A Double Auction Market for Computerized Traders

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1. Introduction

The double auction (DA) is a type of trading institution used by the Chicago Board of Trade, the New York Stock Exchange, and many other securities and commodities exchanges throughout the world. In the DA, buyers and sellers are simultaneously able to call out bids (offers to buy) and offers (offers to sell), and can accept the lowest outstanding offer or highest outstanding bid at any point in the trading process. The rapid flow of information combined with the ability of traders to instantly undercut an outstanding bid or ask makes the DA perhaps the closest embodiment of the economist's notion of a perfect frictionless market.

The efficiency properties of the DA institution have been documented in numerous laboratory experiments using human subjects. By assigning subjects tokens with fixed redemption values, the experimenter can create an artificial market with predetermined supply and demand curves. Economic theory predicts that trading should occur at a price and quantity where supply equals demand: this is the so-called competitive equilibrium solution. The nearly universal finding of more than two decades of experiments is that price and quantity very quickly converge to the competitive equilibrium values.

Surprisingly economists do not yet have a complete theory that explains how and why prices and quantities converge to the competitive equilibrium values. The problem is that while the experimenter knows the supply and demand curves, none of the subjects do. Subjects only know the redemption values of their tokens and may have no technical knowledge of economics. Somehow, through the process of trading, subjects learn enough about the redemption values of their opponents to determine what the "fair market" price must be. Modern rational expectations theory treats DA trading as a continuous-time game of incomplete information. The theory is based on the assumption that smart players use trading strategies that maximize their expected profits given their beliefs about the token values and strategies of their opponents. However, economic and game theorists have not yet succeeded in completely solving or characterizing a set of equilibrium trading strategies, so it is not clear that the rational expectations approach will generate testable predictions, much less "explain" the experimental results. Lacking an understanding of how players form trading strategies, it is difficult to isolate the features of the DA institution responsible for its remarkable convergence and efficiency properties. In particular, current economic theory provides little guidance in predicting the impact of institutional changes such as the Chicago Board of Trade's new AURORA trading system that will ultimately replace the traditional oral double auction pit by a unified "electronic pit" that allows simultaneous trading by brokers who may be located virtually anywhere in the world.

In order to gain a deeper understanding of trading strategies and price formation in DA markets, the Santa Fe Institute is organizing a computerized DA tournament. Instead of human traders, the computerized DA market will consist entirely of program traders using strategies codified in one of the standard computer programming languages. Entrants are allowed to submit trading programs that operationalize their "market intuition," or that use any artificial intelligence, adaptive algorithm, or expert system that is capable of learning about market conditions and trading profitably. In order to encourage entrants to submit the most effective, carefully written, and well-documented strategies, the Institute is offering prize money in the amount of $10,000 to be distributed to entrants in proportion to the profits earned by their trading strategies over the course of the tournament.
2. Background on Experimental Double Auction Markets

In a typical experimental DA market, traders are assigned the roles of buyers or sellers and have induced preferences via allocations of tokens at pre-assigned token costs (sellers) and redemption values (buyers) set by the experimenter. For example, one subject may be assigned the role of seller with the right to sell two tokens, one with a cost of $1.20 and the other with a cost of $2.10. Another subject may be designated a buyer with the right to buy three tokens at redemption values of, say, $3.30, $1.50, and $.90. If the buyer purchases a token from the seller at a price of $2.00, he earns a profit of $1.30 by redeeming the token for $3.30. The seller sells a token costing $1.20 to the buyer at a price of $2.00, yielding a profit of $.80.

A DA market is run for a fixed period of time, typically several minutes, during which traders call out bids and offers attempting to negotiate the most profitable deals. Under the so-called New York Rules, transactions occur only when some trader accepts the current bid or current offer. The current bid is the highest outstanding bid and it remains outstanding until it is either displaced by a higher bid or is accepted by a seller. Similarly, the current offer is the lowest outstanding offer and it remains outstanding until it is either displaced by a lower offer or is accepted by some buyer. New York rules obligate holders of the current bid or offer to transact the instant another trader accepts their bid or offer. At the conclusion of the DA, a player's profit is equal to the sum of the differences between the transaction price and redemption value or token costs of each token traded, or zero if no tokens were traded. Thus, the seller's objective is to sell tokens at prices as high as possible above their token costs, while the buyer's objective is to buy tokens at prices as low as possible below their redemption values.

By controlling the assignment of token costs and redemption values, the experimenter can generate induced supply and demand curves such as those shown in Figure 1, below. The intersection of these curves defines a theoretical equilibrium price and quantity at which trade is predicted to occur in a perfectly competitive market. According to traditional economic theory, a market is deemed to be perfectly competitive only if it satisfies the following requirements: 1) traders are rational, i.e., their sole objective is to maximize trading profits, 2) there are no transaction costs or price limits on trading, 3) there are large numbers of buyers and sellers, and 4) traders have common knowledge of the market clearing price. Typical experimental markets are designed to satisfy requirements 1) and 2), however they rarely satisfy requirements 3) and 4). Experimental DA markets are usually run with a small number of buyers and sellers, some with as few as 3 or 4 each. Furthermore, while the experimenter knows the redemption values of all tokens assigned to traders, this information is almost never given to the subjects, who are only told their own redemption values. Requirement 3) is thought to be necessary to avoid traders from exploiting possible monopoly or oligopoly power present in markets with only a small number of buyers or sellers. Requirement 4) is thought to be necessary to avoid inefficiencies arising from "bluffing" or strategic misrepresentation that can occur in markets with private information. Despite the fact that experimental DA markets don't satisfy all the traditional economic requirements for a perfectly competitive market, observed transaction prices and quantities rapidly converge to the competitive equilibrium values. Charles Plott, a leading experimentalist, summarized the evidence this way:

"The overwhelming result is that these markets converge to the competitive equilibrium even with very few traders. Efficiency levels [the ratio of actual to theoretical profits] tend to converge to near 100 percent. If a change in parameters occurs, such as a shift in demand or supply, the prices converge to the new equilibrium after two or three periods. As long as the institutional structure has [at least] a few buyers and sellers, these convergence and efficiency properties appear to be independent of the basic economic conditions. Different shapes of demands and supplies as systematically examined by Smith (1962) yield no substantial differences. In all cases, after a few periods the market performance was close to that predicted by the competitive model." (Plott, 1982, p. 1493)
Figure 1 (reproduced from Smith 1976) provides a graphical summary of the rapid convergence that is typical of DA experiments.

**Figure 1: Example of experimental DA market.**
Supply and demand curves are shown on left. Contract prices of each transaction are shown on right for four different experiments.
3. Possible Explanations of the Experimental Results

As we mentioned in the introduction, economists do not yet have a good theory of the dynamic process by which prices and quantities converge to the competitive equilibrium (CE) values. Traditional economic theory treats all trading as occurring in equilibrium, i.e., at the market clearing price. However in any real market, including the experimental markets described above, initial trading is done out of equilibrium since traders are initially ignorant of what the market clearing price will ultimately be. Somehow traders are able to learn enough about their opponents' token values during initial stages of trading to rapidly deduce what a "fair" or market clearing price must be.

To date three main types of mathematical models have promised a potentially rich source of hypotheses about traders' learning processes and strategic behavior in the DA: 1) traders are Bayesian learners who employ "rational equilibrium" (RE) strategies, 2) traders use strategies based on fixed, simple behavioral rules-of-thumb, or 3) traders use strategies based on learning algorithms that are continually self-evolving in response to past performance in the marketplace.

The first approach treats the DA as a continuous-time game of incomplete information. The theory is based on the assumption that all traders are rational expected-profit-maximizing Bayesian decision makers, or in more common language, ruthlessly intelligent and extremely greedy. In addition the theory assumes that each trader will adopt a Nash equilibrium trading strategy: i.e., a strategy that maximizes each trader's expected profits given their (mutually consistent) beliefs about the token values and strategies of their opponents. The resulting set of strategies are (sequential Nash) equilibrium strategies in the sense that no trader can find a trading strategy that yields higher expected profits given the strategies of his opponents. Although this theory treats all trading as occurring in equilibrium, the relevant equilibrium here is within the larger framework of the continuous-time trading game rather than the traditional static market game where supply always equals demand at a known market price. The hope is that the equilibrium paths generated using the rational expectation approach might "look like" and hence "explain" the price dynamics observed in experimental DA markets. Unfortunately, owing to the extremely high strategic complexity of the continuous-time DA game, no one has ever succeeded in completely solving for a set of equilibrium trading strategies. Robert Wilson has partially characterized a set of rational expectations equilibrium strategies in a simplified version of the DA, concluding:

"The suggested equilibrium ... offers a concrete explanation of the mechanism by which the dispersed information about traders' valuations is manifested in the prices at which transactions are consummated. The mechanism, according to the present hypothesis, is multilateral sequential bargaining in which the traders are endogenously matched for transactions via a signalling process using delay as the primary signal. Other signalling mechanisms may be possible, but it appears that delay suffices and therefore this provides a presumptive hypothesis from which further studies can proceed. (Wilson, 1984, p. 38)

However, Wilson openly admits some of the problems and limitations of the current version of the rational expectations theory:

"Some of the implications of our model and the specified equilibrium are, in fact, too strong to fit the data well. For example, the equilibrium predicts that traders transact in order of their valuations, and that no traders with extra-marginal valuations [e.g., seller's redemption values above the competitive equilibrium price] succeed in trading. As Easley and Ledyard (1986) report, these properties are often contradicted by experiments." (Wilson, 1984, p. 37)

All of the above remarks must, of course, be taken as speculative until the theory of affiliated random variables is applied ... to determine whether or not the proposed
equilibrium strategies also satisfy the requisite global optimality properties that would be sufficient to establish the validity of the equilibrium." (Wilson, 1984, p. 38)

The fact that Wilson's particular formulation is at odds with experimental findings should not lead us to reject the game-theoretic RE approach. Indeed, a recent result of Ledyard (1986) shows that in principle there exists a game-theoretic model of the DA market with equilibrium paths that are perfectly consistent with the experimental data! Unfortunately Ledyard's result is something of an embarrassment of riches since it also implies that there generally exist many different game-theoretic models that are consistent with the same data: such models fail to generate testable hypotheses about behavior. Ledyard's result relies on three degrees of freedom to provide the flexibility to match the game theoretic outcomes with observed behavior:

1) choice of traders' degree of risk aversion,
2) choice of traders' initial beliefs about their opponents, and
3) choice of a particular RE equilibrium.

If the game theoretic approach is to have empirical content, we will have to restrict the available choices in these dimensions. One can make an a priori argument that traders in a DA should be risk neutral, i.e., they seek to maximize expected trading profits, provided DA earnings are not too large. In addition, in experimental situations traders' initial beliefs can be at least partially controlled by the experimenter, e.g., by generating token values using a publicly announced random generator. Thus, a carefully designed DA experiment ought to offer the theorist less freedom of choice along dimensions 1 and 2. The main problem is in the third dimension: there are generally a vast multiplicity of equilibria in continuous-time DA games. The game theoretic approach must not only assume that traders have the enormous computational capacity to compute even a single set of RE strategies, but that they somehow have the clairvoyance to mutually select the same one. But even then, the theory offers no guidance as to which particular RE will be selected. Different equilibria can have very different empirical implications. Typically game theorists rely on a host of more or less ad hoc criteria for selecting a particular RE which is felt to be most "plausible," but unfortunately this is a subjective judgement which has been a source of disagreement in the proliferating literature on "equilibrium refinements."

In view of this, it seems that the most promising approach is to characterize the behavioral features that are shared by all RE of the DA game. Recent work by Williams (1988) and Satterthwaite and Williams (1989) has followed this approach, yielding considerable insight into the nature of equilibria in a one-shot version of the double auction known as the buyers bid double auction (BBDA). The BBDA is a type of sealed bid double auction in which each seller submits an offer and each buyer submits a bid. The bids and offers are then arrayed to form supply and demand based on the announced bids and offers instead of the true token costs and redemption values. Generally there will be a range of market clearing prices (as in Figure 1), but in the BBDA the closing price is always chosen as the highest market clearing price (the name BBDA arises from the fact that in the bilateral case, i.e., only one buyer and seller, the closing price equals the buyer's bid whenever trade occurs). Once the closing price is determined, trade occurs between the buyers and sellers whose bids and offers are to the left of the intersection of supply and demand at a price equal to the closing price. It is easy to show that sellers cannot influence the closing price in the BBDA; therefore, it is always in their best interest to behave honestly and set their offers equal to their true token costs. Buyers, on the other hand, generally gain by misrepresentation and bid below their true redemption values. This misrepresentation leads to inefficient outcomes: the number of tokens traded is generally less than the number of tokens traded if all buyers had bid

1It is much more difficult (if not impossible) for the experimenter to control traders' initial beliefs about the types of strategies they expect their opponents to use. In game-theoretic terminology, there is no guarantee that traders have common knowledge of the rationality of their opponents.
truthfully. What Satterthwaite and Williams have shown, however, is that in any RE of the
BBDA, the difference between the buyers' bids and their true redemption values goes to zero at
rate O(1/min(n,m)) where n is the number of sellers and m is the number of buyers. This implies
that any RE of the BBDA quickly converges to a CE as the number of buyers and sellers becomes
large. Indeed, Satterthwaite and Williams have numerically computed a specific RE to the BBDA
(where tokens are generated as IID U(0,1) random deviates) and have shown that the RE is close
to CE with as few as 4 buyers and 4 sellers. Experimentalists have generally found (see Smith,
1982b) that the dynamic form of the double auction yields more efficient allocations than the sealed
bid double auction. Thus, the results of Satterthwaite and Williams suggest that we should expect
that RE of the dynamic double auction will produce allocations that are also close to a CE,
providing a possible "explanation" of the experimental findings. The Satterthwaite-Williams
results are only suggestive, however, because they are based on a particular trading rule (the
BBDA) and a particular specification of players' beliefs, i.e., the so-called independent private
values assumption that tokens are IID draws from a continuous distribution with non-zero density
on the unit interval. It is not yet known whether the results will generalize to the case where there
is correlation in token values across traders, or to other trading rules such as the dynamic DA.

While the game-theoretic approach offers considerable promise for understanding DA
markets, there still remain a number of unresolved questions. One apparent puzzle is the game-
theoretic finding that the one-shot sealed bid form of the DA ought to be at least as efficient as the
dynamic form of the DA, contrary to experimental findings. Wilson (1985) used the revelation
principle to show that in his formulation of the DA market, a type of sealed bid DA (very similar to
the BBDA) is an efficient trading mechanism, i.e., it maximizes expected trading profits. Thus the
sealed bid DA ought to perform at least as well as any other trading rule, including the dynamic
DA. We call this an "apparent puzzle" because experimental sealed-bid DA's do not exactly
correspond to Wilson's setup in terms of the precise form of the trading rule, numbers of tokens
per player, or assumptions about traders' beliefs.

Given the difficulty of analyzing equilibria of the dynamic DA, it is still not clear whether the
empirical implications of the rational expectations hypothesis will ever be fully elucidated, and even
so, whether such models are capable of explaining the dynamics of convergence of price and
quantity to CE in experimental markets with small numbers of traders.

The second approach is based on the hypothesis that traders use strategies consisting of
simple behavioral rules-of-thumb. This approach has intuitive appeal, since most humans find the
DA relatively easy to play even though game theorists find it extraordinarily difficult to derive
optimal strategies. An excellent example of this approach is the work of Easley and Ledyard
(1986) who to our knowledge were the first to construct a model of disequilibrium trading that
predicts convergence to competitive equilibrium even in dynamic DA markets with small numbers
of buyers and sellers. They conclude that

"The potential importance of this theory is not just that it seems to describe what
happens in DA experiments, but also that it is the beginning of a positive theory of
how market prices are formed and of how they adjust to changes in supply and
demand conditions. The question of price formation has a long history of ad hoc
and unsuccessful attempts at an answer. Our theory is also ad hoc in the sense that
we make assumptions on individual behavior which are not derived from an
optimizing model. However, our assumptions seem sensible and, more important,
they seem to do a reasonable job of describing actual bids, offers, and transactions.
There is now a target for experimentalists to reject with data or for theorists to
improve on by obtaining a better fit with the data or a better explanation of observed
behavior." (Easley and Ledyard, 1986, pp. 36-37)

The Easley/Ledyard approach has not yet been subjected to rigorous empirical testing. It is not
clear whether it predicts the rapid convergence observed in human experiments, or whether it
explains the experimental "anomalies" in sequencing of trades that Wilson's RE model fails to capture.

The third approach is to use recent developments in learning algorithms and artificial intelligence to model the learning and strategy-formation process of traders. The difficulty with approaches based on rational expectations or fixed rules-of-thumb is that they yield behavior which can be non-robust, non-adaptive, and brittle. Although strategies generated by these approaches can be specially "tuned" to do well in specific environments, relatively small, unforeseen changes in the environment can cause the strategies to break down, yielding very poor performance without any hope of self-improvement.\(^1\) It seems that a *sine qua non* of "intelligent" behavior is the ability to *self-modify*. Self-adaptation of trading programs may turn out to be a critical feature for successful use of machine traders as "guinea pigs" to predict the impact of non-trivial changes in the DA institution.

The checker-playing program of Arthur Samuel (1963, 1967) was one of the first contributions to artificial intelligence. It is instructive to summarize his approach:

"Samuel used both dynamic (look ahead) and static (no look ahead) ways of evaluating any board position. The static method involved a simple mathematical function of several quantities characterizing any board position, and thus could be calculated practically instantaneously, whereas the dynamic evaluation method involved creating a "tree" of possible future moves, responses to them, responses to responses, and so forth. In the static evaluation function there were some parameters which could vary; the effect of varying them was to provide a set of different possible versions of the static evaluation function. Samuel's strategy was to select, in an evolutionary way, better and better values of those parameters.”

(Hofstadter, 1979, p. 604)

The level of play of Samuel's checkers program is extremely high: on the order of the best players in the world. Similar approaches have been applied to chess, with the best programs playing better than 99% of all rated, regularly playing, human chess players. It may turn out that such an approach might also produce extremely effective trading programs for the DA.

In fact, there are now a large variety of competing approaches in the artificial intelligence literature including approaches based on various types of expert systems, means-ends analysis (Newell and Simon, 1972), neural networks (Rumelhart, McClelland, et. al. 1986), learning and adaptive control (Narendra and Wheeler, 1986), *genetic algorithms* (Holland, 1975), and *classifier systems* (Holland et. al., 1986).

Genetic algorithms and classifier systems are based on two fundamental principles of biology: *survival of the fittest* and *genetic recombination*. The principle of survival of the fittest is used to assign a numerical quantity representing *strength* to competing decision rules. Decision rules that have performed well in the past (in terms of profit, for example) become stronger, while those that have performed poorly become weaker. Genetic algorithms can be thought of as a population of independent decision rules (each decision rule being a separate "organism"), whereas a classifier system consists of a single collection of a large number of decision rules operating in parallel, but with a common goal (the classifier system is like a single "organism," with decision

\(^1\)Rational expectations strategies are often brittle and non-robust as a result of the strong common knowledge assumptions that form the foundation of the Bayesian equilibrium concept. For example the RE approach requires players to have specific beliefs about the strategies and the distribution of redemption values of their opponents. If these beliefs are inappropriate, there is no guarantee that the RE equilibrium strategies will, in fact, be good strategies in the environment at hand.
rules as its "cells"). Decision rules are encoded as strings in an alphabet, such as strings of 0's and 1's. In classifier systems these strings take the form of condition/action rules. Information from the outside environment is translated into a list of input messages which are matched to the condition parts of each of the condition/action rules. If a message matches the condition part of a rule, then the action part of the rule can be executed, causing the classifier system to place a bid or accept an offer, for example. If a message matches the condition parts of more than one decision rule, the action that is ultimately taken depends on the strength of the rules involved, with higher strength rules having a higher probability of having their action parts executed.

The principle of genetic recombination is used to discover new, potentially more profitable decision rules. Treating the decision rules as strings of DNA, two parents are selected for "mating" on the basis of their strength and are genetically recombined using the cross-over operation. This results in two offspring, see Figure 2, which replace two of the weakest decision rules in the existing population.

```
Parents

1 0 0 0 0 0 0 0 1 1 0 1 1 1 1

Mating

1 0 0 0 0 0 0 0 1 1 0 1 1 1 1

Offspring

1 0 0 0 0 0 1 0 1 1 1 1 1 1 1
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Figure 2: Application of genetic cross-over to decision rule strings

Genetic algorithms and classifier systems may appear somewhat abstract, but they have been successfully applied to a variety of problems such as the repeated prisoners dilemma (Axelrod, 1987 and Miller, 1988), and regulation of gas pipeline transmissions (Goldberg, 1989). Marimon, McGrattan and Sargent (1989) have recently used classifier systems as traders in a dynamic trading game known as Wicksell's Triangle. Even though the traders are given little prior information about their environment, they eventually "discover" one of the RE equilibria of the trading game. It is possible that classifier-like systems could yield effective trading strategies in the DA tournament. The challenge is to find a convenient way to represent the history of the game in the form of a decision rule string so that application of the cross-over operation will produce sensible new strategies.

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1It is often convenient to also include a third "don't care" symbol, #, in any position of the condition/action rule in order to permit an input message to match the condition string regardless of whether there is a 0 or a 1 in the corresponding position of the input message.
4. Why Sponsor a Tournament?

The DA tournament was organized: 1) to gather scientific data which may offer new insights into the formation of effective trading strategies, 2) to compare the performance of program traders and human traders, and 3) to create an artificial market to help us better understand the operation of the 'invisible hand' in real world markets.

This is not the first tournament to be held exclusively for competing computer programs. Computerized chess tournaments are well-known. In economics, Robert Axelrod (1984) has sponsored a computer tournament for a game known as the Repeated Prisoner's Dilemma. The results of that tournament were surprising and instructive: some strategies supplied by game theorists did poorly in comparison to a very simple computer strategy, Tit for Tat, submitted by Anatole Rapoport.1 Axelrod's tournament succeeded in revealing new insights into how cooperation is established in repeated relationships, insights that were not forthcoming from existing game-theoretic literature. Following Axelrod's lead, the Santa Fe Institute is sponsoring the DA tournament in the hope of gaining new insights into the nature of efficient trading strategies and the role of the market mechanism. As we have shown in section 3, these insights have not been forthcoming from the traditional economics literature.

A natural question arises: why use an artificial market with program traders to study real-world markets with human traders? Our response is that experimental markets are much simpler to understand than real-world markets, and since they are artificial, we can conduct carefully controlled experiments to gather data that could never be obtained from a real market. The reason we want to use program traders instead of human traders is that we can't observe strategies in human experiments: we only observe behavior. Human strategies are unobservable simply because they are the result of some conscious (or even subconscious) thought process going on inside the subject's brain. However, in the DA tournament, we will directly observe entrants' trading strategies as well as the implied behavior. The tournament will provide a new window on the problem of strategic behavior: writing a computer program forces people to "verbalize" their thought processes, in some cases revealing their "real time" trading strategies. A final reason for running experiments with program traders instead of humans is that human experiments are much more costly to run (not only in terms of money outlays to subjects, but more importantly, in terms of the setup costs on the part of the experimenter and programming staff). There are many interesting institutional variations that we would like to test experimentally.

Program traders, on the other hand, don't require real cash payments and the use of fast computers allows one to "accelerate time" to run hundreds or even thousands of experiments overnight. If program traders are actually found to behave "like" or perform "as well" as human traders, and if they really do have good adaptation skills, then the machine traders might possibly serve as very cheap test subjects for various "what-if?" experiments on changes in institutional structure. A sufficiently realistic computerized market would then help us to "beta-test" institutional innovations before trying them out in the real marketplace.

In order to determine whether program traders really do behave like human traders, we plan to collect matching experimental data on the behavior of human subjects in an identical trading environment. We also plan to conduct mixed experiments where some traders are human and others are machines. Once we have the data there are a number of interesting scientific questions we can explore. We list some of them below:

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1It is interesting to note that Axelrod (1987) and Miller (1988) have shown that Holland-style genetic algorithms eventually "discover" strategies very similar to Tit for Tat.
1. What do winning trading strategies look like? Do they share any distinguishing characteristics? What past information do they rely on? Do they look ahead, anticipating future moves of their opponents as good chess players do? How "adaptive" are good strategies? Are there strategies that "make a killing" in a particular set of market conditions, but do poorly in others?

2. What statistical methods are appropriate for discriminating "artificial data" generated by machine traders from "real data" generated by human traders? Are there special experiments that allow us to discriminate human from computer traders, either collectively or individually? Will it turn out that even seasoned experimenters won't be able to distinguish machine traders from human traders?

3. Do DA markets with mixed human/machine traders experience any special problems? In repeated play, will humans be able to discover and exploit computerized trading strategies? Do computer traders generate any market instabilities?

4. Is it the case that lack of information about opponents' token values dominates the lack of information about their strategies? How well does a passive, price-taking strategy do relative to an aggressive, exploitative strategy?

5. What if a single subject is given monopoly power? Will machine traders be able to recognize their monopoly power and optimally price discriminate? What happens if we limit the amount of market information that traders observe: how much information can we delete until market performance starts to seriously degrade? Can we identify any "sufficient statistics" that carry most of the relevant information needed by the market? What happens when subjects are "asymmetrically informed" about market conditions, say, buyer B1 knowing the history of all player's bids and offers but buyer B2 only knowing only current bids and offers? Does the informational asymmetry seriously degrade performance of the market? What happens if we give certain subjects "inside information" about token values of their opponents? Does collusive behavior arise endogenously in these environments?

6. Suppose we succeed in solving for a set of RE strategies, perhaps numerically on a supercomputer. How will this "rational" strategy do in the tournament? How robust is it to changes in market conditions? What is a good measure of the "complexity" of a trading strategy? Is there a simple way to classify strategies and present them in an easily comprehensible form? Among AI-type strategies, what kinds of learning rules and self-adaptation procedures are employed? Can these strategies do as well as special purpose algorithms, or will they need extensive "training" in order to compete effectively?
5. Implementation of the Tournament

The tournament is implemented via a *message-passing protocol*. We have constructed a central *monitor program* that coordinates the trading process by communicating with the trading programs, executing their bids, offers, buy and sell orders, and informing them of the actions of other traders. The trader programs (which could also be an interface to a human trader) communicate only with the monitor, not directly with each other. It is important to note that the monitor is only a clerk: it is not an "auctioneer" and has no market clearing authority. The message-passing protocol synchronizes the traders and specifies the form of allowable messages that programs can send, but all this is transparent to tournament entrants. We have written "skeleton" trader programs in C, FORTRAN, and PASCAL to handle all the message-passing housekeeping, allowing entrants to focus solely on the logic of their strategies (other languages are allowed, but at the cost of having to write a program which handles the message-passing housekeeping).

The structure of the computerized DA game is very similar to the continuous-time experimental DA markets described in section 2. The major differences are 1) time is discretized into alternating bid/offer and buy/sell steps, and 2) the adoption of *Chicago Rules*.

The discretization of time was adopted to simplify the programming of trading strategies and to improve the synchronization of communications between players and the monitor in a multi-processing or network environment with delays that may vary from player to player or moment to moment. In our discrete-time formulation we have better control over how much cpu response time is used by the players, yet at the same time we feel there is little loss in generality over the continuous-time approach since a continuous-time trading environment can be approximated well by a discrete-time model with very many short trading intervals. Indeed, the discrete-time framework allows us to study certain questions (such as the effect of the length and number of trading intervals on market efficiency) that couldn't be answered in a continuous-time framework. Our discrete-time version of the DA consists of a series of alternating *bid/offers* (BO) and *buy/sells* (BS) steps. Traders are allowed to make bids and offers only during a BO step, and are allowed to purchase tokens only during a BS step. A typical DA game consists of initialization steps (during which traders are assigned roles of buyer and seller and are informed of their token values), followed by a series of alternating BO and BS steps. We have found that this setup yields a DA market which performs very much like a continuous-time experimental market, while at the same time substantially simplifying the task of programming a trading strategy.

*Chicago Rules* were chosen to make our implementation of the DA resemble the new AURORA computerized DA market created by the Chicago Board of Trade (CBOT) and to reduce the number of random tie-breaks which occur in a discrete-time trading environment when, for example, multiple traders accept an outstanding bid or offer. Under the Chicago Rules only the *current bidder* or *current offerer* (the holder of the highest outstanding bid or lowest outstanding offer) are allowed to trade.\(^1\) Presumably CBOT adopted these rules in part to guarantee that all transactions are publicly observable, possibly in response to the recent trading abuses that have been uncovered at the CBOT and elsewhere. Having experimented with alternative sets of trading rules, we have found that the Chicago Rules produce a "fairer," more strategically interesting version of DA which reduces the need for random tie-breaking. A criticism of Chicago Rules is that it prevents traders from hanging quietly in the background, watching for a "good deal," and quietly making their trades. Chicago Rules force traders to "show their hands" by posting a current bid or offer before they can trade, promoting the revelation of information in the DA

\(^1\)In the case where there is a current bid but no current offer, any seller is allowed to accept the current bid. Similarly if there is a current offer but no current bid, any buyer is allowed to accept the current offer. If more than one seller or buyer accepts, a winner is chosen at random.
market. We have found, however, that Chicago Rules do not place a significant constraint on traders; they can still stay in the background, quickly "popping up on the board" at the instant they want to make their transactions, after which the current bid and offer are reset to zero. It is still an open question, however, whether Chicago rules improve or degrade market efficiency.

An individual game in the DA tournament consists of a number of periods, each of which is divided into a number of periods. Each period of the DA game consists of a fixed number of alternating BO and BS steps as described above. The reason for structuring games to have multiple rounds and periods within rounds is to control players' abilities to learn about their opponents. Tokens and redemption values are fixed within a single round, but are allowed to change between rounds. Thus DA games with many periods allow players to learn the values of each others' tokens, while DA games with many rounds allow players to learn about each others' strategies.

The DA tournament will be run in March 1990, awarding approximately $10,000 to tournament participants. Players, tokens, and redemption values of DA games will be randomly chosen so that all players have an equal ex ante chance of earning profits. To reduce the effect of randomness caused by the tie-breaking rules and choice of matchings, tokens, redemption values, etc., we will play a large number of DA games and tournament participants will be paid in proportion to the average total profits earned by their strategies in all games in the tournament. In order to maximize earnings in the tournament entrants will have to write strategies that do well in variety of situations rather being "tuned" for a particular environment. A maximum of 100 strategy entries will be accepted by the tournament deadline. Thus, each entrant can expect ex ante earnings of at least $100; however, the actual amount may be more or less depending on the actual number of entrants and the cleverness and generality of their trading strategies.

Entrants need only to write subroutines for the skeleton player programs: bid and offer strategy subroutines for the BO step, and buy and sell strategy subroutines for the BS step. Strategies are allowed to use information obtained from the monitor in any way they choose, and the resulting programs can be as simple or complex as the entrant desires. Skeleton programs are available in C, FORTRAN, and PASCAL; however, if an entrant is not comfortable using one of the above languages, they will have to contact the tournament organizers to see if their favorite language can be supported. If so, the entrant will need to learn the details of the message-passing protocol described in Palmer, Rust, and Miller (1989).

To assist entrants in testing and debugging their trading strategies, the organizers provide copies of the monitor program (C source code, or in executable form for PC-compatibles), source code for an example trader program, and a human interface program for mixed human/computer tournaments. This allows entrants to test their strategies by playing against them.

To encourage further refinement of strategies in advance of the tournament, the organizers have created the Santa Fe Token Exchange (SFTE). The SFTE opens at the start of each hour for (non-profit) token trading. To use SFTE, entrants must have access to a computer which is a "local node" on the Internet (Arpanet) computer network. Their local node computer must have software that supports the standard TCP/IP communications protocols. This allows entrants to establish a communications link to SFTE and participate in DA games remotely from their local node. Traders on SFTE may be either human (using the human interface program) or machine trading programs running on the local node. SFTE allows entrants to test out their strategies in "trial heats." Since trading programs reside on the local node and not on the SFI computer, participation in the trial heats need not reveal one's strategy in advance of the tournament.
6. References


-15-
89-001 "A Double Auction Market for Computerized Traders"
   John Rust, Richard Palmer, and John H. Miller

89-002 "Communication, Computability and Common Interest Games"
   Luca Anderlini

89-003 "The Coevolution of Automata in the Repeated Prisoner's Dilemma"
   John H. Miller

89-004 "Money as a Medium of Exchange in an Economy with Artificially Intelligent Agents"
   Ramon Marimon, Ellen McGrattan, and Thomas J. Sargent

89-005 "The Dynamical Behavior of Classifier Systems"
   John H. Miller and Stephanie Forrest

89-006 "Nonlinearities in Economic Dynamics"
   José A. Scheinkman

89-007 "Silicon Valley' Locational Clusters: When Do Increasing Returns Imply Monopoly?"
   W. Brian Arthur

89-008 "Mutual Information Functions of Natural Language Texts"
   Wentian Li
89-001 "A Double Auction Market for Computerized Traders"
   John Rust, Richard Palmer, and John H. Miller

89-002 "Communication, Computability and Common Interest Games"
   Luca Anderlini

89-003 "The Coevolution of Automata in the Repeated Prisoner's Dilemma"
   John H. Miller

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