement is met if $\operatorname{Tr} A < 0$ and

---

[16]. In reality, these terms are probably nonlinear, since the military's power to *shape threat perceptions* itself may vary with the size of the military establishment, suggesting terms of the form $a_2(x)y$ and $b_1(y)x$. Interesting work along these lines is underway elsewhere. See Benoit Morel, "Modelling U.S.-Soviet Relations," Draft Analysis, May 13, 1991, Carnegie Mellon University.

[17]. The model provides a nice framework for interpreting gross changes in, for instance, Soviet behavior. Thinking of the Soviets as country x, one might argue that the end of the cold war manifests the facts that Gorbachev's $a_2 <$ Brezhnev's a_2, Gorbachev's g < Brezhnev's g, and Gorbachev's $a_1 >$ Brezhnev's a_1. The last of these--Gorbachev's assessment of the economic burden of the arms race--was perhaps crucial. But, in any event, he adjusted the Soviet parameters sharply and, suddenly, arms race dynamics were changed.

$\operatorname{Det} A > 0$. For a_1 and b_2 positive, the trace condition is obviously satisfied by A, and the determinant is positive if the product of economic fatigue terms $(a_1 b_2)$ exceeds the product of reciprocal activation terms $(b_1 a_2)$.

Now, simple linear model in hand, we wish to examine the transition from this, Hobbesian, world to a collective security regime first and, secondarily, to a world characterized by the presence of a world policeman, or "globocop."

As noted at the outset, there has been no attempt to frame the comparison in dynamical systems terms. An immediate issue, then, is how to operationalize collective security and globocop. Recall that under a *perfect* (as against diluted) three-party collective security regime, if x attacks y, z instantly contributes all forces to y. If z attacks x, y instantly allocates all forces to x, and so forth. The odd man out instantly allocates all force to the attacked party. Under globocop, a force of fixed size, C, from outside the three-actor system is instantly allocated to any attacked party. Various imperfections will be discussed below. But *perfect* collective security, and globocop, are operationalized in figure 3.1, as variations on an underlying linear Richardson model.

Figure 3.1. Linear Models: The Three Basic Variants[18]

Linear Richardsonian Competition

$$\Delta x = -a_1 x + a_2(y - x) + a_3(z - x) + g_x$$
$$\Delta y = b_1(x - y) - b_2 y + b_3(z - y) + g_y$$
$$\Delta z = c_1(x - z) + c_2(y - z) - c_3 z + g_z$$

Linear and Perfect Collective Security

$$\Delta x = -a_1 x + a_2(y - (x + z)) + a_3(z - (x + y)) + g_x$$
$$\Delta y = b_1(x - (y + z)) - b_2 y + b_3(z - (y + x)) + g_y$$
$$\Delta z = c_1(x - (z + y)) + c_2(y - (z + x)) - c_3 z + g_z$$

Linear Globocop

$$\Delta x = -a_1 x + a_2(y - (x + C)) + a_3(z - (x + C)) + g_x$$
$$\Delta y = b_1(x - (y + C)) - b_2 y + b_3(z - (y + C)) + g_y$$
$$\Delta z = c_1(x - (z + C)) + c_2(y - (z + C)) - c_3 z + g_z$$

---

18. Although equivalent matrix formulations will be used below, I eschew matrices here for two reasons. First, for the uninitiated, the basic differences between the regimes comes through more clearly with this notation. Second, and more important, the relationships between the linear versions and the corresponding nonlinear variations below will be much clearer in this notation.

There are clearly some differences between these systems and the Eqs. (1) and (2). First, rather than two actors, there are three, the minimum number necessary to examine collective security. Second, these are systems of difference, rather than differential, equations. For notational simplicity, $\Delta x \equiv x_{t+1} - x_t$, and unsubscripted state variables on the right-hand sides represent values at time *t*. From this point on, we will work in discrete time, mainly because major decisions on national armament levels are taken in discrete time (e.g., annually).[19]

Turning now to more substantive issues, notice that the Richardson model is reformulated slightly in that actors respond not simply to adversary military *levels*, as in the Eqs. (1) and (2), but to the *gaps* between their own levels and those of potential adversaries; country x reacts to $(y - x)$ in the first equation rather than to y as before. Here, with Richardson, we "suppose that what moves a government to arm is not the absolute magnitude of other nations' armaments but the difference between its own and theirs."[20]

Under Richardson, when x evaluates the external threat from y, he computes $y - x$. But under perfect collective security, he can count on z's undiluted support. So, he computes $y - (x + z)$. In turn, when x evaluates the potential threat posed by z, he computes $z - x$ under Richardson, but $z - (x + y)$ under perfect collective security. Likewise for all external threat terms in the model. Even this very simple formulation suggests that life under collective security would differ from life under Richardson in nonobvious ways. For instance, put yourself in x's position and ask: am I better off or worse off if y's arsenal grows? In a Richardsonian world, the answer is clear: you are worse off. Under collective security, by contrast, the answer is not clear since y is a threat in one context but is an ally in another (namely, if z attacks). Whether x ultimately rises or falls with an increase in y depends on the quantity $a_2 - a_3$, which might be positive, negative, or zero.

Relative to linear Richardson, however, linear perfect collective security has a systematically depressive effect on the competition. While this result is intuitive, a formal proof (below) will, in fact, lead to counterintuitive results. In particular, it is the *magnitude* of "the collective security effect"--not its sign--which is unexpected, particularly in the nonlinear variants below. But, having found a simple way to import collective security--or reciprocal altruism--into a basic arms race model, let us delay proofs and structural variations for some elementary simulations. Though informal, these can help us develop a "feel" for how collective security may effect dynamics in the perfect linear case.

---

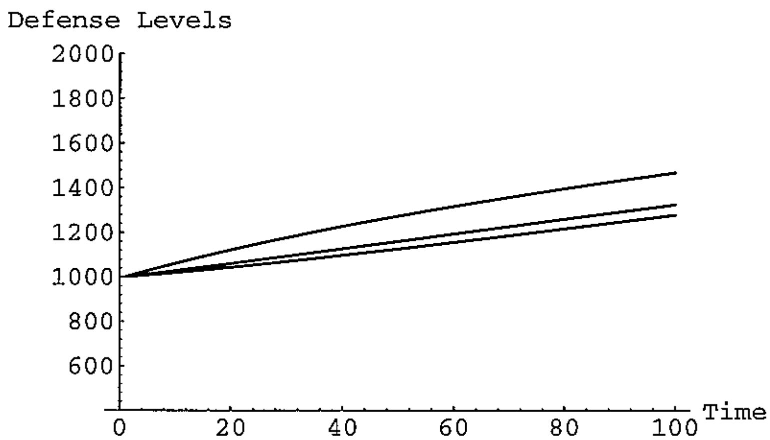
19. In fact, the discrete nonlinear dynamics (below) are considerably richer than the analogous continuous dynamics.

20. Richardson, *Arms and Insecurity*, p. 35.

For illustrative purposes, then, a Base Case simulation of the linear Richardson model is given in figure 3.2. All actors are set at 1000 units initially, and all reaction coefficients, economic fatigue coefficients, and grievances are assumed positive. All assumptions are given in the Appendix.

On those assumptions, the evolution is shown in figure 3.2. The defense levels all increase, leveling off to some equilibrium, $\bar{x} \in \Re^3_+$, given by $\bar{x} = (I - M)^{-1} g$, for appropriately defined M and g.[21]

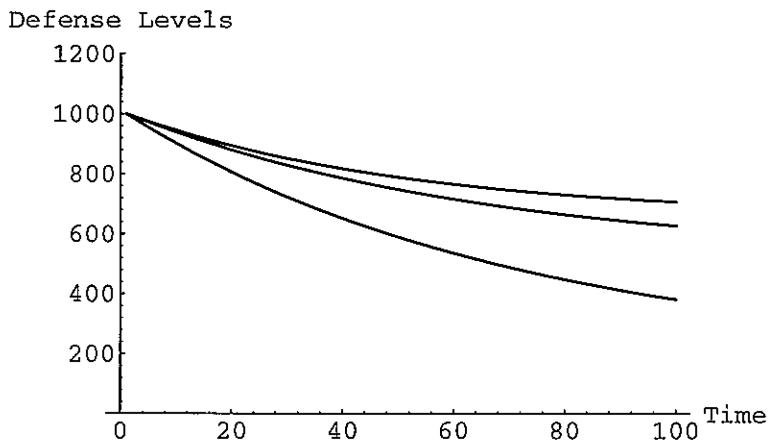
Figure 3.2. Linear Richardson



Now, leaving all initial defense levels and other numbers fixed, what is the effect of the purely *institutional* transition to collective security? How does the change in *rule regime* alter dynamics? Instead of growth, we have decline, as shown in figure 3.3.

[21]. In matrix notation, all three linear models in figure 3.1 share the general form, $x_{t+1} = Mx_t + g$. By definition, an equilibrium, $\bar{x}$, satisfies $\bar{x} = M\bar{x} + g$ so, where it exists (i.e., where $I - M$ is nonsingular), $\bar{x} = (I - M)^{-1} g$.

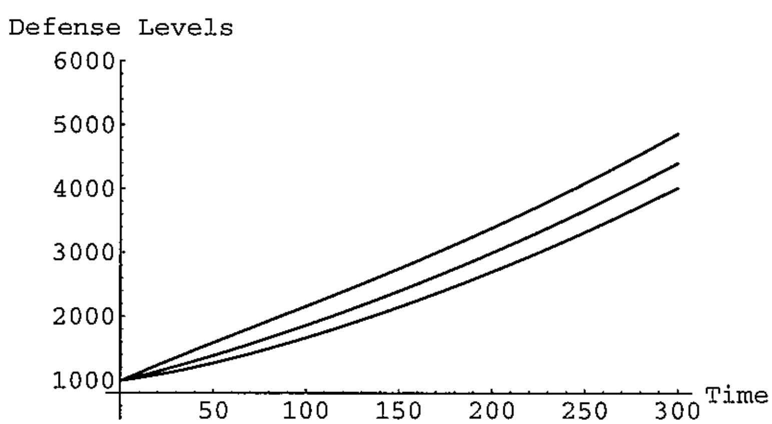
Figure 3.3. Linear Collective Security



In fact, if all coefficients are positive, both systems will attain equilibrium (more on this below).

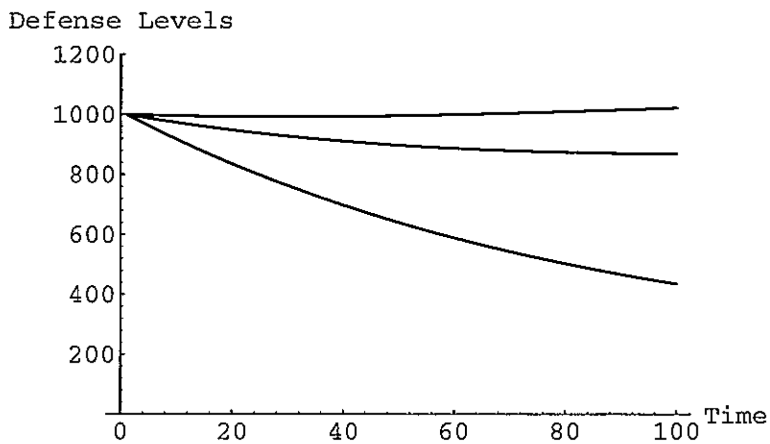
So powerful is the collective security effect, however, that even cases of autocatalytic arms growth (a_1, b_2, c_3 all negative), or "militarism," can be reversed. Figure 3.4 shows an autocatalytic (*negative* economic fatigue) case under Richardson.

Figure 3.4. Richardson Autocatalytic



And, in figure 3.5, we have exactly the same numerical settings, but run under perfect collective security.

Figure 3.5. Collective Security Autocatalytic



These results are systematic. Indeed, if we denote the Richardsonian, collective security, and globocop levels at time t by x_t^R, x_t^{CS}, and x_t^{GC}, then

Theorem. *If* $x_0^R = x_0^{CS} = x_0^{GC} > 0$, *then for all positive* t, $x_t^R > x_t^{CS}$ *and* $x_t^R > x_t^{GC}$. *Here, for* $x, y \in \Re^n$, $x > y$ *iff* $x_i > y_i$ *for all* i.

Proof. Casting the above systems in matrix form, let

$$A = \begin{pmatrix} 1-(a_1+a_2+a_3) & a_2 & a_3 \\ b_1 & 1-(b_1+b_2+b_3) & b_3 \\ c_1 & c_2 & 1-(c_1+c_2+c_3) \end{pmatrix},$$

$$b = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix},$$

$$v = \begin{pmatrix} a_2+a_3 \\ b_1+b_3 \\ c_1+c_2 \end{pmatrix}, \text{ and}$$

$$B = \begin{pmatrix} 0 & a_3 & a_2 \\ b_3 & 0 & b_1 \\ c_2 & c_1 & 0 \end{pmatrix}.$$

For future reference, it is important to note that if $x > 0$, then $Bx > 0$.[22] With these definitions, it is a matter of trivial algebra to show that

$$x_{t+1}^{R} = Ax_t^{R} + b, \tag{4}$$

$$x_{t+1}^{CS} = (A-B)x_t^{CS} + b, \tag{5}$$

$$x_{t+1}^{GC} = Ax_t^{GC} + (b - Cv). \tag{6}$$

If $x_0^R = x_0^{GC}$, then globocop (6) is simply Richardson (4) with grievances reduced by the constant vector $Cv > 0$; the effect is obviously depressive. To prove that collective security is strictly depressive, we establish a simple lemma.

Lemma. If, for some time $\hat{t}$, $x_{\hat{t}}^R > x_{\hat{t}}^{CS}$, then $x_t^R > x_t^{CS}$ for all $t > \hat{t}$.

Proof.

$x_{\hat{t}+1}^{R}$	$= Ax_{\hat{t}}^{R} + b$	by (4)
	$> (A-B)x_{\hat{t}}^{R} + b$	since $Bx_{\hat{t}}^R > 0$
	$> (A-B)x_{\hat{t}}^{CS} + b$	since $x_{\hat{t}}^R > x_{\hat{t}}^{CS}$ by hypothesis
	$= x_{\hat{t}+1}^{CS}$	by (5).

By this Lemma, a proof that $x_t^R > x_t^{CS}$ for all positive *t* will be in hand once we show that $x_1^R > x_1^{CS}$. But this is simple. Since $x_0^R = x_0^{CS}$, call them both $x_0 > 0$. Then,

$$x_1^R - x_1^{CS} = \left[Ax_0 + b\right] - \left[(A-B)x_0 + b\right] = Bx_0 > 0,$$

and we are through. ¤

Now, as noted earlier, these results obtain so long as $Bx > 0$ for $x > 0$. The *specific* B matrix above can be altered considerably while leaving the strictly depressive effect of collective security intact. Connectionist terminology will prove natural for discussing imperfect collective security.

The Connectionism of Collective Security

This perspective emerges from closer scrutiny of the B matrix. The *ij*th entry, b_{ij}, represents the level of altruism that party j shows party i. If we let the symbol "$x \to y$" represent

---

22. To ensure physical realism ($x > 0$), we must stipulate that $a_1 + a_2 + a_3 < 2$, $b_1 + b_2 + b_3 < 2$, and $c_1 + c_2 + c_3 < 2$. I thank Jean-Pierre Langlois for this observation.

the altruism x shows y (i.e., it is $b_{21} > 0$), then, conceptually, the B matrix is

$$B = \begin{pmatrix} 0 & y \to x & z \to x \\ x \to y & 0 & z \to y \\ x \to z & y \to z & 0 \end{pmatrix}.$$

Graphically, this would correspond to the "altruism web" shown in figure 3.6. Pairwise, all altruism is reciprocated; arrows run in both directions. If z attacks x, y allocates force to x and vice versa if z attacks y, and so on. When this is the case, we will say that the collective security system is *maximally connected.*

Figure 3.6. Maximally Connected Altruism Web

$$\begin{array}{ccc} & x & \\ \swarrow\!\nearrow & & \nwarrow\!\searrow \\ y & \leftrightarrows & z \end{array}$$

All off-diagonal elements of the B-matrix are strictly positive; the sign structure is then

$$B = \begin{pmatrix} 0 & + & + \\ + & 0 & + \\ + & + & 0 \end{pmatrix}.$$

The *strength* of any connection (in contrast to the connection *pattern*) is the real number, b_{ij}, which can assume values in $[0,1]$. So, in these terms, perfect collective security entails maximum connectivity and maximum connection strength. In turn, imperfect collective security regimes result from reductions in connectivity, reductions in connection strength, or both.

Maximal Connectivity with Diluted Strength

It is obvious that, if $Bx > 0$ then $\gamma Bx > 0$ for any real $\gamma \in (0,1)$. This is the most transparent case of imperfect or "diluted" collective security. Maximal connectivity--reciprocal altruism--prevails, but, instead of sending *all* of one's forces to the aid of the attacked party, one sends a fraction γ. In fact, the strictly depressive effect is preserved if every off-diagonal entry in the B-matrix is a different $\gamma_{ij} \in (0,1)$. Everyone is better off even if the reciprocal altruism is, in this sense, discriminatory.

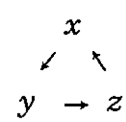
Minimal Connectivity with Diluted Strength

More intriguing, however, the altruism need *not* be reciprocal to leave all parties strictly better off. Specifically, altruism matrices far more sparse than B will fulfill our strictly depressive requirement, $Bx > 0$. Indeed, it is necessary only that each row contain a *single* positive entry. So, for instance, any matrix with the following sign structure will do.

$$B = \begin{pmatrix} 0 & 0 & + \\ + & 0 & 0 \\ 0 & + & 0 \end{pmatrix}.$$

Graphically, this would correspond to the cyclic "altruism web" in figure 3.7.

Figure 3.7. Cyclic Altruism Web



Here, x is unilaterally altruistic to y; y is unilaterally altruistic to z; and z is unilaterally altruistic to x. Everyone is better off, but there is *no reciprocal altruism*. Instead of "you scratch my back and I'll scratch yours," the appeal is "you scratch my back, and I'll scratch Sam's, and Sam will scratch yours." I call this "cyclic altruism."[23] The direction of the cycle is reversed if B has the sign pattern shown below.

$$B = \begin{pmatrix} 0 & + & 0 \\ 0 & 0 & + \\ + & 0 & 0 \end{pmatrix}.$$

It is, in fact, not necessary that these altruism graphs, or "webs," be closed. For instance, any B matrix with the following sign pattern will satisfy our strictly depressive, $Bx > 0$, requirement.

$$B = \begin{pmatrix} 0 & + & 0 \\ + & 0 & 0 \\ 0 & + & 0 \end{pmatrix}.$$

---

23. Obviously, this is a form of diluted altruism, with some γ_{ij}'s equal to zero. But, as it has a different flavor and, because the *position* of the zeros matters, I give it a separate name.

But, its graph is open, as shown in figure 3.8.

Figure 3.8. Open Altruistic Web

$$\begin{array}{ccc} & x & \\ & \swarrow\nearrow & \\ y & \rightarrow & z \end{array}$$

Everyone is strictly better off if x and y are reciprocal altruists and y is unilaterally altruistic to z, even if z is altruistic to no one!

In summary, for the linear models above, there are basically two senses in which collective security can be imperfect and still exert a strictly depressive effect on dynamics. Altruism can be perfectly reciprocal but diluted in strength. It can also be imperfectly reciprocal (as in figure 3.8), even unreciprocated (as in figure 3.7). As we will see, it may in fact be *both* highly diluted and imperfectly reciprocal and still have a profoundly depressive effect. It is in precisely the systems that concern us most--the volatile systems--that such highly imperfect collective security regimes can have dramatic stabilizing effects. Such dynamics, however, really arise only in *nonlinear* systems. Let us turn to these variants.

Part II. Nonlinear Models

Nonlinearities may enter the model in numerous ways. One way is through the balance assessment, or external threat, terms. Modern military establishments do not measure military balances--external threats--by simple subtractions of the form $y-x$, as in the above models. Rather, they often use methods that, at some level or other, embed mutual attrition models implying that *net military advantage is a difference of levels raised to powers*. Where does this come from? Basically, from the attrition stalemate conditions of generalized Lanchester equations. Allow me to derive this quickly.

Generalized Attrition Stalemate

Let R(t) and B(t) be Red and Blue forces at time t, and let r and b (real numbers between zero and one) represent their effectiveness per unit. With constants c_1 through c_4 $(0 \le c \le 1)$, the most general Lanchester attrition system is