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H. Monien
P. Monthous
D. Pines

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On the Application of Antiferromagnetic Fermi Liquid Theory to NMR Experiments in La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\)

H. Monien
P. Monthoux†
and
D. Pines*

Department of Physics and Materials Research Laboratory,
and the Science and Technology Center for Superconductivity,
University of Illinois at Urbana-Champaign
1110 West Green Street
Urbana, IL 61801

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Abstract

NMR experiments on La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) by Kitaoka et al. and Imai et al. are analyzed using the phenomenological antiferromagnetic Fermi theory of Millis, Monien, and Pines, and the results are compared with those previously obtained for YBa\(_2\)Cu\(_3\)O\(_7\) and YBa\(_2\)Cu\(_3\)O\(_6\).\(_{63}\). A one-component model, with hyperfine couplings which are unchanged from those found previously for YBa\(_2\)Cu\(_3\)O\(_7\) and YBa\(_2\)Cu\(_3\)O\(_6\).\(_{63}\), and parameters obtained from experiment, provides a quantitative fit to the data. At all temperatures the antiferromagnetic correlations found in La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) are stronger than those found for the YBCO samples with the result that the characteristic energy for the antiferromagnetic paramagnons which describe the AF spin dynamics is quite

†Address until August, 1990: MS K765, Los Alamos National Laboratory, Los Alamos, NM 87545
*On leave at Santa Fe Institute, 1120 Canyon Rd., Santa Fe, NM 87501 until June 1, 1990.
low (<kT and ~ 20K at T₀). We use the deduced paramagnon energies to calculate the contribution to the electrical resistivity from quasiparticle-antiferromagnetic paramagnon scattering for La₁.₈₅Sr₀.₁₅CuO₄, YBa₂Cu₃O₇, and YBa₂Cu₃O₆.₆₃, and find that it displays a linear temperature dependence for all three materials. Our results support the proposal that the properties of a nearly antiferromagnetic Fermi liquid are genuinely novel, and suggest that both the spin and charge aspects of the normal state properties of the cuprate oxide superconductors can be quantitatively explained in terms of quasiparticles coupled to antiferromagnetic paramagnons whose characteristic energy scale is < kT.
I. Introduction

The phenomenological antiferromagnetic Fermi liquid theory developed by Millis, Monien and Pines [1], hereafter MMP, has been shown to provide a quantitative account of nuclear magnetic resonance relaxation rates for Cu(2), O(2), O(3), and Y nuclei in both YBa$_2$Cu$_3$O$_7$ [1] and YBa$_2$Cu$_3$O$_{6.63}$ [2]. It is natural to inquire whether this one component theory is equally capable of describing the results of NMR experiments on other cuprate oxide superconductors. In the present paper we use the theory to analyze the NMR experiments of Kitaoka et al. [3] and Imai et al. [4] on one member of the LSCO family, La$_{1.85}$Sr$_{0.15}$CuO$_4$. We find that a one-component model with hyperfine couplings of the $^{63}$Cu [2] nuclei and $^{17}$O(2) nuclei which are unchanged from those found previously for YBa$_2$Cu$_3$O$_7$ (1) and YBa$_2$Cu$_3$O$_{6.63}$, (2) provides a quantitative fit to the experimental data. As is the case for YBa$_2$Cu$_3$O$_{6.63}$, the temperature dependence of the $^{63}$Cu relaxation rate reflects in part strong, temperature-dependent antiferromagnetic correlations between the Cu$^{2+}$ spins, and in part a temperature dependent long wavelength static spin susceptibility. We determine the latter from the measurement of the $^{17}$O Knight shift [3]. When this temperature dependence is taken into account, at all temperatures the antiferromagnetic correlations found in La$_{1.85}$Sr$_{0.15}$CuO$_4$ are stronger than those found for the YBCO samples; moreover over much of the measured temperature range the antiferromagnetic correlation length is found to be of the mean field form established earlier for YBa$_2$Cu$_3$O$_7$ and YB$_2$2Cu$_3$O$_{6.63}$.

In La$_{1.85}$Sr$_{0.15}$CuO$_4$ the tendency toward antiferromagnetism is, in fact, so strong that for temperatures above 130K, the system behaves as though it were an itinerant antiferromagnet with a Néel temperature $\sim$ 25K. Below
that temperature, however, the correlation length ceases its rapid increase, and levels off toward a constant value ~ 3 times the interparticle spacing.

A consequence of the stronger antiferromagnetic correlations is that the characteristic energy scale for the antiferromagnetic paramagnons which describe the AF spin dynamics is quite low. It is always less than kT, being ~ 20K for temperatures in the vicinity of Tc, and varying linearly with temperature for temperatures above 130K. Low energy paramagnons provide an effective scattering mechanism for quasiparticles; we calculate the spin fluctuation resistivity for La1.85Sr0.15CuO4 and compare it with our calculated results for YBa2Cu3O7 and YBa2Cu3O6.63. We find that for all three materials the temperature dependence of the resistivity is linear, and that if one assumes that the matrix elements for quasiparticle-paramagnon scattering are similar for the three materials, the increasing tendency toward antiferromagnetic behavior as one goes from YBa2Cu3O7 to YBa2Cu3O6.63 to La1.85Sr0.15CuO4 leads to a comparable progression in the magnitude of the resistivity. These results suggest that a resistivity which varies linearly with temperature is a natural attribute of an almost antiferromagnetic Fermi liquid, and provide support for the view that the anomalous charge and spin properties of the normal state possess a common physical origin.

The plan of the paper is the following. In Section II, we introduce the hyperfine Hamiltonian for the planar nuclei and use it to determine the temperature dependent spin susceptibility from measurements of the 17O Knight shift [3]. As is the case for the YBCO samples, the transferred hyperfine coupling between the Cu2+ spins and the Cu and O nuclei plays a major role in determining the Knight shift of these nuclei. In Section III we review the MMP model for spin-lattice relaxation, and use it to determine the antiferromagnetic enhancement factor from experiment in Section IV.
Because measurements of the $^{17}$O relaxation rate are not yet available, it is necessary to appeal to scaling arguments to determine a key parameter, the correlation length, $\xi_0$ which marks the transition from paramagnon to quasiparticle behavior. We therefore present results for the antiferromagnetic Fermi liquid parameters for La$_{1.85}$Sr$_{0.15}$CuO$_4$ based on a physically reasonable range of choices for $\xi_0$.

We use the above results to analyze two candidate models for antiferromagnetic behavior in Section V: a temperature-dependent interaction between almost localized spins, and Fermi surface nesting of weakly localized itinerant quasiparticles. A comparison of our results for the antiferromagnetic behavior of La$_{1.85}$Sr$_{0.15}$CuO$_4$ with those obtained for the YBCO family leads us to conclude that only the nearly localized moment model is consistent with the NMR experiments on these systems. We present our results for spin-fluctuation resistivity in Section VI, discuss the nature of the normal state of cuprate oxide superconductors, the possible connection between antiferromagnetic behavior and superconductivity, and give our conclusions in Section VII.

II. The hyperfine Hamiltonian and Knight shift experiments

In analyzing the experimental results for the Knight shift and spin-lattice relaxation rates, we use the hyperfine Hamiltonian for the planar Cu and O nuclei adopted by Mila and Rice [5] and Millis, Monien, and Pines [1] to describe NMR experiments in YBa$_2$Cu$_3$O$_{7.5}$.

$$H_{\text{hf}} = \sum_{i,\alpha} ^{63}I_{i,\alpha} A_{\alpha} S_{i,\alpha} + \sum_{ij,\alpha} ^{63}I_{i,\alpha} B_{ij,\alpha} S_{j,\alpha} + \sum_{\langle ij \rangle,\alpha} ^{17}I_{i,\alpha} C_{S_{j,\alpha}} \quad (2.1)$$
in which the $^{63}\text{Cu}$ nuclear spins, $^{63}\Sigma_i$, are assumed coupled to an electron spin $S_i$ (one spin per unit cell composed out of planar Cu and O spins) at the same site, by the direct hyperfine coupling tensor, $A_{\alpha\alpha}$, and to electron spins, $S_i$, at the four nearest neighbor sites, by the transferred hyperfine coupling constant, $B$. In this one-component model, the $^{170}$ nuclear spins $^{17}\Sigma_i$, are assumed to couple only by the transferred hyperfine coupling constant $C$ to the two nearest neighbor electron spins, $S_j$. The spin contribution to the Knight shifts of the planar nuclei are then given by

\[
63K^S_{||} = \frac{(A_{||} + 4B) \chi_0}{63\gamma_n \gamma_e \hbar^2} \quad (2.2a)
\]

\[
63K^S_{\perp} = \frac{(A_{\perp} + 4B) \chi_0}{63\gamma_n \gamma_e \hbar^2} \quad (2.2b)
\]

\[
17K^S_{iso} = \frac{2C \chi_0}{17\gamma_n \gamma_e \hbar^2} \quad (2.2c)
\]

where the $\gamma_n$ are the various nuclear moments and $\gamma_e$ is the electron magnetic moment, and subscripts refer to shifts for magnetic fields applied to parallel and perpendicular to the crystalline c-axis, with corresponding values for the hyperfine couplings.

Monien, Pines, and Takigawa (hereafter MPT) [2] have shown that in YBa$_2$Cu$_3$O$_{7-\delta}$ the hyperfine couplings $A$, $B$, and $C$ are, to within a few percent, independent of the oxygen doping level $\delta$, as one goes from the antiferromagnetic insulator to the 60K superconductor to the 90K superconductor. From their fit to the NMR experimental results for YBa$_2$Cu$_3$O$_7$, and the antiferromagnetic resonance results for YBa$_2$Cu$_3$O$_6$, MMP have determined $A_{||}$, $A_{\perp}$, $B$, and $C$; their results are shown in Table 1. Given the insensitivity of these near hyperfine couplings to the degree of
oxygenation of the YBa$_2$Cu$_3$O$_{7.8}$ compounds it appears reasonable to assume that, to within a few percent, the planar hyperfine couplings for the Cu and O nuclei in the La$_{2-x}$Sr$_x$CuO$_4$ compounds will likewise be independent of the Sr doping level $x$, and that to the same degree of accuracy, there will be no change in these couplings from those found in the Y compounds. Two sets of experimental results support this hypothesis. First, Tsuda et al. [6] find that the antiferromagnetic resonance frequency for the antiferromagnetic insulator, La$_2$CuO$_4$, lies within a few percent of that measured for YBa$_2$Cu$_3$O$_6$ by Yasuoka et al. [7]. Thus the quantity

$$\mu_{\text{eff}}(4B - A_{\perp}) = H_{\text{afr}} \approx 80 \text{ kOe} \tag{2.3}$$

is essentially unchanged. If therefore, as seems plausible, $\mu_{\text{eff}} = (0.62 \pm 0.02)\mu_B$ [8] is changed by at most a few percent on going from YBa$_2$Cu$_3$O$_6$ to La$_2$CuO$_4$, it follows that $A_{\perp}$ and $B$ do not vary. Second, the total $^{63}\text{Cu}$ Knight shift, $^{63}\text{K}_{||}$, found by Kitaoka et al. [3] for a field applied in the $c$ direction is independent of temperature, and is given by

$$^{63}\text{K} = 1.3\% \tag{2.4}$$

It is therefore identical, within experimental error, to the temperature-independent value found for $K_{||}$ in YBa$_2$Cu$_3$O$_7$ [9] and YBa$_2$Cu$_3$O$_{6.63}$ [10]; the latter results reflect the fact that $A_{||} = -4B$, so that the measured total shift is of purely orbital origin. Since one finds the same temperature independent magnitude shift for La$_{1.85}$Sr$_{0.15}$CuO$_4$, it follows that for the latter material one also has $A_{||} = -4B$, and $^{63}\text{K}_{||} = ^{63}\text{K}_{\text{orb}}$. 
If we now assume that C is likewise unchanged from the Y compounds, and take the orbital oxygen Knight shift $^{17}K^\text{orb}_{||}$ to be $-0.0136\%$, the value it possesses for YBa$_2$Cu$_3$O$_{6.63}$, and subtract this value from the experimental results of Kitaoka et al [3], for the temperature-dependent total shift $^{17}K_{||}$, we obtain the result shown in Fig. 1 for the temperature-dependent spin component of the Knight shift, $^{17}K_c(T)$, and the planar spin susceptibility, $\chi_0(T)$. A good fit to $\chi_0(T)$ is obtained with the expression

\[
\frac{\chi_0(T)}{\mu_B} = \{1.01 + 0.41 (T/100)\} \text{ states/ev Cu}^{2+}
\]

As may be seen in Fig. 1, this result is quite similar to that found by MPT for YBa$_2$Cu$_3$O$_{6.63}$. We discuss its physical origin in Sec. IV, and, in the Appendix, compare it with results obtained from an analysis of the experiments on YBa$_2$Cu$_3$O$_{6+\delta}$ of Alloul et al. [11]. Note that the measured temperature independence of $^{63}K_{||}$ over a temperature range (60K $\leq$ T $\leq$ 120K) [3] in which $^{17}K_{||}^c$ and $\chi_0$ change by some 40% can only come about if $^{63}K_{||}$ is of purely orbital origin, as we have argued.

**III. Spin-lattice Relaxation Rates**

In a one component model for the spin-spin correlation function the nuclear spin lattice relaxation rates depend on a product of the form factors, which differ from one nucleus to the other, and the dynamical structure factor $S(q,\omega)$. In the low frequency regime, $\omega \ll T$, in which NMR experiments are done, $S(q,\omega)$ is related to the imaginary part of the spin-spin correlation function $\chi''(q,\omega)$, by:
\[ S(\mathbf{q}, \omega) = \left( \frac{1}{\omega} \right) \chi''(\mathbf{q}, \omega) \]  

(3.1)

The MMP model for spin lattice coupling may be thought of as an antiferromagnetic Fermi liquid model in which the dominant (low frequency) elementary spin excitations are anti-ferromagnetic paramagnons at wavevectors near the antiferromagnetic wave vector \( Q = (\pi/a, \pi/a) \). Thus one assumes that the system is almost, but not quite, antiferromagnetic. In their antiferromagnetic Fermi liquid theory, MMP model the spin correlation function by two separate parts. One describes a quasiparticle part, \( \chi_{QP} \), and the other, more important, part, \( \chi_{AF} \), describes the short wavelength antiferromagnetic correlations,

\[ \chi(\mathbf{q}, \omega) = \chi_{QP}(\mathbf{q}, \omega) + \chi_{AF}(\mathbf{q}, \omega), \]  

(3.2)

The quasiparticle contribution may be written as

\[ \chi_{QP}(\mathbf{q}, \omega) = \frac{\chi_0}{1 - i\omega\pi/\Gamma}, \]  

(3.3)

independent of the wavevector \( \mathbf{q} \). Here \( \Gamma \) is the characteristic spin fluctuation energy for the quasiparticle part, and \( \chi_0 \) is the quasiparticle long wavelength contribution to the measured long wavelength static susceptibility \( \chi_0 \).

The antiferromagnetic part of the spin-spin correlation function is modeled around the antiferromagnetic wavevector \( \mathbf{Q} = (\pi/q, \pi/a) \) by
\[ \chi_{\text{AF}}(\mathbf{q}, \omega) = \frac{\chi_Q}{1 + \xi^2(Q - \mathbf{q})^2 - i(\omega/\omega_{\text{SF}})} \]  

where \( \chi_Q \) is the static spin susceptibility at the antiferromagnetic wavevector \( \mathbf{Q} \), which is related to \( \chi_0 \) by

\[ \chi_Q = \chi_0 (\xi/\xi_0)^2 \]  

where \( \xi \) is the antiferromagnetic correlation length and \( 1/\xi_0 \) is the wavevector at which the antiferromagnetic part of the correlation function, \( \chi_{\text{AF}} \), starts to dominate the quasiparticle contribution. We can relate \( \chi_0 \) to the measured static susceptibility \( \chi_0 \) using Eq. (3.2) and Eq. (3.4),

\[ \chi_0 = \chi_0 \left[ 1 + \frac{1}{2\pi^2} \left( \frac{a}{\xi_0} \right)^2 \right] = \chi_0 \left[ 1 + \frac{R^{1/2}}{2\pi^2} \right] \]  

on introducing the parameter

\[ \beta = \left( \frac{a}{\xi_0} \right)^4 \]  

which measures, \textit{inter alia}, the relative strength of the antiferromagnetic paramagnon contribution to the static spin susceptibility. For a typical value of \( (a/\xi_0)^2 = \pi \) we find that the quasiparticle bit contributes 84% of the static susceptibility, whereas the antiferromagnetic part contributes some 16%.

\( \hbar \omega_{\text{SF}} \) is a typical energy scale for the antiferromagnetic paramagnons which describe the AF spin dynamics. It can be very small, since it is related
to the energy scale of the spin dynamics of the noninteracting quasiparticle system, \( \Gamma \), by:

\[
\omega_{SF} = \frac{\Gamma}{\pi} \times \left( \frac{\xi}{\xi_0} \right)^2 = \frac{\Gamma}{\pi} \frac{\chi_0}{\chi_Q} = \frac{\Gamma}{\pi} \frac{\chi_0}{\chi_Q} \left[ \frac{1}{1 + \beta^{1/2}/2\pi^2} \right]. \tag{3.8}
\]

On comparing the spin lattice relaxation rate for a non-interacting Fermi gas with that calculated from Eq. (3.3), one can easily show that

\[
\Gamma \equiv E_F \tag{3.9}
\]

the quasiparticle Fermi energy. Thus when one has appreciable AF correlations (and we shall see that \( \chi_Q/\chi_0 \) can be as large as 50 for \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \)), the paramagnon energy scale can be as low as 20K.

The relaxation rates are determined by a \( \mathbf{q} \) average over the Brillouin zone of the structure factor multiplied with the appropriate form factors. MMP have obtained the following explicit expressions for the relaxation rates:

\[
63W_{\perp} = \frac{3}{4} \frac{1}{\mu_B^2} \lim_{\omega \to 0} \sum_q \left[ A_{\parallel} - 2B (\cos(q_y a) + \cos(q_y a)) \right]^2 S(q, \omega) \tag{3.10a}
\]
\[ \begin{aligned}
{^{63}\text{W}}_{\parallel} &= \frac{3}{8} \frac{1}{\mu_{B}^{2}} \lim_{\omega \to 0} \sum_{q} \left\{ \left[ A_{\parallel} - 2B(\cos(q_{z}a) + \cos(q_{y}a)) \right]^{2} \right. \\
&\quad + \left. \left[ A_{\perp} - 2B(\cos(q_{z}a) + \cos(q_{y}a)) \right]^{2} \right\} S(q,\omega) \quad (3.10b)
\end{aligned} \]

\[ \begin{aligned}
{^{17}\text{W}} &= \frac{3}{4} \frac{1}{\mu_{B}^{2}} \lim_{\omega \to 0} \sum_{q} \left[ 2C^{2}(1 - \cos(q_{z}a)) \right] S(q,\omega) \quad (3.10c)
\end{aligned} \]

Here \( \vec{q} \) is measured from the antiferromagnetic wavevector \( \vec{Q} = (\pi/a, \pi/a) \).

Whereas the Cu relaxation rate picks up all the antiferromagnetic correlations the O relaxation rate is not strongly enhanced by the AF correlations since the hyperfine field of the Cu spins cancels at the oxygen site. We remind the reader that the connection between the above defined relaxation rates and the measured relaxation times, \( T_{1} \) depends on whether one is considering a nuclear magnetic resonance experiment, for which one has:

\[ \left( \frac{1}{T_{1}} \right)_{\text{NMR}} = \frac{2}{3}W, \quad (3.11a) \]

or a nuclear quadrupolar resonance experiment, for which

\[ \left( \frac{1}{T_{1}} \right)_{\text{NQR}} = 2W = 3 \left( \frac{1}{T_{1}} \right)_{\text{NMR}} \quad (3.11b) \]

Since we have seen that the hyperfine couplings for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_{4}\) are within some 5\% of those measured for YBa\(_{2}\)Cu\(_{3}\)O\(_{7}\) and YBa\(_{2}\)Cu\(_{3}\)O\(_{0.63}\), we use the YBCO result, \( A_{\perp} = 0.84B \) [1,2], in determining \(^{63}\text{W}(T)\) and \(^{17}\text{W}(T)\).
further assume that the AF correlations peak at $Q = (\pi/a, \pi/a)$, and hence find, on making use of the MMP results for the integrals over $q$ in Eqs. (3.10)

\begin{align}
6^3W_{\parallel}(T) &= \frac{12\pi}{\mu_B^2 k_B T} \left[ \frac{\chi_0(T)}{\Gamma(T)[1 + \beta^{1/2}/2\pi^2]} \left\{ 0.294 + \frac{\beta}{\pi^2} \left[ 0.49 \left( \frac{\xi(T)}{a} \right)^2 - 0.62 \log \left( \frac{\xi(T)}{a} \right) + 0.0175 \right] \right\} \right] \quad (3.12a) \\
6^3W_{\perp}(T) &= \frac{12\pi}{\mu_B^2 k_B T} \left[ \frac{\chi_0(T)}{\Gamma(T)[\beta^{1/2}/2\pi^2]} \left\{ 0.772 + \frac{\beta}{\pi^2} \left[ 1.83 \left( \frac{\xi(T)}{a} \right)^2 - 1.10 \log \left( \frac{\xi(T)}{a} \right) - 0.297 \right] \right\} \right] \quad (3.12b) \\
17W(T) &= \frac{3\pi}{2\mu_B^2 k_B T} \left[ \frac{\chi_0(T)}{\Gamma(T)[1 + (\beta^{1/2}/2\pi^2)]} \left\{ 1 + \frac{\beta}{\pi^2} \left[ 0.39 \log \left( \frac{\xi(T)}{a} \right) + 0.17 \right] \right\} \right] \quad (3.12c)
\end{align}

As MPT have noted, the above expressions make it easy to separate out the contribution made by antiferromagnetic paramagnons to the spin-lattice relaxation rate; the "quasiparticle" result is obtained if one takes $\beta = 0$ in Eqs. (3.11), while the importance of antiferromagnetic correlations is measured by defining the quantity

\[
6^3(R_{af})_{\parallel} = \frac{6^3W_{\parallel}(\beta)}{6^3W_{\parallel}(0)} = \frac{1 + \frac{\beta}{\pi^2} \left[ \frac{5}{3} \left( \frac{\xi(T)}{a} \right)^2 - 2.1 \log \left( \frac{\xi(T)}{a} \right) + 0.059 \right]}{1 + \beta^{1/2}/2\pi^2} \quad (3.13)
\]
The relative importance of antiferromagnetic correlations, as one goes from
$^{63}$W to $^{17}$W, is seen clearly in Eqs. (3.12). As MMP have emphasized, for $\xi^2/a^2$
$>> 1$ antiferromagnetic correlations play a dominant role in determining
$^{63}$W(T), with the leading term being proportional to $\xi^2$, and a logarithmic
contribution of antiferromagnetic origin (which is of opposite sign from the
quasiparticle contribution), playing a more significant role than the latter for
$(\xi/a) \geq 1.5$.

Given a knowledge of $\chi_0(T)$, and with the assumption that $\Gamma$ is only
weakly dependent on $T$, it is straightforward to use Eq. (3.12a) to obtain the
temperature dependence of $\xi(T)$ from experiment, provided one is in the limit,
$(\xi(T)/a)^2 >> 1$. Indeed, to the extent that $\xi(T)$ takes the mean-field form,

$$\left(\frac{\xi(T)}{a}\right)^2 = \left(\frac{\xi(0)}{a}\right)^2 \frac{|T_x|}{T + T_x}$$

the quantity $\left[63W||/T \chi_0(T)\right]^{-1}$ should display a temperature dependence of
the form

$$\left[63W||/T \chi_0(T)\right]^{-1} \sim T_1 T \chi_0(T) = a + b T$$

where $a$ and $b$ are constants. Hence a plot of $T_1 T \chi_0(T)$ as a function of $T$
provides a test of the applicability of the MMP model with $\xi^2/a^2$ displaying the
temperature dependence, Eq. (3.14). In Fig. 2 we give the results of Imai et al.
[4] and Kitaoka et al. [3] for the relaxation rate $^{63}[1/T_1(T)]$, while in Fig. 3 we
use our result for $\chi_0(T)$, Eq. (2.5), to plot $\left[T_1 T \chi_0(T)\right]$ as a function of
temperature for the two sets of experimental measurements.
We see that over the temperature range (100K ≤ T ≤ 200K) in which the results of the two groups are in good agreement, the form, Eq. (3.14) is well obeyed. There are, however, significant departures at both lower and higher temperatures. At the high temperatures, the "non-leading" terms [i.e. those other than \( \xi^2/a^2 \) in Eq. (3.12a)] begin to play a significant role, while at lower temperatures, departures from the mean field form Eq. (3.14), become significant. To take into account the presence of the non-leading terms in \( 63W_{\parallel} \) it is necessary to know, separately, \( \Gamma, \beta, \) and \( \xi/a, \) and we consider next the extent to which reliable estimates for these basic parameters of antiferromagnetic Fermi liquid theory may be obtained from experiment.

IV. Determination of antiferromagnetic Fermi liquid parameters

We begin by determining \( 63(R_{af})_{\parallel} \) from experiment. Inspection of Eqs. (3.12a) and 3.13) shows that in the limit, \( (\xi/a)^2 \gg 1, \) one has

\[
63(R_{af})_{\parallel} (T) = \frac{63W_{\parallel}(T)\Gamma(T)}{\chi_0(T)T} \left( \frac{\mu_B^2 \hbar^2}{3.53\pi^2 k_B T} \right)
\]  

(4.1)

For La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) we do not at present have any way of determining \( \Gamma(T) \) directly. Since, however, as discussed in the Appendix, the result we have obtained for its static susceptibility, \( \chi_0(T) \), maps well onto those obtained by MPT for the YBCO family from the experiments of Takigawa et al. [10] and Alloul et al. [11], and, since as shown there,

\[
\Gamma \equiv 0.4eV
\]  

(4.2)
for the entire range of Alloul et al. samples, it seems reasonable to adopt this value for LSCO samples, and in particular for La$_{1.85}$Sr$_{0.15}$CuO$_4$. On doing so, we obtain the results for $(R_{af})_\parallel(T)$ shown in Fig. 4. There we see that at all temperatures the antiferromagnetic correlations found in La$_{1.85}$Sr$_{0.15}$CuO$_4$ are stronger than those encountered in YBCu$_3$O$_{6.63}$ and YBCu$_3$O$_7$.

We consider next the choice of $\beta$. For YBa$_2$Cu$_3$O$_7$ and YBa$_2$Cu$_3$O$_{6.63}$, it is possible to determine $\beta$ directly from experiment by combining the results for $^{63}W_\parallel$, $^{17}W$, and $^{89}W$ at a given temperature. In this way, MMP and MPT find that $\beta = [a/\xi_0]^4 \equiv \pi^2$ for both the O$_7$ and the O$_{6.63}$ samples, and that within experimental accuracy it is independent of temperature. However, for La$_{1.85}$Sr$_{0.15}$CuO$_4$ there is not sufficient experimental information to determine $\beta$ from experiment. In the absence of experimental constraints, it is, of course, tempting to assume that $\beta \equiv \pi^2$ for all cuprate oxide superconductors and to use this value in calculating $\xi/a$ and other quantities of interest. If one does so, one is led to what appears to be unreasonably large values for $\xi/a$, $\lambda_Q$, etc. The reason is simple. Since, as we have seen, $^{63}(R_{af})_\parallel \sim \beta \xi^2/a^2$, is some three times larger for La$_{1.85}$Sr$_{0.15}$CuO$_4$ than it is for YBa$_2$Cu$_3$O$_{6.63}$, in taking $\beta \equiv \pi^2$, one is attributing that increase entirely to an increase in $\xi/a$, and so finds $\xi/a \sim 7$ at $T \equiv 100$K, a result which appears substantially larger than the estimate, $(\xi/a) \sim 3$, obtained from neutron scattering experiments on the LSCO family. We are therefore led to ask whether some, or all, of the increase in $^{63}(R_{af})_\parallel$, might be due to a change in $\beta$. In other words, since $R_{af}$ changes considerably as one goes from the YBCO materials to the LSCO materials, it is perhaps not unreasonable to expect that the correlation length $\xi_0$, (and hence $\beta$) which marks the transition from antiferromagnetic paramagnon to quasiparticle behavior might also change.
In deciding on a candidate range for $\beta$, we begin by comparing further the results we have obtained from our analysis of the NMR data for $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$ with the corresponding results for the two members of the YBCO family for which extensive experimental data is presently available, $\text{YBa}_2\text{Cu}_3\text{O}_7$ and $\text{YBa}_3\text{Cu}_3\text{O}_{6.63}$. As we have seen in Fig. 4, there is a steady progression in the magnitude of the antiferromagnetic enhancement of the $^{63}\text{Cu}$ relaxation rate as one goes from $O_7$ to $O_{6.63}$ to $\text{La}_{1.85}\text{SrCuO}_4$. This is a bit surprising since if one uses, say, the method of Kokura et al. [12] for matching $\text{La}_{2-y}\text{Sr}_y\text{CuO}_4$ to $\text{YBa}_2\text{Cu}_3\text{O}_x$,

$$y = \frac{x-6.5}{2},$$  \hfill (4.3)

then $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$ should correspond to $\text{YBa}_2\text{Cu}_3\text{O}_{6.8}$; hence its properties might be expected to be intermediate between those of the $O_7$ and $O_{6.63}$ materials. Indeed, this is the case for the corresponding values of $\chi_0(T)$, shown in Fig. 13, and discussed further in the Appendix. How then to understand the apparently anomalously large value of $^{63}(R_{af})_\parallel$ for $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$?

A plausible explanation is that $\beta$ is substantially larger in the latter material. Since, according to Eq. (3.13),

$$R_{af} \sim \beta \frac{\xi^2}{a^2} = \frac{\xi^2}{\xi_0^2} \frac{a^2}{\xi^2} = \frac{\xi^2}{\xi_0^2} \left(\frac{a^2}{\xi_0^2}\right) \xi^2$$

it follows that if $(\xi_0/a)$ is smaller in the LSCO series than in the YBCO series, one could, for comparable values of $\xi/a$, obtain a much larger
antiferromagnetic enhancement ratio, \(^{63}(R_{af})_\parallel\). We are therefore led to explore the consequences of choosing a value of \(\beta\) such that the corresponding values for \(\xi/a\) fall between those found for YBa\(_2\)Cu\(_3\)O\(_7\) and YBa\(_2\)Cu\(_3\)O\(_6.63\). As may be seen in Fig. 5, if we take

\[
\beta = (4.5 \pm 1.5) \pi^2 \tag{4.4}
\]

we achieve this goal. In what follows, we shall adopt this value, giving results for \(\beta = 4.5 \pi^2\), and indicating the range of changes in the various quantities of interest associated with taking values of \(\beta\) as small as \(3\pi^2\) or as large as \(6\pi^2\). We discuss in the following section the possible physical origin of changes in \(\beta\) in going from the YBCO family to the LSCO family.

Our results are presented in Figs. 6-10. We comment on these briefly. We note first that, as may be seen in Fig. 6a, one obtains quite similar results for \(\xi(T)\) from the two sets of experimental data, and that \((a^2/\xi^2)\) now displays a linear temperature dependence for \(T \geq 130\text{K}\), while departures from the form, Eq. (3.14) are evident at lower temperatures. Above 130K, the tendency toward antiferromagnetism is so strong that \(\xi(T)\) follows the approximate temperature dependence.

\[
\left(\frac{\xi(T)}{a}\right)^2 = (25) \left[ \frac{25}{T-25} \right] \quad [T \geq 130\text{K}] \tag{4.5}
\]

Thus as the temperature is lowered from 250K, the system appears to be on its way toward becoming an itinerant antiferromagnet with a Néel temperature \(~ 25\text{K}\), but at \(T \sim 130\text{K}\), this behavior begins to change, as though the system starts to become aware, even at this elevated temperature, of the fact that it is
going to become a superconductor, not an antiferromagnet. As shown in Fig. 6b, quite similar behavior is seen in YBa$_2$Cu$_3$O$_{6.63}$, where MPT note that since $\xi/a$ is a constant below $T_c$, the transition, which begins at 130K, of $\xi/a$ from Néel behavior to superconducting behavior may perhaps be a precursor effect. It is perhaps not an accident that this change in character of $\xi/a$ is more striking in those materials, which resemble incipient itinerant antiferromagnetics for $T > 130K$, than it is for YBa$_2$Cu$_3$O$_7$, which shows much less in the way of antiferromagnetic tendencies ($T_x$ being $\sim 113K$ rather than being negative, and $\xi/a$ being considerably smaller at all temperatures).

Having obtained $\xi(T)$, it is a straightforward matter to calculate $\chi_Q(T)$ and $\chi_Q(T)/\chi_0(T)$; the resulting values are compared with those obtained for the three YBCO samples in Fig.7a and 7b, where again the two sets of experimental results on La$_{1.85}$Sr$_{0.15}$CuO$_4$ yield quite similar values for these quantities. These results enable us to explore a proposed correlation between $\chi_0(T)$ and $\chi_Q/\chi_0$. One of us [13,14] has suggested that it is the temperature dependent antiferromagnetic correlations in YBa$_2$Cu$_3$O$_{6.63}$ and other cuprate oxides which are responsible for the measured decrease in $\chi_0(T)$ with decreasing temperature, the idea being that in an antiferromagnetic Fermi liquid, the strong short-range correlations responsible for the enhancement of $\chi_Q$ necessarily inhibit the ability of the spin system to respond to a uniform field, and so lead to a decrease in $\chi_0$. MPT have examined this proposal for YBa$_2$Cu$_3$O$_{6.63}$, and find a threshold, $(\chi_Q/\chi_0) \equiv 12$, beyond which AF correlations begin to suppress $\chi_0(T)$. This effect is illustrated in Fig. 8, where we compare their results for $[\chi_0(300K)/\chi_0(T)]$ as a function of $\chi_Q(T)/\chi_0(T)$ for YBa$_2$Cu$_3$O$_{6.63}$, with those calculated here for La$_{1.85}$Sr$_{0.15}$CuO$_4$. We see that with $\beta = 4.5\pi^2$, for values of $\chi_Q/\chi_0$ such that

\[15 \leq \chi_Q/\chi_0 \leq 30,\]
corresponding to $T \geq 150K$, the temperature dependent antiferromagnetic correlations in $\text{YBa}_2\text{Cu}_3\text{O}_{6.63}$ and $\text{La}_{1.85}\text{Sr}_{0.15}\text{Cu}_4$ produce a comparable substantial reduction in $\left[ \chi_0(T)/\chi_0(300K) \right]$ for the two materials. However, for lower temperatures, and larger values of $(\chi_Q/\chi_0)$, the influence of the short-range antiferromagnetic fluctuations on $\chi_0(T)$ would seem less significant for $\text{La}_{1.85}\text{Sr}_{0.15}\text{Cu}_4$ than for $\text{YBa}_2\text{Cu}_3\text{O}_{6.63}$. Since for $\beta = 4.5\pi^2$ our deduced values for $(\chi_Q/\chi_0)$ agree for the two materials down to $T \sim 80K$ (corresponding to $(\chi_Q/\chi_0) \equiv 50$) this departure from "universality" may be traced to the behavior of $\chi_0(T)$ in the two materials. As may be seen in Fig. 13, for $T < 150K$, $\chi_0(T)$ falls off more rapidly than linearly for $\text{YBa}_2\text{Cu}_3\text{O}_{6.63}$, while it maintains its linear decrease for $\text{La}_{1.85}\text{Sr}_{0.15}\text{Cu}_4$.

A measurement of $\chi^W(T)$ would provide a useful further constraint on the parameters we have deduced from experiment, as would quite accurate measurements of $63\chi^W(T)$. To that end we give in Figs. (9) and (10) our predicted values for $\chi^W(T)$ and for the anisotropy ratio $\chi^W(T)/\chi^W(T)$. We see that measurements of $\chi^W(T)$ in $\text{La}_{1.85}\text{Sr}_{0.15}\text{Cu}_4$ should make it possible to determine $\beta$ rather accurately, while the calculated variations with $\beta$ in both the magnitude and temperature dependence of $[63\chi^W/63\chi^W]$ are at the $\leq 10\%$ level, and hence will be difficult to determine from measurements of $63\chi^W(T)$.

It should also be noted that in the above calculation of $\chi^W(T)$ for $\text{La}_{1.85}\text{Sr}_{0.15}\text{Cu}_4$ we have assumed that the antiferromagnetic correlations peak at the commensurate wavevector, $Q = (\pi/a,\pi/a)$. Since a departure from commensurability introduces a term $\sim \beta(\xi^2/a^2)$ in $\chi^W$, which would dominate the logarithmic term for even quite small departures from commensurability, it is evident that a measurement of $\chi^W(T)$ for this system will provide valuable information about such a departure, as well as about $\beta$. 
Our results for the paramagnon energy, \( \omega_{\text{SF}} \), are given in Fig. 11, where they are compared with values previously obtained by MMP for O7 and by MPT for O6.63. Again one sees a consistent progression; the more antiferromagnetic the system is, the lower the paramagnon energy. For all three systems, the paramagnon energy increases linearly with \( T \) over a wide range of temperature as is to be expected from the proportionality of \( \omega_{\text{SF}} \) and \( a^2/\xi^2 \). These results demonstrate that \( \hbar \omega_{\text{SF}}(T) \leq k_B T \leq k_B T_c \) for all three materials, and that \( \hbar \omega_{\text{SF}} \) is dramatically smaller than \( k_B T \) in the case of \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \).

Given the fact that the relaxation rates are known to some 10\%, that we have not taken into account the possible (\( \leq 20\% \)) variation in \( \Gamma(T) \) between 250K and 40K, that \( \chi_0(T) \) is likely not known to be much better than 20\%, and that our heuristic choice of \( \beta \) may not be more accurate than, say, 30\%, it seems reasonable to assign an uncertainty of some \( \pm 30\% \) to all the values we have deduced here.

V. The Physical Origin of Antiferromagnetic Behavior

We now consider briefly the information which our analysis of the NMR experiments on \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \) provides on the physical origin of the measured very strong antiferromagnetic correlations. For an almost localized set of spins, the correlations arise as a consequence of a temperature-dependent interaction between the (barely) itinerant quasiparticles, while for a weakly localized spin system, these would arise from Fermi surface nesting of the (quite) itinerant quasiparticles. The MMP description of antiferromagnetic correlations makes no distinction between these alternatives, since it only assumes that the static susceptibility in the vicinity of \( Q \) may be written as
\[ \chi_{\text{AF}}(Q - q, 0) = \frac{\chi_Q}{1 + \xi^2 q^2} = \frac{\chi_0 \left( \frac{\xi^2}{\xi_0^2} \right)}{1 + \xi^2 q^2} \quad (5.1) \]

without inquiring as to the physical origin of \( \xi_0 \) or \( \xi(T) \). If, however, we consider the mean field expression,

\[ \chi_{\text{AF}}(Q - q, 0) = \frac{\chi_{Q-q}}{1 - F_{Q-q} |q|} \quad (5.2) \]

where \( \chi_{Q-q} \) is the static susceptibility of non-interacting quasiparticles or local moments, and

\[ F_{Q-q} = J(Q - q) \chi_{Q-q} \quad (5.3) \]

is the dimensionless measure of the degree by which \( \chi_{Q-q} \) is enhanced by the interaction, while \( J(Q - q) \) measures the strength of the interaction between the quasiparticles (or local moments), then we can consider separably the role played by \( \chi \) and by \( J \) in determining the antiferromagnetic enhancement.

Thus we may write:

\[ \bar{\chi}_{Q-q} = \frac{\chi_0}{\alpha_Q + \alpha_\xi^2 q^2} \quad (\alpha = \pm 1) \quad (5.4a) \]

\[ F_{Q-q} = F_Q - q^2 \xi_F^2 \quad (5.4b) \]

\[ J_{Q-q} = J_Q - q^2 \xi_J^2 \quad (5.4c) \]
and then combine the above expression with Eq. (3.5) to obtain

\[
\frac{\xi^2}{\xi_0} = \alpha Q [1 - F_Q]
\]

(5.5)

and

\[
\xi_0^2 = \alpha Q \xi_F^2 + \xi_\chi^2 [1 - F_Q] = \alpha \xi_\chi^2 + \bar{X}_0 \xi_J^2 .
\]

(5.6)

We thus see that \( \xi_0 \) is determined by the interplay between the wavevector dependence of \( \bar{X}_Q \) in the vicinity of \( Q \) and that of \( J_Q \). The sign of \( \alpha \), and hence of the contribution made by \( \xi_\chi^2 \) to \( \xi_0^2 \) reflects clearly the difference between Fermi surface nesting behavior and barely itinerant local moments, since to the extent that nesting is significant, we expect that \( \bar{X}_Q > \bar{X}_0 \), with \( \bar{X}_Q \) maximum at \( Q \), so that, according to Eq. (5.4a)

\[
(\alpha_Q)_{\text{nest}} < 1
\]

(5.7a)

\[
(\alpha)_{\text{nest}} = +1
\]

(5.7b)

and the consequences of the wavevector dependence of \( \bar{X}_Q \) and \( J_Q \) in the vicinity of \( Q \) are additive. For local moment behavior, on the other hand, we have

\[
(\alpha_Q)_{\text{loc}} < 1
\]

(5.8a)

\[
(\alpha)_{\text{loc}} = -1
\]

(5.8b)

so that

\[
\left( \frac{\xi_0^2}{\xi_J^2} \right)_{\text{local}} = \bar{X}_0 \xi_J^2 - \xi_\chi^2 .
\]

(5.9)
It follows that for nearly localized local moments both significantly smaller values of $\xi^2_0$, and a greater variation of $\xi_0$ from one family of cuprate oxide superconductors to another are to be expected than would be the case for nearly nested Fermi liquids. From this perspective both the comparatively small value, $(\xi_0/a) = 0.56$, found by MMP for the YBCO family, and the still smaller value, $(\xi_0/a) = 0.38$, found here with $\beta = 4.5\pi^2$, are physically reasonable. On the other hand, it is difficult, if not impossible, to explain the insensitivity of $\xi_0$ to oxygen concentration for YBCO samples from a nesting perspective, and equally difficult to understand the proposed variation in $\xi_0$ from YBCO samples to La$_{1.85}$Sr$_{0.15}$CuO$_4$.

Our conclusion that the antiferromagnetic behavior is primarily associated with barely itinerant local moment interaction is consistent with the physical picture put forth by Monien, Pines, and Slichter [15] for YBa$_2$Cu$_3$O$_7$ and the results of MPT for YBa$_2$Cu$_3$O$_{6.63}$, since it is only in such a picture that one would find that not only is the transferred hyperfine interaction, $B$, between a $^{63}$Cu nucleus and its nearest neighbor Cu$^{2+}$ spins unchanged by doping, but also that the dynamical consequences of this interaction are unaffected.

VI. Spin-fluctuation resistivity

We have seen that the anomalous magnetic behavior of the normal state of the cuprate oxide superconductors can be simply explained using antiferromagnetic Fermi liquid theory. In this section we explore the possibility that a major aspect of the anomalous normal state charge behavior of these systems, a resistivity which varies linearly with temperatures for electric fields applied in the planes, likewise finds an explanation using antiferromagnetic Fermi liquid theory. We do so by using our results for the
temperature-dependent antiferromagnetic paramagnon energy to compute the contribution to the electrical resistivity from quasiparticle-paramagnon scattering. Moriya, Takahashi and Ueda [16] have given an expression for the resistivity produced by quasiparticle-spin fluctuation scattering which is well-suited to this purpose. When one makes the translation from their notation to our own, one obtains a resistivity which can be written as:

$$\rho = 10^{-2} \alpha_\rho \bar{\varphi}(T)$$  \hspace{1cm} (6.1)

where

$$\bar{\rho}(T) = a^2 \sum_q \int_{-\infty}^{\infty} d\omega \frac{e^{\omega/T}}{(e^{\omega/T} - 1)^2} \chi_{AF}'(q,\omega)$$  \hspace{1cm} (6.2)

and $\alpha_\rho$ is a constant ($\leq 1$) which depends on the matrix element for quasiparticle-paramagnon scattering, the density of quasiparticles, etc. On carrying out the sum over $q$, one then obtains:

$$\bar{\rho}(T) = \frac{\beta^{1/2}}{2\Gamma} T \int_0^\infty dx \frac{x e^x}{(e^x - 1)^2} \left\{ \tan^{-1} \frac{xT}{\omega_{SF}} - \tan^{-1} \frac{xT}{\omega_{SF} + 4\Gamma/\beta^{1/2}} \right\}$$  \hspace{1cm} (6.3)

In this form, we see that as long as $\omega_{SF} \sim T$, one is guaranteed a linear resistivity, no matter what the magnitude of $\omega_{SF}$. (Moriya et al. reached a similar conclusion using the linearity with temperature of $(a^2/\xi^2)$ as their criterion.)

This condition is a sufficient, but not a necessary one, as a numerical evaluation of the integral in (6.3) reveals. In Fig. 12a we give our results for $\bar{\rho}(T)$ for several choices of $\beta$ for La$_{1.85}$Sr$_{0.15}$CuO$_4$, and for YBa$_2$Cu$_3$O$_7$ and YBa$_2$Cu$_3$O$_{6.63}$. We see that the temperature dependence of the resistivity is
remarkably linear, for all three materials, despite the fact that the form for \( \omega_{SF}(T) \) at low temperature is far from linear in the case of both \( \text{O}_{6.63} \) and \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_{4} \). These results suggest that the resistivity linearity results in part from the fact that the significant low frequency spin excitations, the paramagnons, have energies which are less than \( kT \).

We have examined this possibility by calculating \( \bar{\rho}(T) \) for Debye paramagnons, taking two examples in which \( \omega_{SF} \) is independent of temperature; \( \hbar\omega_{SF} = 30\text{K} \) and \( \hbar\omega_{SF} = 60\text{K} \). Our results, using \( \beta = \pi^2 \) to facilitate comparison with those we obtained for the \( \text{O}_{6.63} \) sample, are given in Fig. 12b. We see there that at low temperatures one finds a quadratic temperature dependence which, however, for \( kT \gg \hbar\omega_{SF} \) (in the case of the Debye paramagnons) switches over to a linear temperature dependence. Comparison of the 30K Debye paramagnon result with our results for \( \text{YBa}_{2}\text{Cu}_{3}\text{O}_{6.63} \) shows that the calculated resistivities begin to differ for \( T \geq 100\text{K} \), where the \( \text{O}_{6.63} \) paramagnon energy begins to increase linearly with temperature. Because the latter energy is larger than 30K, the resistivity curve falls below the Debye paramagnon result; moreover, because \( (\hbar\omega_{SF})_{6.63} \) increases linearly with temperatures for \( T \geq 120\text{K} \), we find \( \bar{\rho}(T) = bT \), rather than an "apparent" high temperature resistivity of the form \( \bar{\rho}(T) = b'T - a \). Put another way, for \( T < 120\text{K} \), the transition to \( (\hbar\omega_{SF})_{6.63} = \text{const} \), leads to an inflection in \( \bar{\rho}(T) \), and an apparently larger slope, \( b' > b \); however because \( T_c > \hbar\omega_{SF} \), one doesn't see the beginnings of a transition to quadratic behavior found in the model calculations.

Two other features of these results deserve mention. First, for a given material, as \( \xi_0 \) decreases (\( \beta \) increases), the slope of the linear resistivity increases. If \( \beta \) becomes too large, the result is a departure from linearity at high temperature, which may be traced to the contribution to the integral
coming from the \[ \tan^{-1} \frac{x T}{\omega S F + 4\Gamma/\beta^{1/2}} \] term. Second, for a fixed value of \( \beta \), one sees clearly the influence of increased antiferromagnetic correlations on the slope of the resistivity; for \( \beta = \pi^2 \), as one goes from \( O_7 \) to \( O_{6.63} \) to \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \), the slope of the resistivity curves increases. It is not, however, simply proportional to either \( \beta^{1/2} \) or to \( 63(R_{af})_\parallel \), as a close inspection of the results depicted in Fig. 12a shows.

We do not here attempt a quantitative comparison with experiment for these materials, primarily because we do not yet possess a method for obtaining a reliable quantitative estimate of the constant, \( \alpha_p \), which we expect to vary from material to material, and which strong coupling (i.e. vertex) corrections may reduce considerably from the "weak coupling" estimate, \( \alpha_p \approx 1 \), given by Moriya et al. [16]. Thus it is not at present possible to use results for the resistivity to pin down the value of \( \beta \) for \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \). It is hoped that future work might make this possible.

VII. Discussion and Conclusion

Because the experimental data presently available for \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \) is considerably more limited than that available, say, for \( \text{YBa}_2\text{Cu}_3\text{O}_7 \) (for which \( 63K_s, 17W, \text{and } 63W_\perp \) have also been measured), it is not possible to obtain all of the antiferromagnetic Fermi liquid parameters of interest directly from a fit to experiment. Thus while there can be no question that \( 63(R_{af})_\parallel \) for \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \) is greater at all temperatures than the corresponding quantity for \( \text{YBa}_2\text{Cu}_3\text{O}_{6.63} \), whether that greater enhancement results from a lower value of \( \xi_0 \), as the scaling arguments presented in Section IV suggest, or reflects a larger value of \( \xi/a \) (as would be the case if \( \xi_0 \) for the two systems remains the same), remains to be determined. As we have seen, a
measurement of $^{17}\text{W}(T)$ would not only provide information on $\xi_0$, but would also supply valuable information on whether the near antiferromagnetism is commensurate or incommensurate.

Our results support the proposal [14] that the properties of a nearly antiferromagnetic Fermi liquid are genuinely novel, and suggest that both the spin and charge aspects of the normal state properties of the cuprate oxide superconductors can be quantitatively explained in terms of quasiparticles coupled to antiferromagnetic paramagnons whose characteristic energy scale is $\lesssim kT$. The low-frequency magnetic properties of a nearly antiferromagnetic liquid differ markedly from those of the usual Landau Fermi liquid, in that the dominant low frequency excitations are not found at long wavelengths, but rather for wavevectors in the vicinity of $Q; these paramagnons in turn give rise to a quasiparticle lifetime which, for $T > T_c$, we expect to be given by [cf MPT],

$$\left(\frac{\hbar}{\tau}\right)^{-T}$$

rather than the usual Landau result, $$(\hbar/\tau) \sim T^2$$. In similar fashion, for $T \approx T_c$, the contribution to the resistivity from quasiparticle-quasiparticle interaction (once Umklapp processes are taken into account) will not be $\sim T^2$, as is the case for a normal Fermi liquid, but rather $\sim T$, as we have shown above.

The precise form of the resistivity depends on the temperature dependence of $\omega_{\text{SR}}$, and hence on $(a^2/\xi^2)$. As we have seen NMR experiments provide direct information on $a^2/\xi^2(T)$, and show that the mean field form, Eq. (3.14), is well-obeyed above $T \approx 130\text{K}$ for all three cuprate oxide superconductors thus far studied in detail. The results of the preceding section show that once one gets the mean-field form for the correlation lengths, one obtains a resistivity which varies linearly with temperature.
A quite unexpected feature of nearly antiferromagnetic cuprate oxide superconductors is the strongly temperature dependent static magnetic susceptibility found for YBa$_2$Cu$_3$O$_{6.63}$ [10] and here for La$_{1.85}$Sr$_{0.15}$CuO$_4$. It is natural to attribute this behavior to the increasingly important role of antiferromagnetic correlations as the temperature decreases [13], [14], and the results presented in Fig. (8) support a correlation between the "reduced" long wavelength static susceptibility, $[\chi_0(T)/\chi_0(300K)]$ and the "enhanced" antiferromagnetic static susceptibility, $[\chi_Q(T)/\chi_0(T)]$. This explanation for $\chi_0(T)$ will, however, remain only a plausible conjecture, until born out by microscopic model calculations which show that the presence of low frequency antiferromagnetic paramagnons can lead to a depressed value for $\chi_0(T)$. [In passing, we note an appealing candidate for the "dangerous" diagrams responsible for this anomaly are vertex corrections to the local field Fermi liquid parameters, $F^a_0$, which might yield $F^a_0(T)$. Calculations are underway to verify this conjecture.]

It is likewise reasonable to expect that the antiferromagnetic paramagnons will give rise to a temperature-dependent quasiparticle effective mass, and the recent work of Moriya et al. [16] supports this idea.

For systems which display a temperature-dependent long wavelength static magnetic susceptibility, we do not expect that a straightforward local mode coupling application of Ginzburg-Landau theory to obtain, say, $a^2/\xi^2(T)$, (cf [16]) will be successful; rather a self-consistent, "strong-coupling" version will need to be developed and applied to obtain the results found by MPT for YBa$_2$Cu$_3$O$_{6.63}$ and by us for La$_{1.85}$Sr$_{0.15}$CuO$_4$.

The results we have obtained for the antiferromagnetic correlation parameters for the three materials provide a tantalizing hint of the possible connection between antiferromagnetic behavior and the transition to
superconductivity. As one of us noted [13], on the basis of a very preliminary and approximate fit of MMP theory to the NMR experiments on O_{6.63} and La_{1.85}Sr_{0.15}CuO_{4}, it is evident that the closer the system is to becoming antiferromagnetic, the lower its superconducting transition temperature. The hierarchy we have established for 63R_{af} (a quantity which is independent of our choice of β) is thus inverted for transition temperatures. As may be seen in Table 2, one can establish an equally good qualitative correlation between, say, ħω_{SF}(T_c) and T_c. However, the results presented there show that there does not exist a simple proportionality, T_c ~ ħω_{SF}(T_c), which the preliminary analysis [13] suggested. Given the fact that the quasiparticle lifetimes are comparatively short, and that ħω_{SF} < kT, it is clear that any calculation of superconductivity based on an attraction produced by the Coulomb correlations responsible for the antiferromagnetic behavior will have to go well beyond taking into account the attraction between a pair of quasiparticles produced by the exchange of a single antiferromagnetic paramagnon.

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Appendix 1

The experimental results of Alloul, Mendels, and Ohno [11] for the Knight shift and relaxation rate for $^{89}$Y nuclei in a number of different samples of YBCO provide valuable information of the influence of oxygen content on both $\chi_0(T)$ and the antiferromagnetic Fermi liquid parameter, $\Gamma(T)$. In this appendix we present an analysis of the data of Alloul et al. based on taking the hyperfine Hamiltonian for the interplanar Y atoms (compare Eq. 2.1) to be

$$H_Y = \sum_{<ij>,a}^{89}{I_i,a} D S_{j,a}$$

where $^{89}I_{i,a}$ is the $\alpha$th component of the $^{89}$Y nuclear spin, $S_{j,a}$ is the $\alpha$th component of the planar Cu spin, and $<ij>$ is the sum over nearest neighbors; the Y Knight shift is then given by

$$^{89}K^{s}_{iso} = \frac{8D}{^{89}\gamma_Y e\hbar^2} \chi_0$$

while the corresponding relaxation rate (see MMP) is

$$^{89}W = \frac{3}{4} \frac{1}{\mu_B \hbar} \lim_{\omega \to 0} \sum_{q} [16D^2(1 - \cos(q_x a))] [1 - \cos q_y a] S(q,\omega)$$

which upon carrying out the integral over $q$ becomes
\[
89W = \frac{6D^2\pi}{\mu_B^2\hbar} \frac{\chi_0(T)k_B T}{\hbar \Gamma(T)} \left[ \frac{1 + 0.2 \frac{\beta}{\pi^2}}{1 + \beta^{1/2}/2\pi^2} \right]
\]  

We first obtain \(\chi_0(T)\) from the Y Knight shift data of Alloul et al., by assuming, as a number of authors [Walstedt et al. (17), Butaud et al. (18) Imai (19)] have previously done, that the \(^{89}Y\) chemical shift is \(-200\) ppm, and taking the hyperfine coupling, \(D\), to be that obtained by MPT,

\[D = -3.0 \text{ kOe}/\mu_B\]  

Our results are shown in Fig.13, where they are compared with those for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) and YBa\(_2\)Cu\(_3\)O\(_7\). We see that the result we have obtained for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) is consistent with the correspondence between LSCO samples and YBCO samples proposed by Torrance, Eqtn. (4.3), which leads one to expect that the results for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) should map onto samples of YBa\(_2\)Cu\(_3\)O\(_6.80\). Thus the temperature dependent susceptibility, \(\chi_0(T)\), we have derived from the Knight shift experiments of Kitaoka et al., [3] falls in-between the results obtained by MPT for O\(_{6.63}\) and that of MMP for O\(_7\), \(\chi_0 = 2.62\) states/ev.

We next combine our result for \(\chi_0(T)\) with the results of Alloul et al. for \(^{89}W(T)\) to plot, in Fig. 14, \(^{89}\left[T_1T\chi_0(T)\right]\) as a function of oxygen concentration.

We use for all samples, the value, \(\beta = \pi^2\) found by MPT for O\(_{6.63}\), and normalize the expression (cf Eq. A3b) so that we obtain agreement with the results of MPT for \(\Gamma(T)\) for O\(_{6.63}\). As may be seen there, for all the Alloul et al. samples with oxygen content equal or greater than O\(_{6.63}\), we find
(A5)

\[ \Gamma(T) = 0.4 \pm 0.04 \text{ ev}, \]

and this is the value we use in our calculations on La\textsubscript{1.85}Sr\textsubscript{0.15}CuO\textsubscript{4}. 
References


13) D. Pines, Proc. of the ICPHCES 89, eds. J. Smith and A. M. Boring, to be published in Physica B.


19) T. Imai, unpublished.
Legends for Figures

Fig. 1. The spin component, $^{17}$K$_s$(T) of the planar oxygen Knight shift and the planar spin susceptibility, $\chi_0$(T).

Fig. 2. The $^{63}$Cu nuclear spin relaxation rate as measured by two different experimental groups: (•) Imai et al. [4]; (□) Kitaoka et al. [3].

Fig. 3. The experimentally determined product, $(T_1)_{NQR}$T$\chi_0$(T), from the experimental measurements of Imai et al. [4] (•) and Kitaoka et al. [3] (□) is plotted versus the temperature.

Fig. 4. The calculated values of antiferromagnetic enhancement factor, $(\frac{^{63}R_{af}}{\Omega})_{\parallel}$ are plotted as a function of temperature for three cuprate oxide superconductors: La$_{1.85}$Sr$_{0.15}$CuO$_4$ (crosses); YBa$_2$Cu$_3$O$_{6.63}$ (points); YBa$_2$Cu$_3$O$_7$(squares). The data of Imai et al. [4] has been used for La$_{1.85}$Sr$_{0.15}$CuO$_4$; very nearly the same result is obtained from the data of Kitaoka et al. [3].

Fig. 5. The antiferromagnetic correlation length as a function of temperature for La$_{1.85}$Sr$_{0.15}$CuO$_4$ (x), YBa$_2$Cu$_3$O$_{6.63}$ (•), and YBa$_2$Cu$_3$O$_7$ (□). For La$_{1.85}$Sr$_{0.15}$CuO$_4$ we have used three different values of $\beta$: $\beta = 4.5\pi^2$(x), $\beta = 3\pi^2$ (upper dashed line); $\beta = 6\pi^2$ (lower dashed line). For La$_{1.85}$Sr$_{0.15}$CuO$_4$ we have used our values calculated from the data of Imai et al. [4]. As may be seen in the insert, essentially the same result is obtained using the data of Kitaoka et al. [3] (□).

Fig. 6(a). The inverse correlation length squared, $(a^2/\xi^2)$ determined from fits to the experiments of Kitaoka et al. [3] (□) and Imai et al. [4] (•) plotted as a function of temperature for $\beta = 4.5\pi^2$. The dotted lines represent linear fits to the data above T = 130K. The corresponding values for $\beta = 3\pi^2$ (lower dashed line) and $\beta = 6\pi^2$ (upper dashed line) are also given.
6(b). The inverse correlation length squared, \( (\alpha^2/\xi^2) \), determined for \( \beta = 4.5\pi^2 \) from the experiments of Imai et al. [4] (x), is compared with the corresponding values calculated by MPT for YBa\(_2\)Cu\(_3\)O\(_7\) (square) and YBa\(_2\)Cu\(_3\)O\(_{6.63}\) (circle). The dotted lines represent the best linear fits to the data.

Fig. 7. (a) The calculated values of \( \chi_Q(T) \) for \( \beta = 4.5\pi^2 \) are plotted as a function of temperature for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) (crosses), YBa\(_2\)Cu\(_3\)O\(_{6.63}\) (points), and YBa\(_2\)Cu\(_3\)O\(_7\) (squares).

(b) The calculated values of \( \chi_Q(T)/\chi_0(T) \) for \( \beta = 4.5\pi^2 \) are plotted as a function of temperature for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) (crosses), YBa\(_2\)Cu\(_3\)O\(_{6.63}\) (points), and YBa\(_2\)Cu\(_3\)O\(_7\) (squares). In these calculations for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) we have used the values of \( \xi/\alpha \) determined from the experimental results of Imai et al. [4]; again, similar results are obtained from the data of Kitaoka et al. [3].

Fig. 8. The value of \( \chi_Q(T)/\chi_0(T) \) calculated for three different choices of \( \beta \) are plotted against \( \left[ \chi_0(300)/\chi_0(T) \right] \) and compared with the proposed form of a "universal" relation between \( \left( \chi_0(300K)/\chi_0(T) \right) \) and \( \chi_Q(T)/\chi(T) \) derived from the MPT analysis of NMR experiments on YBa\(_2\)Cu\(_3\)O\(_{6.63}\) (circle). The results for \( \beta = 4.5\pi^2 \) are denoted by square; the upper dotted line is \( \beta = 6\pi^2 \), the lower dotted line is \( \beta = 3\pi^2 \).

Fig. 9. Predicted value of the ratio \( ^{63}W_{||}(T)/^{17}W(T) \) as a function of temperature for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\), using three different values of \( \beta \): \( \beta = \pi^2 \) (circle), \( \beta = 4.5\pi^2 \) (triangle), \( \beta = 6\pi^2 \) (square).

Fig. 10. Predicted value of \( ^{63}W_{\perp}/^{63}W_{||} \) as a function of temperature for La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) three different values of \( \beta \): \( \beta = \pi^2 \) (circle), \( \beta = 3\pi^2 \) (triangle), \( \beta = 6\pi^2 \) (square).
Fig. 11. The spin fluctuation temperature, $\omega_{SF}$, is plotted as a function of temperature for La$_{1.85}$Sr$_{0.15}$CuO$_4$ with $\beta = 4.5\pi^2$ (crosses), and compared to results obtained previously by MPT for YBa$_2$Cu$_3$O$_{6.63}$ (●), and YBa$_2$Cu$_3$O$_7$ (□). For La$_{1.85}$Sr$_{0.15}$CuO$_4$ we also give our results for $\beta = 3\pi^2$ (lower dotted line) and $\beta = 6\pi^2$ (upper dotted line).

Fig. 12(a). The reduced resistivity due to quasiparticle-antiferromagnetic scattering calculated using the formula of Moriya et al. [16] for La$_{1.85}$Sr$_{0.15}$CuO$_4$, using the value, $\beta = \pi^2$ [□] $\beta = 4.5\pi^2$ [x] and $\beta = 6\pi^2$.

Also shown are comparable results for YBa$_2$Cu$_3$O$_7$ [x] and YBa$_2$Cu$_3$O$_{6.63}$ [●].

(b). The temperature dependence of the reduced resistivity calculated using $\beta = \pi^2$, $\omega_{SF} = 30K$, denoted by a dotted line, $\omega_{SF} = 60K$ denoted by a dashed line. Shown for comparison is the result obtained for O$_{6.63}$ for which $\omega_{SF} = 30K + 38(T/100) K$ (□).

Fig. 13. A comparision of $\chi_0(T)$ for La$_{1.85}$Sr$_{0.15}$CuO$_4$ with that obtained by MPT from the data of Alloul et al. [11] for the Y Knight shift in the YBCO family.

Fig. 14. Calculated values of $\Gamma(T)$ for YBCO samples obtained from the Knight shift and relaxation rate experiments on Y nuclei by Alloul et al. [11].
Table 1. Hyperfine Couplings (in kOe/μB) for planar Cu and O nuclei

<table>
<thead>
<tr>
<th>Hyperfine couplings</th>
<th>$A_\parallel$</th>
<th>$A_\perp$</th>
<th>$B$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deduced values</td>
<td>$-163\pm3$</td>
<td>$34\pm1$</td>
<td>$40.8\pm1$</td>
<td>$69\pm2$</td>
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</table>

Table 2. Antiferromagnetic Fermi liquid parameters for three cuprate oxide superconductors

<table>
<thead>
<tr>
<th>Superconductor</th>
<th>$\langle 63^{R_{af}} \rangle_\parallel(T_c)$</th>
<th>$(ξ/\alpha) \text{ at } T_c$</th>
<th>$\omega_{SF}(T_c)[°K]$</th>
<th>$T_c[°K]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>YBa$_2$Cu$_3$O$_7$</td>
<td>8</td>
<td>2.25</td>
<td>94</td>
<td>90</td>
</tr>
<tr>
<td>YBa$_2$Cu$<em>3$O$</em>{6.63}$</td>
<td>27</td>
<td>4.0</td>
<td>36</td>
<td>60</td>
</tr>
<tr>
<td>La$<em>{1.85}$Sr$</em>{0.15}$CuO$_4$</td>
<td>73</td>
<td>3.5</td>
<td>16</td>
<td>38</td>
</tr>
</tbody>
</table>
Fig. 1
Fig. 3

$(T_1)_{COF}$, $T\chi_0(T)$ [arb. units]

$T [K]$
Fig. 5
Fig. 6b
Fig. 7a

χ(T) [eV⁻¹]

T [°K]
Fig. 7b

Graph showing the relationship between $\chi_T / \chi_0(T)$ and $T^\circ K$. The data points are plotted against the temperature axis ($T^\circ K$) and the y-axis represents the normalized susceptibility $\chi_T / \chi_0(T)$. The graph includes both circular and cross symbols.
Fig. 8
Fig. 9
Fig. 12a
Fig. 12b
Fig. 13

\[ \chi_0(T) / \mu_B^2 \text{[eV]} \]

Temperature [K]
Fig. 14