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New Physics of Metals: Fermi Surfaces Without Fermi Liquids

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ABSTRACT

We relate the historic successes, and present difficulties, of the renormalized quasi-particle theory of metals. ("AGD" or "Fermi Liquid Theory"). We then describe the best-understood example of a non-Fermi liquid (NFL), the normal metallic state of the cuprate superconductors.

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For some 40 years almost all electronic phenomena in metals have been interpreted in terms of a general theoretical framework which one could variously call renormalized free particle theory, Fermi liquid theory, or "AGD" after the best-known book on the subject.[1]

I came to the conclusion a few years ago that this theory is, in very many of the most interesting cases, basically a failure. For the first 20 years of its history, until the mid-70's, it served us very well; but then as we began to focus on the most interesting (or the most anomalous) cases more and more of the copious literature of our subject came to be engaged in fitting the proverbial square peg into a round hole. It is not that there are no instances which fit the framework, but that, contrary to the claims for universality which are been made for it, it seems that for systems with strong interactions, it often is completely misguiding.

In order to make my point I must first describe the nature of this conventional theory. It arose in the '50's, just after the successes of the Schwinger-Feynman-Dyson theory in quantum electrodynamics, and it borrows the techniques which were so successful in that theory. In quantum electrodynamics, the scheme was to map the properties of the real "physical" vacuum and the real physical particle excitations onto the corresponding entities of a supposed "bare" vacuum with "bare" particles, by the process of renormalization. One defines a "propagator" or Green's function, $G(r-r', t-t')$, which is the amplitude for finding a particle at point r and time t if it was inserted at point r' and time t' into the real vacuum. The particle can encounter various interactions with vacuum fluctuations on the way, which are sorted out into a series using Feynman diagrams. If this series is well-behaved, its sum can be written in terms of a "self-energy" which merely renormalizes the unperturbed propagator without changing its essential character.

In the condensed matter physics of metals there is no vacuum, but there is a Fermi sea if the electrons are non-interacting. This is treated formally as a vacuum in which both hole and particle excitations can propagate, in parallel to the treatment in QED of the Dirac sea of negative-energy electrons as a vacuum for positrons. There is a surface in p-space of zero energy, the Fermi surface. The unperturbed Green's function (Fourier transformed into momentum (p) and energy (ω) space) is

$$G(\omega, p) = \frac{1}{\omega - (\epsilon_p - \mu)}$$

where ϵ_p is the single-particle band energy, and μ is the chemical potential E_F. Positive ω refers to electron-like propagators, negative (backwards-moving) ω to holes. The Feynman diagram series can, if convergent, be resummed in terms of a "self-energy" which is the sum of all "self-energy parts" and appears in the exact Green's functions' denominator:

$$G = \frac{1}{G_0^{-1} - \Sigma} = \frac{1}{\omega - (\epsilon_p - \mu) - \Sigma(\omega, p)}$$

The assumption is that Σ is sufficiently regular that the only singularities of G are poles at a modified p-dependent energy $E_p - \mu$ of strength $0 \leq Z_p = \frac{1}{1 - \frac{\partial \Sigma}{\partial \omega}} \leq 1$

$$G = \frac{Z_p}{\omega - (E_p - \mu)} + \text{incoherent part}$$

These poles are the renormalized quasiparticles.

This theory was made useful and meaningful by a series of theorems proved in the late '50's, which depend on the idea that quasiparticles at E_F do not decay, because the exclusion principle blocks off all states into which they can decay, to order $\omega^2 = (E_p - E_F)^2$.

Migdal : If Z is finite there is a jump at p_F in n_k of magnitude Z: there is a real, measurable Fermi surface.

Landau : The dynamics can be completely described at low energies by the quasiparticles, except for a small finite number of collective modes near $q = 0$. (The Fermi Liquid Theory).

Luttinger : The Fermi surface contains a number of p-states exactly equal to the number of electrons.

Finally, Migdal again: Phonons (lattice vibrations) can be added in simply to the theory including only the lowest-order diagrams, (the buzzword is "neglect vertex corrections") because the ion's mass is much heavier than the electron's.

The very elegant final form of the theory, though invented by three groups simultaneously, is expressed in the book by "AGD"[1]. Its greatest achievement almost coincided with its birth: it turned out to require only a formally trivial (if conceptually profound) redefinition of the "vacuum" and the theory as revised by Schrieffer, Nambu and Eliashberg elegantly encompassed BCS superconductivity.[2] By 1965 Schrieffer, I, and later W.L. McMillan, working with the beautiful experiments of Giaever and Rowell, had made the theory quantitative, dealing with the real complexities of real materials so efficiently that the superconducting T_c of metallic elements like Pb, Hg and Al may be the best predicted of all condensed matter phase transitions.[3] Triumphs in such fields as "Fermiology", the measurement of complex Fermi surfaces of real metals, led us to feel that the problem of the electron liquid in metals was finished in principle, with only quantitative or marginal problems left, some of the simpler of which were solved in the late '60's and early '70's—like magnetic impurities in metals, the so-called "Anderson model" which led to the "Kondo effect", which turned out to be the Fermi Liquid in a new guise. Finally, our confidence was bolstered by understanding much about the superfluidity in ^3He, the original "Fermi liquid" referred to by Landau, as a consequence of Landau's theory supplemented by the spin fluctuation theory of Schrieffer and Doniach, in '73–'74.[4]

Two more developments contributed to the general sense of accomplishment of these years. First, there was the development of many useful and accurate experimental probes such as tunneling spectroscopy, photoemission with spectacularly enhanced resolution and other similar high-energy probes, etc. Second was the development of methods of electronic energy band and energy level calculations which were extraordinarily successful and accurate for semiconductors and ordinary metals, so that an "electronic structure" even for a complex material could be calculated, though often little attention was paid to its experimental reality, if any.

It was, ironically, in the triumphant field of superconductivity that this beautifully clear picture began to waver and lose focus. Superconductors were finding more and more technological uses starting from the discovery of high-field superconductivity. But the superconductors of practical value, with high critical fields and T_c's between 15 and 25° K, were not simple metals but outlandish intermetallic compounds of transition metals with formulas like V_3Si, Nb_3Sn or Ge, $Pb(Mo_6S_8)$,[5] etc. B.T. Matthias, the paladin of the field, taunted theorists with their inability to understand these more complex and interesting metals, which came to be called the "bad actor" superconductors.[6] In the same period of the '70's and -80's, Matthias and his experimental friends and collaborators in the world of exotic materials, for instance T.H. Geballe, devised or brought under study a number of metals which tested the limits of the theory of metals in various other ways: two-dimensional layer materials such as the "dichalcogenides" $NbSe_2$ and TaS_2,[7] quasi-one-dimensional chain metals such as $NbSe_3$ and the tungsten bronzes,[8] "mixed valence" metals where electrons from the inner f-shells of the rare earths and actinides break out of their shells, at least at low temperatures, and hybridize with Fermi sea electrons; and the "organic" superconductors or metals such as the TCNQ compounds, or the Bechgaard salts, where stacks of aromatic molecules form metallic chains or layers.[9] There had also been considerable interest in metals with metal-insulator transitions, such as the metallic oxides of vanadium and titanium.[10] The variety of nature is inexhaustible, but this list will do.

All of these materials are, for one reason or another, cases in which the interactions between electrons in the metal are particularly strong, effective, or both. There came into existence a field of physics which specialized in these "strongly-interacting electrons", of which I was, God help me, a happy and active participant throughout the '70's and '80's. Like all of my colleagues in the field, I assumed that eventually some clever reworking of the time-worn diagrammatic technique would solve every problem: I was, as I have come to realize, "brainwashed by Feynman" into believing that these diagrammatic, perturbative, particle-based techniques were all of physics. (Not implying with this slogan anything negative about Feynman himself: he was the most flexible of theorists). It was only with the discovery of the "high-T_c" cuprate superconductors in 1986–7[11] that I began to realize that for almost 20 years this type of theory had not had a single unequivocal success, and to speculate that the reason might lie not in our lack of skill or in the complexity of the physics of these new materials, but in a fundamental breakdown of the canonical theory: new ideas and concepts were needed. The one great success of the past decade reinforces this point: the quantum Hall effect is the case par excellence in which perturbation techniques are not used at all and the entire system is dominated by impurities (in the integer effect) and interactions (in the fractional one.) In the latter case one finds elementary excitations completely unlike renormalized free electrons, having, for instance, fractional charge and statistics.

Let me describe a few of the anomalies exhibited by these materials, before settling on the cuprates as, actually, the simplest and most unequivocal case of a non-Fermi liquid metal (NFL, an abbreviation we shall use often). One may count no less than 5 classes of superconductors which do not resemble the classic BCS, elemental metals. The characteristics of the "BCS" class are easily understood in terms of the dynamic screening theory developed in the early '60's: (1) they are polyelectronic metals with large Fermi surfaces. Matthias[6] developed a set of empirical correlations of free electron density with T_c which work very well and which make mechanistic sense. (2) They are non-magnetic: magnetism anticorrelates with T_c, and magnetic impurities are deadly to T_c. This is easily understandable: magnetism usually results from dominance by the repulsive Coulomb interactions between electrons as opposed to the attraction caused by phonon-electron coupling. (3) They are good conductors, well below the Mott limit of $\ell/\lambda_{\text{de Broglie}} \simeq 1$. (4) They tend to have stable, symmetrical structures. (5) T_c is limited to a fraction of the lattice vibration energy θ. $T_c \leq 1/3 - 1/4\theta_D$. In no particular order, I list the new classes of superconductors which have been observed in the past decade or two.

(1) The "organic" superconductors BEDT, Bechgaard salts, etc.[9] These are layer-or chain-like arrays of stacked, charged aromatic molecules. (An early suggestion by Little

motivated their discovery but has no predictive or explanatory relevance.) They violate several of the rules: superconductivity is closely associated with antiferromagnetic insulating phases as well as with various other rather confusing magnetic phase transitions, and the electron density is very low (less than 1 per large molecule). No plausible suggestion as to a mechanism for the T_c's, which range up to 12°K, has been advanced, but the resemblance to the cuprates in their association with magnetic insulators, and in their low-dimensional, anisotropic structures suggests that the mechanism may be the same.

(2) The heavy-electron superconductors.[12] These are mixed valence metals such as UBe_{13}, $CeCu_2Si_2$, UPt_3, with low T_c's ($\lesssim 1°K$) but very high electron specific heats so that the total entropy of condensation can be thousands of times that in conventional metals. The superconducting electrons come from an f band no more than .01 ev wide or less, which is magnetic in the room-temperature state. Most of these have magnetic spin density wave phase transitions closely associated with superconductivity and affecting electrons from the same bands. No mechanism for superconductivity has been suggested, but it has been plausibly proposed on experimental grounds that they are not isotropic s-wave and are therefore not phonon driven. Much investigation of all kinds of transport anomalies, magnetic phase transitions, and other anomalies continues in this field.

(3) The "layer" superconductors $NbSe_2$, TaS_2 etc.[7] Here the anomaly is not only the low electron-density and unusual structures, but particularly the association with charge density wave distortions which are not plausibly explained on the basis of "nesting Fermi surfaces" which give anomalous responses at the spanning vectors. Such nesting Fermi surfaces should cause phase transitions at a temperature scale comparable with the Fermi energy $\sim$ 1–2 ev, not well below room temperature ($<$ .01ev).

(4) "Cluster compounds, a vaguely defined category including C_{60}^{---},[13] "Chevrels", i.e., $(X)\ Mo_6S_8$,[14] $Ba-K\ BiO_3$[15] and similar materials, and among others perhaps the A_{15}'s. All have moderately high (15–40°K) T_c's. All of these seem plausibly motivated by phonons but have various puzzling anomalies indicating that straightforward theories

don't apply. The Chevrels, for instance, are almost immune to magnetic constituents. The Bismuthates have highest T_c when near a metal-insulator transition. The electron density of C_{60}^{---} is low, its bands very narrow; $K_4C_{60}^{4-}$ is an insulator for no obvious reason. The A_{15}'s undergo mysterious low-temperature density wave transitions.

(5) Finally, there are by now some two to three dozen chemically distinct cuprates with T_c's ranging up to 150°K, which I will discuss shortly.

I have focussed on superconductors mainly because that is such a striking and easily measured electronic property, but in all the above cases there are other anomalies which are often even farther from theoretical explanation. I am not claiming that I know an explanation for all of these anomalies, I am rather trying to express the sense of almost complete incapacity, of what was supposed to be a complete and perfectly general theory, to deal with any of the problems being posed by the experimentalists. This does not mean that there is not a massive theoretical literature, but this seemed not to deal with the real world of experiment but with artificial models, too far from reality to be relevant. Another subculture seemed content to calculate electronic energy bands without any examination of whether they are relevant to the real materials—as for instance, a full three-dimensional Fermi surface was claimed to have been obtained for cuprate materials which were known to have no metallic conduction along one direction in space. Most disturbing was the experimentalists' claim to have verified this Fermi surface experimentally, when a cursory look at their data convinced me that the correlation between experiment and theory was no better than with a randomly chosen band structure, possibly not even constrained to have the right number of electrons. Basically, false confidence in the validity of renormalized quasiparticle theory is delaying progress in this field, not enhancing it.

Let me now turn to a brief discussion of the cuprates as the best example for the failure of the old theory, and as a case in which the outlines of a new theory are clear.

First let me set up the basic phenomenology. Few will not be familiar with the typical structures of these materials (Fig. 1), but not as many will appreciate the oversimplified, but correct, theorists' picture of them. (Fig. 2) Mobile electrons live (for practical purposes) only in the $CuO_2^{(--(+x))}$ layers, between which are essentially inert layers of "stuff" which carries out two function: (1) "doping": neutralizing the charge of the $Cu^{+(2+x)}O_2^{(-2)}$ layers, i.e., a charge reservoir function; (2) providing a transmission medium, more or less effective, for quantum hopping of electrons between the CuO_2 layers.

The CuO_2 layers are remarkably stable as compared to the flexible structure and stoichiometry of the "stuff". They are in every case of a square planar structure (Fig. 3) which may be slightly deformed but never in such a way as to modify the basic energy level structure seriously. Clearly the square planar bond of Cu to its four O neighbors is the strongest structural element in the problem.

There is an almost universal "generalized phase diagram" describing the materials, of which not all pieces have been found for all compounds, but no contradictory data exist. This is plotted in the temperature-"doping percentage" plane. (Fig. 4) Doping percentage δ is the difference of the numerical Cu valence from 2+. In general δ is positive, but for one or two unique cases it is negative, though these are not as clear -cut as the more common $Cu^{2+\delta}$.

The striking facts are (1) the narrow region of $\delta = 0 \pm 1 - 2\%$ of stable, insulating (Mott) antiferromagnetism. J is large and does not vary by more than 20%.

(2) A transition region characterized by defects and/or instabilities of $1\% < \delta <\sim 10\%$, where the material is a poor conductor with low (if any) T_c, or an insulator. (3) A region of considerable stability $\delta \sim 10-30\%$ (the material often self-dopes—adjusts its stoichiometry to a concentration within this range) which is optimum for superconductivity, though T_c may vary from 10–150°K. There are characteristic anomalous behaviors in this state; I am indebted to N.P. Ong for the concept that the normal metal in this range is in some sense an "ideal" 2-dimensional metallic system, from which deviations are seen as one varies the doping from the ideal range.

Finally, for $\delta >\sim 30\%$ the material becomes a somewhat more conventional metal, and T_c drops rapidly to zero.

The existence of the square planar structure and the narrow region of antiferromagnetic phase suggests, and much additional data confirms, that the basic electronic structure is determined by a very simple, one-band, two-dimensional Hubbard Model. The orbital of the d-shell of Cu which interacts most strongly with the oxygen neighbors is the $d_{x^2-y^2}$ orbital, which will hybridize strongly with the "p_σ" orbitals on the O^{--}. The bonding linear combination is deep (4–6ev) below the Fermi level, while the antibonding linear combination is pushed up above the other d levels and forms the basic function for the single partially occupied band, which can be adequately represented with only two parameters, t and t', the nearest-and next-nearest neighbor hopping integrals.

$$H_{1-\text{electron}} = \sum_{i,j,\sigma} t_{ij}\, c^{+}_{i\sigma}\, c_{j\sigma}$$

To this energy the Hubbard model adds only one extra parameter, the repulsive energy U which prevents double occupancy of a site:

$$\mathcal{H} = \sum_{ij\sigma} t_{ij}\, c^{+}_{i\sigma}\, c_{j\sigma} + U \sum_{i} n_{i\uparrow}\, n_{i\downarrow}$$

We will find that superconductivity involves one other parameter, at least, the hopping integral between planes, which will have a complex and variable structure depending on the "stuff"; we will refer to it in terms of a hopping term

$$\sum_{n\neq n'} \sum_{k} t_{\perp n,n'}(k)\, c^{+}_{k\sigma}(n)\, c_{k\sigma}(n')$$

where n refers to the various planes, and $c_{k\sigma}(n)$ are the electron operators in momentum space. But it is known that $t_\perp$ is an order of magnitude smaller than t or t', and in fact it plays no role in the ideal 2D region. Superconductivity also brings in "residual interactions" in the Landau Fermi liquid sense, which affect T_c slightly and the symmetry of the gap a great deal; but this is not our concern here. In the true sense these are "irrelevant" parameters in the normal state.

The central qualitative fact about this ideal metallic phase is that it has two independent energy scales: a region of energies and temperatures > about 50–150°K (region "A") and a region below this, "B", where superconductivity occurs—and occasionally other phenomena such as the "spin gap". Region A is characterized by (1) Power law transport and electromagnetic properties ("scale-free", i.e., with only one scale); (2) 2-dimensionality; (3) rough quantitative universality for all cuprate planes. Region B has a widely variable scale—T_c—and superconductivity is clearly 3-dimensional, nor are the superconducting properties very universal.

To show the contrast between the two scales, I borrow a graph originated by B. Batlogg[16] and updated for me by Ong. In Fig. 5 I plot a characteristic measurement on the 2D metallic planes, the temperature coefficient of planar resistance $\frac{d\rho_{ab}}{dT}$, against T_c, the superconducting transition temperature. The two parameters seem to be absolutely independent of each other. Many theorists persist in assuming that the properties of the planes determine T_c, but such theories (anyons, gauge theories, spin fluctuation theories) have little relation to reality, if they cannot explain this independence.

The most striking power law of the higher energy region is the conductivity itself. Fig. 6 shows how strikingly linear it is for a pure single crystal of YBCO, while Fig. 7 shows that over a very broad range of energies, from $\sim 100\text{cm}^{-1}$ to nearly 10^4 cm^{-1}, the complex conductivity is proportional to $(i\omega)^{-1+2\alpha}$.[17] This observation also has an implication about the type of theory which is relevant. By many authors the Hubbard model is transformed, by a canonical transformation valid at low energies, into the "t-J" model, which introduces an exchange parameter $J \sim \frac{t^2}{U}$ of order 500–1000 cm^{-1}. This artificial low energy scale is indeed the correct one for the spin degrees of freedom of the insulating antiferromagnet, where no particle motion is possible; but in the metal there is no sign of it and the physics is uniform over a much wider range of energy. J is introduced artificially by projecting the kinetic energy term on that of an infinite U model, and has the effect of correcting the resulting errors. The effective U in the metal is not as large as in the insulator and the transformation to infinite U is a poor approximation except at very low energy. Unfortunately, most attempts at gauge theories have started out from $t - J$ rather than Hubbard physics and are not relevant, as Fig. 7 demonstrates.

The striking power law which is even more universally observed than the "linear" resistivity is the T^{-2} power law of the Hall angle $\theta_H = \omega_c \tau_H$ (Fig. 8).[18] This strange and beautiful behavior shows unequivocally that electrical conduction is a composite process carried out by a complex entity not simple quasiparticles. That it is uniform around the Fermi surface is shown by the T^{-4}-dependence of magnetic resistance (Fig. 9) which is the variance of the Hall angle. τ_H is a qualitatively <u>different</u> quantity from τ_c.

A third "power" law, in a sense, is the <u>non</u>-existence of metallic conductivity along the c-axis, in the presence of large conductivity in the ab plane. This is strikingly shown in infrared reflectivity measurements (Fig. 10) for c-polarized radiation; the crystals of $(La - Sr)_2CuO_4$ reflect like lossy insulators ($R \sim .5$) above T_c, but are good superconductors ($R \equiv 1$) below T_c. The power we predict is $\sigma_c(\omega) \propto \omega^{2\alpha}$ ($2\alpha \simeq 1/4$) and some measurements (Fig. 11) suggest we are right. Note that $\sigma \rightarrow 0$ as $\omega \rightarrow 0$, at least at $T = 0$.

What kind of theory can we use to understand this anomalous behavior? The quasiparticle theory fails generically in one dimension, where in fact there exists an exact solution of the Hubbard model (among others) in 1D, by Lieb and Wu,[19] as well as a considerable tool bag of techniques for one-dimensional electronic models. Initially I simply began to use these solutions as a "template" for the NFL case, but as time went on both experimental and theoretical reasoning led me to realize that the basic features reappear in 2D at least.

Let me describe the features of the Lieb-Wu solution. The key to this solution, as pointed out by Haldane,[20] is that it is a "Luttinger Liquid", in the sense that it is not a Fermi liquid but that it has a Fermi surface satisfying Luttinger's theorem, in that excitation energies go to zero at a surface in momentum space. But Migdal's construction does not work: $Z = 0$, so there are no quasiparticles and no jump in n_k at the Fermi surface.

If there are no quasiparticles, what is there? We find two kinds of excitations, which in physical situations must be created in pairs but are independent once made (see Fig. 12). There are charge excitations called "holons" which have charge e, no spin, and carry momenta near $2k_F$, and have a velocity v_c: their dispersion curve looks like that of an electron, crossing through zero at $2k_F$, and there are holons and antiholons.

There are spin excitations called spinons which have no charge, S=1/2, velocity v_s, and which go to zero at k_F, but do not extend through the zero: there are no antispinons; these behave like Majorana Fermions. If we try to make an antispinon we create a spinon at a different momentum. Exactly the same things were found in Bethe's 1931 solution of the Heisenberg model. Haldane and students have exhaustively studied their properties.

When we add or take away an electron we must always make at least one of each: the electron decays very rapidly into a spectrum, and is not a stable particle: this we take to be the definition of NFL. The spectrum of electron-like states near the Fermi surface at k_F is shown in Fig. 13. The Green's function for the electron is:[21] (we give that for free electrons for comparison)

$$\text{F.L.} \quad : \quad G \simeq e^{ik_F x} \, \frac{1}{x - vt}$$

$$\text{NFL} \quad : \quad G \simeq e^{ik_F x} \, \frac{1}{(x - v_s t)^{1/2} \, (x - v_c t)^{1/2} \, (x^2 - v_c^2 t^2)^{\alpha/2}}$$

We see in Fig 13 that the spectrum's breadth is $\propto \omega$. The three parts of G are: (1) the spin moving at velocity v_s; (2) the charge at velocity v_c; (3) a backflow due to the electron's modification of all the other electrons' wave functions as it passes through the sea of opposite spins. $\alpha \simeq 1/8$; though small, it is this part which causes the Fermi surface to smear.

What we find theoretically and experimentally is that the 2D system, at least with strong interaction, is just a tomographic superposition of effective 1D models, one for each point on the Fermi surface. It was discovered by Luther,[22] and recently enlarged on by several authors, that this is the case for Fermi liquid theory, because of the exclusion principle's restriction to only forward ("non-diffractive") scattering which is exploited in Landau's theory. What I have discovered is that the same applies to NFL as well.

As Haldane points out,[23] the Fermi surface can be thought of as the order parameter of a critical point at $T = 0$, the excitations as fluctuations of this order parameter, and the power laws as valid throughout the neighbourhood of this critical point (whether they be F.L. or NFL.) What happens as we approach this critical point is that suddenly the interlayer interactions grow to relevance and cause superconductivity; but that part of the theory—region "B"—is not our subject here.

The power laws we observe are straightforwardly explained within our NFL theory: (1) the linear dependence of $1/\tau_{cond}$ on T or ω. This is the decay of the accelerated electron into spinon and holon. This is not a resistivity process unless something else is added: impurity or phonon scattering of the holons (spinons are scattered only by magnetic, time-reverse breaking, impurities). This is exactly analogous to phonon resistivity: under "phonon drag" conditions, phonon scattering is not a resistivity process, but phonons are, normally, scattered and if they are they control resistivity. So we call this the "holon non-drag" regime. Most Luttinger liquids are in the holon drag regime which is quite different. The slight deviation of the power from 1 is also predicted by the theory. (2) Hall angle. A magnetic field only rotates the Fermi surface, so it does not affect the electron-spinon-holon process and does not cause any additional electron decay. Reciprocally, electron decay cannot affect the Hall angle. Thus the underlying spinon-spinon scattering which, like electron-electron processes, is $\propto T^2$, is the Hall angle controlling process. The current is mostly carried as a backflow by spinons, not by the holon "charge" carriers, and in fact the holon current will not be colinear with that of the spinons.

Finally, the absence of c-axis conductivity is an effect of $Z = 0$: the direct, coherent hopping is ineffectual, and in general hops will take place incoherently to high-energy states.[24] The proof is a bit subtle but the principle (and the fact) is obvious.

There is no reason to suppose that the NFL phenomenon is restricted to cuprates, and in fact infrared spectra resembling Fig. 7 are common to many of the weird materials mentioned earlier. Most of the anomalies have possible explanations; for instance, the density-wave responses of the Luttinger liquid are much more singular than those of the Fermi liquid. What is important is to have one case firmly tied down.

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