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The Economic Productivity of Urban Areas: Disentangling General Scale Effects from Local Exceptionality

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Abstract

The factors that explain differences in the economic productivity of urban areas have remained difficult to measure and identify unambiguously. Here we show that a synthesis of the classical representation of economic activity in a city in terms of a production function, together with a scaling perspective that accounts for the systematic effects of population size, leads to a new expression for the Total Factor Productivity (TFP) of urban areas. We empirically demonstrate that there is a systematic dependence of urban productivity on population size, resulting from the mismatch between the size dependence of wages and labor, so productivity increases by about 11% with each doubling in population. Moreover, deviations from the scale dependence, capturing the effect of local factors (including history and other contingencies) also manifest surprising regularities. Although productivity is maximized by the combination of high wages and low labor input requirement, high TFP cities show invariably high wages and high levels of employment relative to their size expectation. Conversely, low TFP cities show both low wages and employment. Finally we show that how educational attainment relates to these patterns and derive how it can naturally parameterize the TFP. We believe that these results shed new light on the important problem of establishing the determinants of urban productivity and inform the development of economic theory related to growth.

Keywords: Urban Scaling, Production Function, Total Factor Productivity.

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1. Introduction

Much research has been carried out over the past two decades trying to elucidate the causes of productivity differences across urban areas in the United States. The prevalent approach has been to utilize a variant of the growth accounting method (Solow, 1957) in order to statistically examine which of the myriad characteristics of urban areas affect their productivity (see, for example, Drennan et al., 2002; Florida, Mellander and Stolarick, 2008; Glaeser et al. 1992, 1995; Henderson, 1988; Lobo and Rantisi, 1999; Lobo and Smole, 2002). Among the many possible determinants of location-specific productivity, agglomeration economies — a set of phenomena ultimately dependent on the size and density of urban populations — have been highlighted in the literature (e.g., Carlino, Chatterjee and Hunt, 2007; Harris and Ioannides, 2000; Knudsen et al., 2008; Puga, 2010; Rosenthal and Strange, 2004). An earlier literature documented the positive effects of urban (population) size on productivity measured as average wage (Carlino, 1979; Moomaw, 1981; Segal, 1976; Shefer, 1973; Sveikauskas, 1975). The relationship between urban size and productivity is indeed a central fact of urban economics (Glaeser and Resseger, 2010).

The privileged role of cities as centers for the generation, recombination and exchange of knowledge—a role rediscovered by the new economic growth theory (Lucas, 1988)—provides a mechanism through which urban economies can become differentiated with regards to their productivity. As Florida (2005) and Glaeser (2011) point out, those cities which succeed in attracting skilled and creative individuals, responsible for the generation of new ideas and the application of existing ideas in novel ways, are bound to be more productive. Jones and Romer (2010) remind us—in a discussion centered on economic growth at the national level but which is also relevant for urban economies—of the possibly virtuous cycle for the acceleration of growth between size of population and the generation of ideas. A larger agglomeration of individuals can be expected to sustain a larger repertoire of intellectual capabilities, thereby facilitating the creation and recombination of ideas, and increasing the likelihood that interactions among individuals will occur through which new ideas are generated and shared.¹

The importance of population size as a major determinant of the intensity of socio-economic activity in urban areas has recently been re-emphasized by research applying scaling analysis to a diverse spectrum of urban indicators (Bettencourt et al. 2007, 2010; Bettencourt, Lobo and Strumsky, 2007). Scaling analysis, which has been a powerful tool across many science domains, represents how measurable characteristics of a system respond to a change in the size of the system. Its analytical punch stems from the observation that this response is often a simple, regular, and systematic function over a wide range of sizes, indicating that there are underlying generic constraints at work on the system. Cities are one such system: on the average, they manifest non-trivial scaling across many metrics, whether infrastructural or socio-economic, and appear to scale in the same way across a variety of urban systems. Indeed, simple power law scaling, discussed below, seems to be an approximately universal characteristic of cities world-wide, suggesting that a common dynamic has been at play in the development of cities and their economies, independent of local history, geography and culture.

¹ The argument that increases in urban scale generate greater positive externalities was eloquently made by Marshall (1890) and Jacobs (1969).

The methodological hallmark of investigations into the sources of economic growth and the determinants of productivity, whether at the national or sub-national levels, has been the use of a production function as a compact description of how economic output is generated. The framework of scaling provides a different perspective on metropolitan productivity from that implicit in the production function framework as it does not necessarily require that urban characteristics be outputs of a productive process, and as such can be applied very generally to characterize economic quantities, social metrics and infrastructural urban properties in the same manner. Measured deviations from the idealized average scaling relationships capture the effect of location-specific variations—due to history, geography, environment, culture, or contingency—on the performance of cities (Bettencourt, Lobo, Strumsky, West 2010). Deviations from the scaling relationship in effect represent the true unique “essence” of any given city. In this sense scaling analysis offers a complementary perspective on urban properties from a production analysis.

The integration of the production function and scaling analysis frameworks presented here rests on four “stylized” facts about urban economies.

1. Population size matters for urban productivity.
2. The share of total urban income accrued by urban labor is approximately the same (~ 0.7) across the urban economies of the United States, and has remained the same across the four decades for which data is reliably available.
3. The generation, recombination and exchange of ideas by and among individuals is the primary engine of economic growth.
4. Important indicators of urban economic life exhibit non-linear scaling behavior.

Bringing together the scaling and production function frameworks under the empirical cover of these facts makes it explicit how size constrains metropolitan *total factor productivity* (TFP). If it is the case that larger cities are more productive by virtue of their larger population size, then this systematic scale-dependence should be incorporated into a model of urban economic production that is common across cities of different sizes.

The decomposition of urban productivity effects in terms of their systematic population scale-dependence from other factors is an analytically important step towards accurately identifying the causes of economic under- or over-performance. The expression for urban TFP derived here explicitly incorporates the effects of scale on productivity by accounting for the systematic variation of urban characteristics with population size. To achieve this, we adopt a parameterization of urban indicators of productivity and economic inputs in terms of both systematic dependences on city size, which are common to all cities, and scale-independent local deviations from this general trend in terms of indicators we call *Scale Adjusted Metropolitan Indicators*, or *SAMIs* (Bettencourt et al., 2010). The final result is an expression for urban TFP which explicitly controls for the effects of population size. With this scale-adjusted productivity metric it is therefore possible to disentangle the effects on productivity of urban characteristics, many of which can themselves be expected to be scale-dependent. As an example of this exercise we explicitly consider the effects of human capital, measured in the usual way as educational attainment and which also scales with population size, on urban TFP.

The discussion is organized as follows. The next section briefly introduces scaling analysis. Section three builds upon the scaling relationship to construct a scale-adjusted indicator of metropolitan performance. Section four utilizes the framework of a production function to derive a scale-adjusted measure of metropolitan productivity while section five presents a parametrization of the measure. The scale-adjusted effects of human capital on productivity are quantified in section six. Section seven decomposes the scale-adjusted productivity measure along two dimensions, total wages and employment. Section eight presents our conclusions and discusses their implications for further research.

2. Scaling Analysis

Scaling characterizes how a given systemic quantity of interest, Y , depends on a measure of the size of a system, N . A common feature of scaling is *scale invariance*, which corresponds to a relationship formalized as:

$$Y(N) = Y_0 N^\beta \quad (1)$$

where Y_0 is a normalization constant and β is the scaling exponent (which can also be interpreted as an elasticity as usually defined in economics). The significance of this “power law” relation becomes evident when we consider an arbitrary scale change by a factor λ from N to λN . (The use of a power-law functional form in equation (1) is not essential to the main argument.) This induces a change in Y from $Y(N)$ to $Y(\lambda N)$ that, in general, can be expressed as

$$Y(\lambda N) = Z(\lambda, N)Y(N) \quad (2)$$

This equation expresses the relation between Y for a system of size N , to Y for a system λ times larger. When the scale factor Z depends only on λ , i.e. $Z(\lambda, N) = Z(\lambda)$, equation (2) can be solved *uniquely* to give the scale-invariant result of equation (1) with $Z(\lambda) = \lambda^\beta$. Scale-invariance implies that such a relationship—the ratio $Y(\lambda N)/Y(N)$ —is parameterized by a single dimensionless number β , usually referred to as the *scaling exponent*. The quantity $Y(\lambda N)/Y(N)$ is independent of the particular system size N but is dependent on the ratio between sizes λ ; such systems are often referred to as “self-similar.” (Non-interacting systems are extensive and are characterized by $\beta = 1$.)

The observation of scale invariance implies that the effects of increasing population size are general and can be observed by comparing any two cities, regardless of their size. If, for example, Y measures economic output, and two metropolitan areas have population sizes of N and λN , respectively, scaling implies that the ratio of their outputs is a function of the proportion of their population sizes λ , but not of N . As remarked by Barenblatt (2003), scaling relations manifest an important empirical property: the phenomenon, so to speak, repeats itself (albeit nontrivially) on changing scales. Such repetition strongly suggests that there are underlying dynamical processes generating and maintaining the same relationship among structural and functional variables over the range of the scale — typically many orders of magnitude. The existence of approximate scaling phenomena for urban areas — documented in Bettencourt et al. (2007) and Bettencourt et al. (2010) using a variety of socio-economic metrics — is an indication that there are generic social mechanisms at play across an entire urban system, integrating

together in a single swoop many of the complex interactions among the individuals, households, firms, and institutions living, residing and operating in these spaces.²

Note that equation (1) bears a close resemblance to an urban production function with Y denoting total output of urban areas and N the size of urban populations (see, e.g., Glaeser et al. (1995)). Dividing both sides of the equal sign by N we get $y = Y_0 N^{\beta-1}$, which can be interpreted as an equation for output per person as a function of the maximal number of people sharing ideas with each other (see, e.g., Jones and Romer, (2010)). In a sense then the mathematical machinery of production functions and scaling analysis are very similar (especially considering that in both cases the functions are homogenous).

The value of the scaling exponent can be expressed using the log-transformed function:

$$\ln Y_i = \ln Y_0 + \beta \ln N_i + \xi_i, \quad (3)$$

where the ξ_i represent deviations from the scale-invariant form. The simplicity of a straight line when $\ln Y$ is plotted against $\ln N$ conveys graphically the striking result of self-similarity: as the size of the metropolitan area changes, the relationships among its different components and processes must adjust so that the relationship between size and Y is maintained. Ordinary least squares estimation (OLS) of equation (3) — correcting for heteroskedasticity, and using data on Gross Metropolitan Product (*GMP*) and population for the 363 continental metropolitan statistical areas (MSAs) of the United States smoothed over the 2005-2007 period — gives the following result:

$$\ln(GMP_i) = 8.961 + 1.151 * \ln(Pop_i), \quad R^2 = 0.96, \quad (4)$$

with *p-values* virtually zero.³ It is no surprise that larger metropolitan areas produce more output, or that population size explains most of the variability in output, but what is surprising is that the scaling relationship is systematically superlinear: a 1% increase in population is associated on average with a 1.15% increase in output, regardless of city size. These self-similar and increasing returns to scale establish quantitatively the economic advantages of large cities.

The scaling relationship in equation (3) can be hypothesized to apply for other socio-economic phenomena likely to be affected by agglomeration economies. Two examples are provided by the number of patent applications submitted by metropolitan residents to the U.S. Patent Office (a broad measure of inventive activity), and the number of individuals employed in

² Examples of scaling relationships in the socio-economic realm include the well-known “Zipf’s Law”, which states that a city’s size decreases in inverse proportion to its rank among other cities within the same urban system (Zipf 1949); the rank-size distribution of firms (Steindl 1965; Ijiri and Simon 1977); the distribution of executive compensation (Walls 1999); and “Pareto’s Law” for the distribution of personal income, (Mandelbrot 1963). For a review of scaling analysis in economics see Brock (1999) and Stanley and Plerou (2001).

³ Data on Gross Metropolitan Product, and on metropolitan employment and population, is provided by the Commerce Department’s Bureau of Economic Analysis (www.bea.gov/regional/index.htm#gsp).

“creative” occupations (as defined in Florida (2002)).⁴ The estimation results, also using data for the 363 continental metropolitan statistical areas (MSAs) of the United States smoothed over the 2005-2007 period, are:

$$\ln(\textit{patent applications}_i) = -13.103 + 1.372 \ln(\textit{population}_i), \quad R^2 = 0.72, \quad (5)$$

$$\ln(\textit{creative employment}_i) = -4.006 + 1.121 \ln(\textit{population}_i), \quad R^2 = 0.91, \quad (6)$$

with *p-values* also virtually zero. Again, it is to be expected that larger metropolitan areas would generate more patent applications, or have more creative employment, than smaller-sized urban areas. But the pronounced superlinearity of the scaling coefficients and the consistency of their values across quantities might again be considered surprising, and clear evidence for how scale brings about systematic disproportionate increases in human capital and inventive activity. It is worth pointing out that population size alone accounts for most of the metropolitan variability in wealth, inventive activity and creative employment.

3. A Scale-Adjusted Metropolitan Indicator

The scaling relationship between urban economic output and population has an immediate practical implication when it comes to measuring and comparing metropolitan productivity levels. The most common productivity measure, output per worker (or its close relative, output per capita), implicitly assumes that output, which is a function of population, increases *linearly* with either population or employment. In other words, the implicit assumption of per capita measures is that $Y \propto N$ (Uslaner, 1976), thereby ignoring an essential feature of cities, namely agglomeration, that produces the non-linearity inherent in scaling laws. Given the scaling manifested by the data, it is clearly more meaningful to subtract the effects of urban size that are independent of location so as to focus on size-independent exceptions to these general dynamics. Such a scale-independent indicator is easy to construct if one uses the average scaling relations discussed in the previous section as the *null model* against which urban areas are to be compared. For any given metropolitan area the actual value of Y typically deviates from its idealized value as given by the scaling relationship.

To be more specific, consider the scaling equation (1) as a starting point. With each fractional increase of population size, $\Delta N/N$, the relative increase in per capita output, $y = Y/N$, is given by

$$\frac{\Delta y}{y} \approx (\beta - 1) \frac{\Delta N}{N}. \quad (7)$$

If $\beta \approx 1$, then, on average, $\Delta y \sim 0$, y is constant and Y is linear in N . In this case, a standard per capita measure would be an appropriate baseline for ranking urban areas. However, for $\beta > 1$ the baseline is itself a function of size (N). Furthermore, for a given value of $\Delta N/N$, y depends only

⁴ Data on patent applications was obtained from the U.S. Patent and Trademark Office; creative class employment counts were compiled by the Martin Prosperity Institute (Rotman School of Management, University of Toronto) using data provided by the Bureau of Labor Statistics (BLS).

on β but not on initial population size, expressing the principle that a meaningful comparison of productivity among urban areas should rely on relative magnitudes rather than on their absolute values.

Equation (4) expresses the average productivity for a city of size N . Deviations from this average behavior capture the characteristics of each individual urban area not accounted for by the general agglomeration effects of population size. These deviations can be quantified by the residuals, equation (3):

$$\xi_i = \ln \frac{Y_i}{Y(N_i)} = \ln \frac{Y_i}{Y_0 N_i^\beta}, \quad (8)$$

where Y_i is the observed value of output for each metropolitan area.⁵ We refer to ξ as a *Scale-Adjusted Metropolitan Indicator* (SAMI). Unlike per capita indicators, SAMIs are dimensionless and, by construction, independent of urban size (Bettencourt et al. 2010). SAMIs can be constructed for any variable capturing features of urban life, which are subject to agglomeration effects. The scaling equation for an urban metric can be re-written as:

$$Y_i = Y_0 N_i^\beta e^{\xi_i}, \quad (9)$$

where as before Y and N denote, respectively, a measure of urban economic performance and size of the urban population.

4. Urban Total Factor Productivity

It is often assumed, but rarely verified, that there is a production function that applies to all cities within an urban system, regardless of population size and other salient characteristics. A necessary condition for this assumption to hold is that the factors' share of total output (or income) are constants, independent of time and common throughout all cities in an urban system. An expression for urban total factor productivity (TFP) can then be obtained if a production function for urban output can indeed be specified.

We find it useful to recapitulate the derivation of an urban production function. We proceed by first stating an accounting truism: at any time, t

$$Y_i(t) = W_i(t) + R_i(t), \quad (10)$$

with Y signifying the pecuniary value of the total output generated in the i th metropolitan area, W denoting total labor income, and R the total capital income (all three variables being time and place dependent). Defining the factor shares as:

$$1 - \alpha = \frac{W_i(t)}{Y_i(t)}, \quad \alpha = \frac{R_i(t)}{Y_i(t)}, \quad (11)$$

⁵ The construction of SAMIs is similar to other uses of the method of residues (Batty and March, 1976).

the assumption of the constancy of factor shares in time and across population size requires that

$$\frac{\partial \alpha}{\partial N} \Big|_t = 0, \quad \frac{\partial \alpha}{\partial t} \Big|_N = 0. \quad (12)$$

The share of total income accruing to labor ($1 - \alpha$) can be calculated for both Metropolitan Statistical Areas (MSAs) and Micropolitan Statistical Areas, which together constitute the urban areas of the United States and which represent a wide variety of geographic, demographic and socio-economic characteristics.⁶ Figures 1 and 2 show the time series, from 1969 to 2009, for the economy-wide value and the urban mean of the ratio of labor income to total income ($1 - \alpha$); the urban mean is calculated for all 942 urban areas.⁷ As shown in Figure 1, urban labor's share of total income displays the same temporal trend as the national labor's share of income, both hovering around a value of 0.70. As indicated by the standard error bars in Figure 2, the ratio of urban labor income to total income ($1 - \alpha$) exhibits little variation across urban areas.⁸ (Strictly speaking, however, the value of $1 - \alpha$ is not precisely constant over time, as clearly shown by Figure 2, but is slowly varying; note the granularity of the vertical scale in the figure.) Reassuringly, there is little systematic relationship between labor's share of total income and urban population size as the correlation between location-specific values for $1 - \alpha$ and urban population is around 0.10 for every year between 1969 and 2009.

Returning to equation (10), we now differentiate it relative to time and divide by output to obtain

$$\frac{1}{Y_i(t)} \frac{dY_i(t)}{dt} = \frac{W_i(t)}{Y_i(t)} \frac{dW_i(t)}{dt} + \frac{R_i(t)}{Y_i(t)} \frac{dR_i(t)}{dt} = (1 - \alpha) \frac{dW_i(t)}{dY_i(t)} + (\alpha) \frac{dR_i(t)}{dY_i(t)}. \quad (13)$$

Similarly, equation (10) can be differentiated with respect to population size. The constancy of α is now invoked in order to integrate the relation in (13) to obtain

$$Y_i(t) = C W_i(t)^{1-\alpha} R_i(t)^\alpha. \quad (14)$$

The integration constant, C , is independent of time but would, in general, have been expected to be a function of N . However, an identical equation can be derived by using population as the dependent variable, in which case, integrating with respect to N leads to an integration constant,

⁶ MSAs and Micropolitan Areas are defined by the U.S. Office of Management and Budget and are standardized county-based areas having at least one urbanized area (with 50,000 or more population in the case of MSAs or at least 10,000, but less than 50,000, in the case of Micropolitan Areas), plus adjacent territory with a high degree of social and economic integration with the core as measured by commuting ties. Both MSAs and Micropolitan Areas are in effect unified labor markets. There are 366 MSAs and 576 Micropolitan Areas as of June 2011.

⁷ Total personal income is calculated as the sum of wage and salary disbursements, supplements to wages and salaries, proprietors' income, rental, dividend and interest income, and personal current transfer receipts, less contributions for government social insurance. Labor income is the sum of wage and salary disbursements and supplements to wages and salaries. Data for metropolitan personal income is provided by the Commerce Department's Bureau of Economic Analysis (BEA) – Table CA04 “Personal income and employment summary” (<http://www.bea.gov/regional/reis/default.cfm?selTable=CA04>).

⁸ Calculating $1 - \alpha$ using data for Gross Metropolitan Product (available only for the 2001 – 2009 period) yields comparable results to those using data on labor income and total income.

C , independent of N but potentially dependent on t . Because the two identical forms must match, we conclude that the integration constant C must be independent of both time and population size. Furthermore, inserting the definition of α into equation (14) shows that C is, in fact, completely determined in terms of only α :

$$C(\alpha) = (1 - \alpha)^{\alpha-1} \alpha^{-\alpha}. \quad (15)$$

Equation (14) can be related to the familiar Cobb-Douglas production function, which is a widely-used model for national and urban economies.⁹ This requires the conversation factors:

$$w_i(t) = \frac{W_i(t)}{L_i(t)}, \quad r_i(t) = \frac{R_i(t)}{K_i(t)}; \quad (16)$$

w and r are, respectively, the marginal product of labor (*MPL*) and the marginal product of capital (*MPK*). We then have that:

$$Y_i(t, N) = C(\alpha) W_i(t, N)^{1-\alpha} R_i(t, N)^\alpha = A_i(t, N) L_i(t)^{1-\alpha} K_i(t)^\alpha, \quad (17)$$

with the A term a measure of the location-specific technology, or “Total Factor Productivity” (TFP) of the *ith* urban area. Technology can be interpreted broadly so that it can encompass all the social, demographic, technological, environmental, policy and even cultural factors which determine the overall productivity of an urban area.

From equation (17) we get the following expression for metropolitan TFP as a function of the marginal productivity of labor and capital:

$$A_i(t, N_i) = C(\alpha) \left(\frac{W_i(t, N_i)}{L_i(t, N_i)} \right)^{1-\alpha} \left(\frac{R_i(t, N_i)}{K_i(t, N_i)} \right)^\alpha = C(\alpha) w_i(t, N_i)^{1-\alpha} r_i(t, N_i)^\alpha. \quad (18)$$

Equation (18) conveys the familiar economic logic under which technology improvements are manifested through the more effective utilization of the factors of production. Next, we show how scaling analysis provides constraints on the form of the TFP term (A) resulting in its systematic parameterization as an explicit function of general population size effects and specific local deviations.

5. Parameterization of Urban TFP from Scaling Analysis

We first note that both the numerator and denominator in the expressions for the marginal product of labor (*MPL*) and marginal product of capital (*MPK*) exhibit scaling behavior so that the marginal productivity of the two production factors can be recast using their associated SAMIs as:

⁹ See, for example, Glaeser et al. (1992, 1995), Lobo and Smole (2002), and Abel, Dey and Gabe (2010).

$$w_i(t) = \frac{W_i(t, N_i)}{L_i(t, N_i)} = \frac{W_0 e^{\xi_i^W(t)} N_i(t)^{\beta_W}}{L_0 e^{\xi_i^L(t)} N_i(t)^{\beta_L}} = \frac{W_0}{L_0} e^{\xi_i^W(t) - \xi_i^L(t)} N_i(t)^{\beta_W - \beta_L}, \quad (19)$$

$$r_i(t) = \frac{R_i(t, N_i)}{K_i(t, N_i)} = \frac{R_0 e^{\xi_i^R(t)} N_i(t)^{\beta_R}}{K_0 e^{\xi_i^K(t)} N_i(t)^{\beta_K}} = \frac{R_0}{K_0} e^{\xi_i^R(t) - \xi_i^K(t)} N_i(t)^{\beta_R - \beta_K}. \quad (20)$$

An expression for A then takes the general form:

$$A_i(t) = A_0(t) e^{\xi_i^A} N_i(t)^{\beta_A}, \quad (21)$$

with

$$A_0(t) = C(\alpha) \left[\frac{W_0(t)}{L_0(t)} \right]^{1-\alpha} \left[\frac{R_0(t)}{K_0(t)} \right]^\alpha, \quad (22)$$

$$\xi_i^A = (1-\alpha)(\xi_i^W - \xi_i^L) + \alpha(\xi_i^R - \xi_i^K), \quad (23)$$

$$\beta_A = (1-\alpha)(\beta_W - \beta_L) + \alpha(\beta_R - \beta_K). \quad (24)$$

Equations (21) – (24) make explicit how urban TFP depends on both population scale, through the scaling exponents, and on local, scale-independent fluctuations (through the SAMI's). Evaluating A , requires knowledge of how K , the metropolitan capital stock, scales with urban size but unfortunately reliable data on urban capital stocks in the U.S. is not available at present. We can, however, estimate the value of the scaling coefficient for urban TFP. Given the observed values for the scaling coefficients for total wages and labor, $\beta_W \approx 1.13$ and $\beta_L \approx 1$, and with $(1-\alpha) \cong 0.7$, the first term to the right of the equal sign on equation (24) has a value of 0.09. What about the value of the $\alpha(\beta_R - \beta_K)$ term? Under the widely-made assumption that the rental price of capital (r) is constant, or nearly so, across metropolitan areas, and given that $R = r \times K$, or equivalently, $R_0 N^{\beta_R} = r K_0 N^{\beta_K}$, then $r = (R_0 / K_0) N^{\beta_R - \beta_K}$. For r to be a constant, $\beta_R = \beta_K$ must hold. Another line of reasoning we can invoke for ignoring the term $(\beta_R - \beta_K)$ is that the capital to labor ratio must remain nearly the same across urban areas in order for MPK to remain invariant across those areas—thus metropolitan capital income and capital stocks must scale similarly. Therefore $\beta_A \cong 0.09$; urban productivity increases, on average, at about 9% with each doubling of population.

The systematic (i.e., average) dependence of A on urban population size originates in the mismatches of the scaling of total wages (W) versus labor (L), and, potentially, of capital income (R) versus capital returns (K). Given the observed values for the scaling coefficients for total wages and labor, their difference can generate an average increase in productivity resulting from a self-similar wage premium for the same amount of labor (and also, potentially, a savings in the amount of labor input). The scale-adjusted measure for urban TFP can be approximated by:

$$\xi_i^A \approx (1 - \alpha)(\xi_i^W - \xi_i^L). \quad (25)$$

According to equation (25), the systematic dependence of urban productivity on population size can be represented as the differential scaling of total wages and employment.

6. What About Human Capital?

Much research has identified “human capital” (captured through a measure of educational attainment or professional occupations) as the principal reason why some cities are more economically productive than others (see, for example, Glaeser and Saiz (2004)). In this section we modify the derived scale-free expression for urban TFP so that it incorporates the possible effects of human capital. A general expression for urban TFP is given by:

$$A_i(N_i) = A_i[H_i(N_i), N_i, \bullet], \quad (26)$$

with productivity depending not only on population size (N), but also on the number of educated or skilled individuals, denoted by H , which itself scales with population (urban TFP may, of course depend additionally on other characteristics).

We take equation (21) as our point of departure and specify the following exact equation for metropolitan human capital:

$$H_i = H_0 e^{\xi_i^H} N_i^{\beta_H}, \quad (27)$$

where H measures the number of educated (or skilled or creative or inventive) individuals in the i th metropolitan area. (Using data from 2009 β_H is approximately 1.11.) From equation (27) we get an expression for N :

$$N_i = \left(\frac{H_i}{H_0} \right)^{\frac{1}{\beta_H}} e^{-\frac{1}{\beta_H} \xi_i^H}, \quad (28)$$

which is substituted for the population size variable in (21):

$$A_i(H_i) = \left[\frac{A_0}{H_0^{1/\beta_H}} \right] H_i^{\beta_A/\beta_H} e^{\xi_i^A - \frac{\beta_A}{\beta_H} \xi_i^H}. \quad (29)$$

Equation (29) is admittedly not very elegant but dissecting it reveals a surprising result regarding the effects of human capital on urban productivity once scaling effects are controlled for. Note that the ratio of scaling coefficients for TFP and for human capital is quite small:

$$\frac{\beta_A}{\beta_H} = (1 - \alpha) \left(\frac{\beta_W - \beta_L}{\beta_H} \right) = (0.7) \left(\frac{0.13}{1.11} \right) \cong 0.08. \quad (30)$$

Turning to the other term on the right-hand side of equation (29):

$$\xi_i^A - \frac{\beta_A}{\beta_H} \xi_i^H = (1 - \alpha) (\xi_i^W - \xi_i^L) - \frac{\beta_A}{\beta_H} \xi_i^H, \quad (31)$$

which shows that the correction due to H , even if ξ_i^H is high (meaning that a given urban area has an unusually high concentration of human capital for its population size), is bound to be a very small number.

Thus, the dependence of metropolitan TFP on H is in general quite slow, slower than for population size in fact. Note however that if we had instead used a per capita measure of educational attainment, $h_i = H_i / N_i$, we would have found a much stronger dependence since $\beta_{h_i} \approx 0.8$. This observation highlights the importance of searching for the dynamical mechanisms that propel economic productivity and that we see as being largely proxied by population size. Most socio-economic variables, from violent crime to the incidence of certain infectious diseases, increase with population size in similar ways as educational attainment but are almost certainly not causally linked to increased economic productivity. While the case for educational attainment is certainly much more suggestive, its use in the context of a production function still leaves unexplained the mechanisms of how it is incorporated into economic production.

7. Decomposition of Urban TFP

We calculated the scale-adjusted TFP using equation (25) and data for both Metropolitan and Micropolitan Areas averaged over the period 2001-2005, and setting $1 - \alpha$ (labor's share of income), to equal 0.7.¹⁰ For this decomposition we only use data on metropolitan wages and employment as these two variables are "directly" and unambiguously measurable, and the effects of human capital can be expected to be subsumed under accrued wages.

The top fifty urban areas, ranked according to the values of their scale-adjusted productivity (ξ^A), are shown on Table 1, while Table 2 shows the rankings for the top fifty Metropolitan Areas (MSAs). One result immediately stands out: the absence of the large metropolitan areas from the top ranks of the most productive urban centers (in contrast to a ranking generated by using output per worker as the measure of productivity).

Figure 3 shows all urban areas in terms of their two performance metrics, wages, ξ^W , and labor, ξ^L plotted as coordinates on a two-dimensional graph, including their population size denoted by the size of the circles, and their scale adjusted productivity ξ^A as their color. From equation (25) we easily see that the 45-degree solid green line divides the plane into two regions: above the line, where $\xi^A > 0$, urban areas display above average TFP and are denoted in warm colors (green to red); below the line, where $\xi^A < 0$, and denoted in cold colors (green to dark blue) appear urban areas with below average TFP.

¹⁰ Data on total wages, employment and population were obtained from the Regional Economic Accounts produced by the Commerce Department's Bureau of Economic Analysis (<http://www.bea.gov/regional/reis/>). Wage data was deflated using the Federal Reserve's chain-type price index and is expressed in 2005 dollars (<http://research.stlouisfed.org/fred2/series/GDPCTPI?cid=21>).

The tabular and graphical results taken together show an interesting trend in the exceptionality of urban TFPs, once population size has been accounted for (recall that, by using the SAMI's, the effects of population size have already been factored out). While the way to maximize TFP is to have exceptionally high wages and exceptionally low labor input (employment), few cities with such properties exist (they appear in the 2nd quadrant of Figure 3, which is hardly populated). The urban area with the highest productivity, by far, is Los Alamos (a Micropolitan Area in the state of New Mexico not shown in Figure 3 because it is so far off-scale), a location very familiar to some of us. Los Alamos, with a population of about 18,000 inhabitants, receives an annual injection of approximately \$2.2 billion in federal funds allocated to Los Alamos National Laboratory. Los Alamos shows both exceptionally high wages and levels of employment. The second highest urban TFP, even after accounting for population size, corresponds to Silicon Valley (the San Jose-Santa Clara, Metropolitan Area in California). San Jose also shows exceptionally high wages, and to a lesser extent high levels of employment. All other urban areas with highest TFP (dark red in Figure 3) share most of the same general characteristics. An exception is Harriman (TN), which shows a high TFP as a result of low levels of employment, and not particularly exceptional wages.

As already mentioned there are two a priori independent ways in which an urban area can obtain greater than expected productivity, given its size,, corresponding to a 45-degree line on the ζ^W - ζ^L plane: $\zeta^W = C + \zeta^L$, where the intercept $C = \zeta^A/\alpha$ is set for different values of ζ^A . The red solid line in Figure 3 maps the space of equal TFP at varying ζ^W and ζ^L for Silicon Valley. Note how no other urban area approaches the performance of San Jose, and no urban areas even come close among those with employment less than average (2nd and 3rd quadrants). Similarly the lowest possible TFP would correspond to low wages and high employment (4th quadrant of Figure 3, which again is hardly populated at all). The line in dark blue in Figure 3, tracks the TFP of the lowest ranked metropolitan area: Rio Grande City-Roma (TX). Most actual cities with very low TFP, including the metropolitan areas of McAllen and Brownsville (TX), show similar patterns of low wages and low employment. However there are some exceptions, such as Vermillion (South Dakota), which shows exceptionally large employment ($\zeta^L = 0.44$) but only average total wages ($\zeta^W = 0.03$). While arguably this is a symptom of a functioning community it is penalized in terms of an exceptionally low TFP because its marginal product of labor (MPL) is small, resulting in low productivity.

Thus we see that most actual cities lie in the first and third quadrants of Figure 3. The most productive actual urban areas show exceptionally high wages and high employment, whereas those least economically successful tend to show both low wages and low employment. This is expressed in terms of a linear regression ($\zeta^W = -0.02 + 1.17 \zeta^L$, $R^2 = 0.74$, black solid line), which is close to a 45-degree line but also shows a slightly greater slope emphasizing the trend for higher wages and lower employment in high TFP cities and lower wages and higher employment for those with lower TFP. These results suggest that, unlike firms, the principal objective of cities is not to maximize their productivity alone. In fact as decentralized economies, the key property of economically successful cities is that they appear to maximize wages, and this in turn may lead to general high levels of employment. This close relationship between high wages and high levels of employment and vice versa seems to be a general feature of urban economies.

8. Conclusions

Understanding cities has remained difficult because there are many important variables that are interdependent and that strongly co-vary. As Jane Jacobs remarked “Real cities present...situations in which several dozen quantities are all varying simultaneously and in subtly connected ways.” (Jacobs, 1969, pp. 433) It is therefore essential to develop methodologies that make it possible to evaluate the separate effects of different characteristics on urban metrics, and on urban economic productivity in particular. Few characteristics are as consequential for urban life as population size, which is both a facilitator and consequence of socio-economic activities.

Here we have shown that an integrated consideration of the standard approach to urban areas as production devices, and of the systematic dependence of the main factors of production on population size (via urban scaling analysis) results in a more general form of a Cobb-Douglas production function. The resulting functional form manifests explicitly dependences of urban productivity on population size and local factors in terms of size-independent deviations (SAMIs). In particular, the analysis leads to a new expression for the total factor productivity (TFP) in terms of an explicit scale-invariant dependence on population size and on size-independent deviations due to the mismatch between labor income and employment (as well as capital income and capital stock).

The decomposition of urban productivity through scaling analysis shows that the productivity of urban areas is actually a fairly low dimensional quantity characterized not only by a systematic dependence on population size but also by a close relationship between exceptions to population size expectations in terms of wages and labor. Empirical estimation shows that there are two main effects that determine, in practice, the differentials in economic productivity of urban areas. First there is a general effect associated with population size. This effect results from different scaling between total wages (which scales super-linearly) and labor (which scales linearly). This difference results on a systematic scale invariant effect such that urban TFP increases by approximately 11% with each doubling in population size of an urban area, whether from 10 to 20 thousand or from 1 to 2 million. Once expressed in terms of human capital, instead of total population, this dependence becomes somewhat slower, though still positive. Second, deviations from this trend are themselves regular, with most urban areas manifesting high TFP showing exceptional high wages and high employment while those with low TFP showing low wages and low employment. It is the fact that larger deviations in magnitude occur in terms of wages than of employment that makes this co-variation be positive or negative. The local effects of higher wages are partially accounted for by higher human capital, but only to a limited extent. These results suggest that the economies of cities are not maximizing total productivity per se, as might be the case for a firm, but instead providing environments for economic development and productivity enhancements than when successful lead to growth in both wages and employment. Economic theory aimed at explaining the productivity of urban areas (in the U.S., at least) should be aimed at these clear and regular empirical relationships.

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Figure 1. Ratio of urban labor income to total income ($1 - \alpha$).

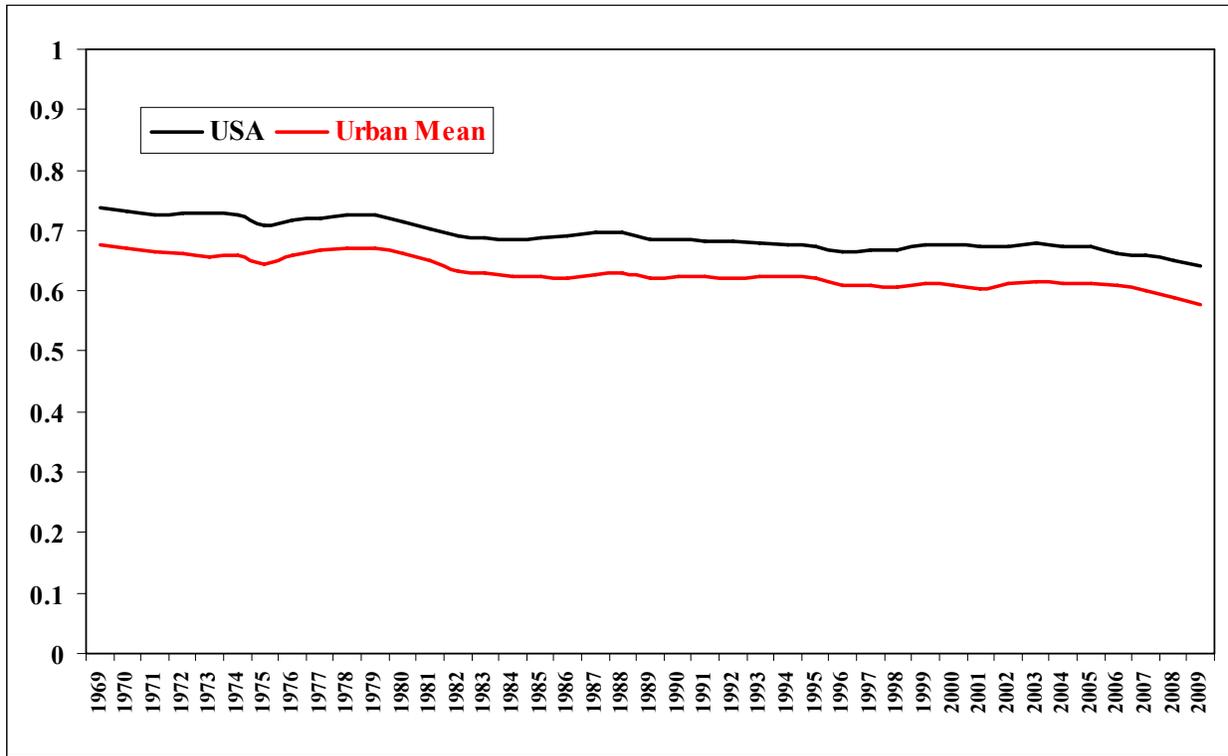


Figure 2. Variability across urban areas in the ratio of metropolitan labor income to total income. (Bars above the mean line indicate standard errors.)

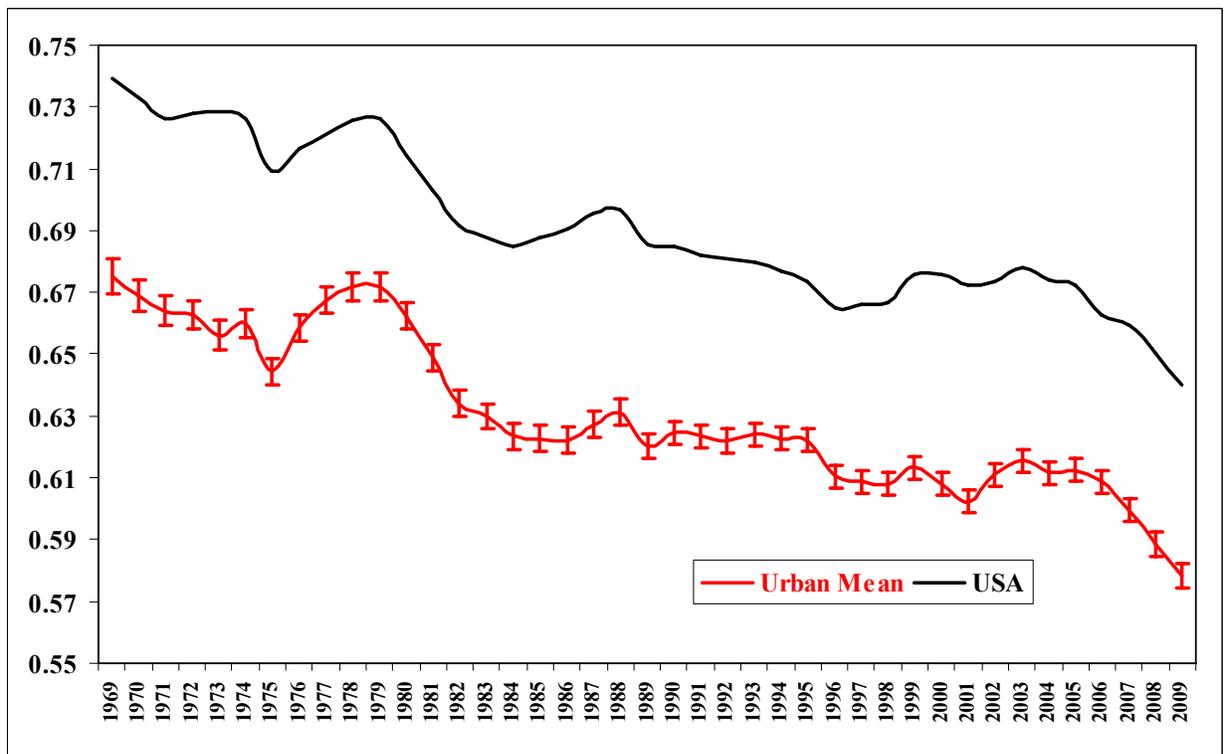


Figure 3 The SAMIs for urban areas' TFP (color) in the ζ^W - ζ^L plane. The size of each symbol denotes its population (smallest cities are shown at the same small symbol size). The solid green line divides the space into TFPs above (positive) and below (negative) the expected value for each city's population. The solid red line is the equal TFP parameter space for Silicon Valley, while the solid blue line is the equal TFP space for the least productive city in the sample (Rio Grande City-Roma, TX). The black solid line shows the linear best fit to the data $\zeta^W = -0.02 + 1.17 \zeta^L$ ($R^2=0.74$).

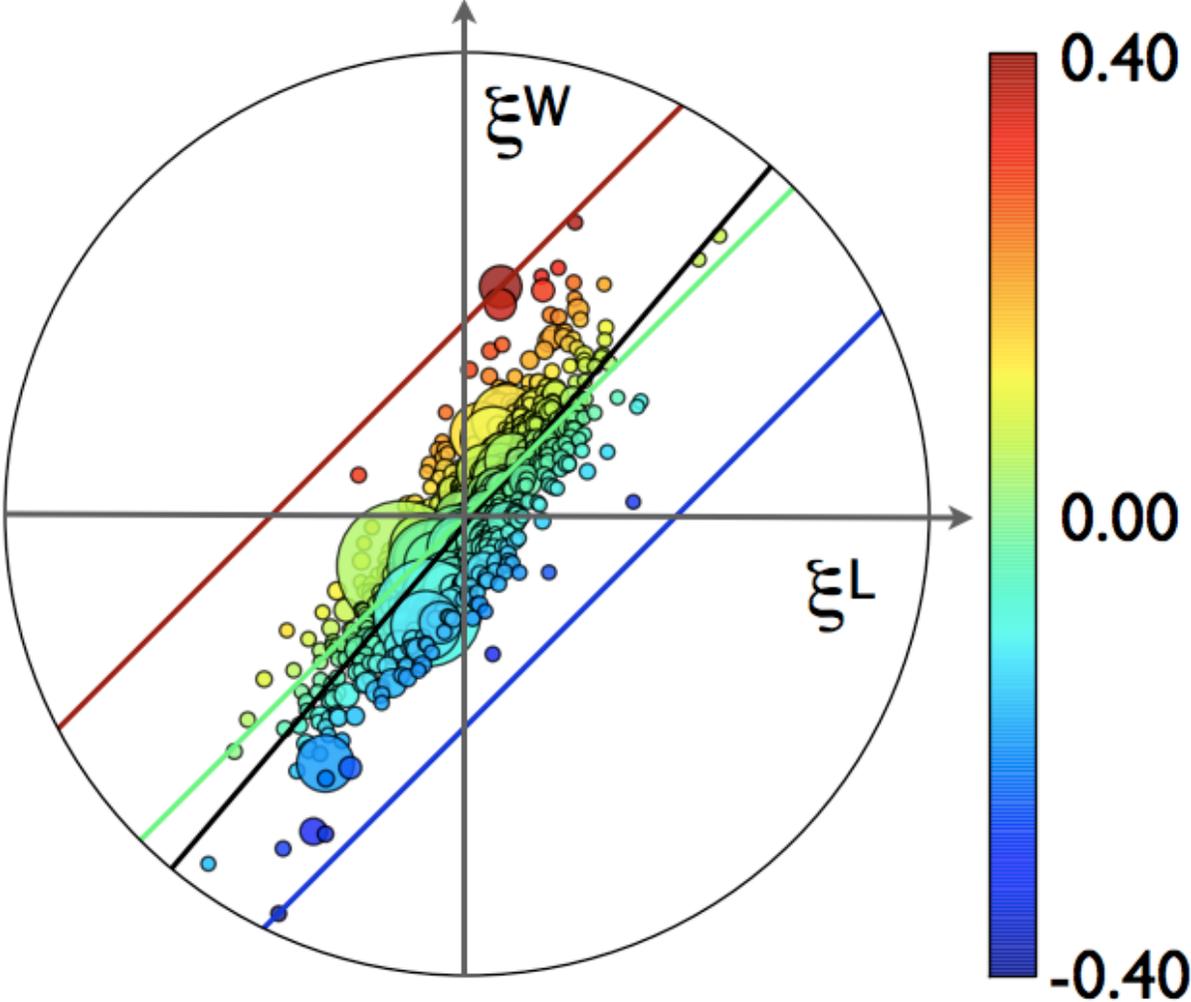


Table 1. Top 50 urban areas, ranked by their scale-adjusted measure of TFP (ξ^A).

	Urban Area	ξ^A	ξ^W	ξ^L
1	Los Alamos, NM (Micropolitan Area)	0.6964	1.7771	0.7822
2	San Jose-Sunnyvale-Santa Clara, CA (Metropolitan Area)	0.3674	0.6155	0.0907
3	Gillette, WY (Micropolitan Area)	0.3480	0.7895	0.2923
4	Bridgeport-Stamford-Norwalk, CT (Metropolitan Area)	0.3342	0.5672	0.0898
5	Rock Springs, WY (Micropolitan Area)	0.2937	0.6664	0.2467
6	Trenton-Ewing, NJ (Metropolitan Area)	0.2799	0.6054	0.2056
7	Harriman, TN (Micropolitan Area)	0.2791	0.1053	-0.2934
8	Midland, MI (Micropolitan Area)	0.2691	0.3906	0.0061
9	Kokomo, IN (Metropolitan Area)	0.2652	0.4415	0.0627
10	Elko, NV (Micropolitan Area)	0.2544	0.4585	0.0950
11	Sidney, OH (Micropolitan Area)	0.2369	0.6268	0.2884
12	Borger, TX (Micropolitan Area)	0.2328	0.2749	-0.0576
13	Marshfield-Wisconsin Rapids, WI (Micropolitan Area)	0.2196	0.5390	0.2253
14	Lexington Park, MD (Micropolitan Area)	0.2189	0.3729	0.0602
15	Wilmington, OH (Micropolitan Area)	0.2045	0.5831	0.2909
16	Columbus, IN (Metropolitan Area)	0.1995	0.5330	0.2480
17	Connersville, IN (Micropolitan Area)	0.1845	0.1965	-0.0671
18	Columbia, TN (Micropolitan Area)	0.1783	0.3424	0.0878
19	Boulder, CO (Metropolitan Area)	0.1776	0.5536	0.3000
20	Hinesville-Fort Stewart, GA (Metropolitan Area)	0.1762	0.1730	-0.0787
21	Oshkosh-Neenah, WI (Metropolitan Area)	0.1731	0.4166	0.1694
22	Ann Arbor, MI (Metropolitan Area)	0.1728	0.4689	0.2220
23	Durham-Chapel Hill, NC (Metropolitan Area)	0.1715	0.4795	0.2344
24	Bellefontaine, OH (Micropolitan Area)	0.1676	0.2733	0.0340
25	Auburn, IN (Micropolitan Area)	0.1652	0.4951	0.2590
26	Bloomington-Normal, IL (Metropolitan Area)	0.1643	0.4435	0.2089
27	Defiance, OH (Micropolitan Area)	0.1640	0.3351	0.1008
28	Corning, NY (Micropolitan Area)	0.1636	0.1331	-0.1006
29	Battle Creek, MI (Metropolitan Area)	0.1612	0.1723	-0.0579
30	Andrews, TX (Micropolitan Area)	0.1559	0.1135	-0.1092
31	Pahrump, NV (Micropolitan Area)	0.1546	-0.0364	-0.2573
32	Fort Leonard Wood, MO (Micropolitan Area)	0.1542	0.2880	0.0677
33	Carson City, NV (Metropolitan Area)	0.1540	0.5265	0.3065
34	Norwich-New London, CT (Metropolitan Area)	0.1534	0.3287	0.1095
35	Decatur, IL (Metropolitan Area)	0.1533	0.2927	0.0736
36	St. Marys, GA (Micropolitan Area)	0.1511	0.1630	-0.0529
37	Rochester, MN (Metropolitan Area)	0.1511	0.4771	0.2613
38	Warsaw, IN (Micropolitan Area)	0.1510	0.2754	0.0597
39	Manchester-Nashua, NH (Metropolitan Area)	0.1471	0.2958	0.0857
40	Wilson, NC (Micropolitan Area)	0.1450	0.2973	0.0902
41	Fort Valley, GA (Micropolitan Area)	0.1395	-0.0795	-0.2787
42	Hartford, CT (Metropolitan Area)	0.1357	0.2802	0.0864
43	Crawfordsville, IN (Micropolitan Area)	0.1351	0.2644	0.0714
44	LaGrange, GA (Micropolitan Area)	0.1321	0.3561	0.1674
45	Owatonna, MN (Micropolitan Area)	0.1316	0.4748	0.2869
46	Warner Robins, GA (Metropolitan Area)	0.1313	0.2055	0.0178
47	Findlay, OH (Micropolitan Area)	0.1304	0.4602	0.2739
48	Racine, WI (Metropolitan Area)	0.1285	0.0224	-0.1612
49	Kennewick-Pasco-Richland, WA (Metropolitan Area)	0.1281	0.1230	-0.0600
50	San Francisco-Oakland-Fremont, CA (Metropolitan Area)	0.1241	0.2166	0.0394

Table 2. Top 50 metropolitan areas, ranked by their scale-adjusted TFP (ξ^A).

	Area	ξ^A	ξ^W	ξ^L
1	AreaName	SAMI A	SAMI W	SAMI L
2	San Jose-Sunnyvale-Santa Clara, CA	0.4743	0.7609	0.0834
3	Bridgeport-Stamford-Norwalk, CT	0.4433	0.7178	0.0845
4	Trenton-Ewing, NJ	0.3917	0.7567	0.1972
5	Kokomo, IN	0.3784	0.5597	0.0192
6	Columbus, IN	0.3140	0.6575	0.2088
7	Hinesville-Fort Stewart, GA	0.2920	0.3145	-0.1026
8	Oshkosh-Neenah, WI	0.2860	0.5504	0.1418
9	Ann Arbor, MI	0.2856	0.6314	0.2235
10	Boulder, CO	0.2852	0.6337	0.2263
11	Durham-Chapel Hill, NC	0.2839	0.6480	0.2424
12	Bloomington-Normal, IL	0.2789	0.6014	0.2030
13	Battle Creek, MI	0.2742	0.3022	-0.0895
14	Carson City, NV	0.2709	0.6742	0.2871
15	Norwich-New London, CT	0.2659	0.4771	0.0973
16	Rochester, MN	0.2657	0.6396	0.2599
17	Decatur, IL	0.2652	0.3952	0.0164
18	Manchester-Nashua, NH	0.2588	0.4495	0.0798
19	Warner Robins, GA	0.2489	0.3947	0.0392
20	Hartford-West Hartford-East Hartford, CT	0.2449	0.4428	0.0930
21	Kennewick-Pasco-Richland, WA	0.2445	0.3191	-0.0303
22	Racine, WI	0.2420	0.1725	-0.1733
23	Huntsville, AL	0.2343	0.4667	0.1320
24	Vineland-Millville-Bridgeton, NJ	0.2321	0.1744	-0.1572
25	San Francisco-Oakland-Fremont, CA	0.2292	0.3744	0.0469
26	Napa, CA	0.2287	0.5025	0.1757
27	Ithaca, NY	0.2151	0.4459	0.1386
28	Washington-Arlington-Alexandria, DC-VA-MD-WV	0.2146	0.4588	0.1522
29	Monroe, MI	0.2146	-0.0639	-0.3705
30	Saginaw-Saginaw Township North, MI	0.2130	0.2330	-0.0712
31	Longview, WA	0.2101	0.1487	-0.1515
32	Springfield, IL	0.2081	0.4361	0.1389
33	Sheboygan, WI	0.2079	0.4470	0.1500
34	Atlantic City-Hammonton, NJ	0.2050	0.4705	0.1776
35	Dalton, GA	0.2048	0.4995	0.2069
36	Boston-Cambridge-Quincy, MA-NH	0.1980	0.3698	0.0870
37	Sandusky, OH	0.1974	0.3751	0.0931
38	Elkhart-Goshen, IN	0.1925	0.5796	0.3046
39	Janesville, WI	0.1899	0.2121	-0.0591
40	Corvallis, OR	0.1840	0.4342	0.1713
41	Burlington-South Burlington, VT	0.1837	0.4833	0.2209
42	Mansfield, OH	0.1828	0.2294	-0.0318
43	Peoria, IL	0.1783	0.2633	0.0086
44	Rome, GA	0.1781	0.2529	-0.0015
45	New Haven-Milford, CT	0.1779	0.2168	-0.0374
46	Holland-Grand Haven, MI	0.1757	0.2310	-0.0201
47	Cheyenne, WY	0.1749	0.4234	0.1736
48	Cedar Rapids, IA	0.1722	0.3898	0.1438
49	Spartanburg, SC	0.1717	0.2269	-0.0183
50	Harrisburg-Carlisle, PA	0.1716	0.4782	0.2330