Metropolitan Patenting, Inventor Agglomeration and Social Networks: A Tale of Two Effects

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We investigate the separate effects on metropolitan patenting in the United States of inventor agglomeration and the structure of social networks linking inventors within and across metropolitan areas. Using patent data we have been able to assign a metropolitan location to individual inventors, link inventors who have co-authored patents, and characterize the structural features of the network of connections linking inventors. We find that inventors are disproportionately agglomerated in the larger metropolitan areas. Our findings also indicate that while agglomeration of inventors and agglomeration of co-patenting relationships both have a positive effect on metropolitan patenting output, inventor agglomeration is a much stronger determinant. Once important socio-economic characteristics of metropolitan areas are controlled for, structure features of inventor networks have small effects on metropolitan patenting.

Keywords: metropolitan patenting, inventor agglomeration, social networks of inventors, network effects.

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1. Introduction

Urban settings matter for innovation. Social networks matter for innovation. These two claims have by now achieved the status of truisms. Generating a third truism seems a simple exercise in concatenation: urban social networks of inventors matter for innovation. But this is a truism in need of disentangling. How do urban environments and networks matter *together*? Can the positive effects of networks on invention and innovation be clearly separated from the agglomeration effects that urban environments have on creative activity? What are the features of metropolitan networks of inventors and innovators that matter? The present discussion is an initial attempt at illuminating the extent to which the characteristics of social networks forged by inventors help to explain metropolitan inventive productivity.

The crucial role that cities have played in the development of science and technology, and more broadly, in the generation of inventions and innovations—intellectual and material, cultural and political, institutional and organizational—is well documented by historiographical work (see for example Bairoch 1988, Braudel 1992, Hall 1998, Jacobs 1984, Landes 1999, Mokyr 2002, Mumford 1968, Rosenberg and Birdzell 1986, Spufford 2003). The role of cities as centers for the integration of human capital and as incubators of invention was rediscovered by the "new" economic growth theory, which posits that knowledge spillovers among individuals and firms are the necessary underpinnings of growth (Lucas 1988, Romer 1986). As Glaeser (1996) points out, the idea that growth hinges on the flow and exchange of ideas naturally leads to recognition of the social and economic role of urban centers in furthering intellectual cross-fertilization. Moreover the creation and reposition of knowledge in cities increases their attractive pull for educated, highly skilled, entrepreneurial and creative individuals who, by locating in urban centers, contribute in turn to the generation of further knowledge spillovers (Feldman and Florida 1994, Florida 2002, 2004, Glaeser 1999). This seemingly spontaneous process, whereby knowledge produces growth and growth attracts knowledge, is the engine by which urban centers sustain their development through unfolding innovation.

Awareness about the importance of knowledge spillovers for innovation and about the social and communicative nature of knowledge spillovers, easily leads to an appreciation of the role of "social networks." Economic sociologists argue that economic interactions cannot be fully understood without attention to the web of social relationships in which these interactions are embedded (*e.g.*, Granovetter 1985, Polanyi 1957, Swedberg 2003, Uzzi 1996, White 2002, Zuckerman 2003). One can similarly argue that the process of invention cannot be well understood without paying attention to the social interactions among inventors (Arora and Gambardella 1994, Orsenigo, Pammolli and Riccaboni 2001, Powell, Doput and Smith-Doerr 1996, Walker, Kogut and Shan 1997). Inventors do not operate in isolation; the creation of new ideas is a process that often involves the integration and recombination of existing knowledge originating from different individuals, locations, institutions and organizations. The creation of new ideas is a process that often involves the integration and recombination of existing knowledge originating from different individuals, locations, institutions and organizations.

Social networks play an important role in the diffusion of information and knowledge since they provide the formal connections and informal linkages through which information and ideas flow among individuals. These knowledge spillovers occur without the mediation of market mechanisms, transcend the institutional and workplace settings in which individuals operate, and cut across organizational boundaries. The spread of knowledge and ideas tends to be local rather than global, and for early stage innovating, when tacitness is high, face-to-face contact becomes essential for effective knowledge transfer. Close proximity is thus likely to be not only helpful in capturing knowledge spillovers but necessary.

The collaborative and informal, and even serendipitous, interactions among inventors and innovators in effect generate a social network among innovators. Concentrating people engaged in related activities in a particular location thus creates an environment that facilitates the rapid and effective diffusion of ideas. Social networks have been highlighted as an important facet of regional innovation (Piore and Sabel 1984**,** Breschi and Lissoni 2001, Owen-Smith and Powell 2004) and are believed to be vital mechanisms for transferring knowledge and ideas between firms.[1] To the extent that inventing and innovating are predominantly urban phenomena and to the extent that there are networks linking metropolitan innovators, we feel justified in speaking of *metropolitan networks of inventors*.

It is a compelling inquiry to ascertain which features of urban societies foment, or hinder, invention and innovation. Historical evidence not withstanding, it isn't easy, however, to measure location-specific invention and knowledge spillovers (a difficulty eloquently discussed by Krugman (1991)) and this difficulty hampers the formal understanding of the relationship between urban characteristics and innovation. A similar hurdle is encountered when considering what features of metropolitan inventive networks are associated with higher levels of inventive activity. It is one thing to provide persuasive theoretical arguments and suggestive narrative evidence for the importance of metropolitan networks of inventors for location-specific invention; it is another to make these networks visible and thus amenable to measurement and formal modeling. Using patent data to both record metropolitan inventive activity and construct metropolitan inventor networks provides a plausible solution to these twin empirical difficulties.

Some knowledge flows do nevertheless leave an evidentiary trail in the form of patented inventions (Acs and Audretsch 1989, Jaffe and Trajtenberg 2002, Jaffe, Trajtenberg and Fogarty 2000, Malerba and Orsenigo 1999, Sorensen and Fleming 2004). By using patent data to identify individual inventors, assigning these inventors a metropolitan location, and viewing patent data as a source of relational data (*i,e.*, patent co-authorships), one can identify metropolitan networks of inventive collaboration. Once these networks have been rendered visible we can inquire as o their effects on inventive activity. Do some inventor networks possess characteristics that result in higher rates of invention? Are larger networks of innovators indeed more productive than smaller-sized networks? Besides size of a network, that is, how many inventors constitute the network, does internal network structure matter?

Anticipating our principal results, we find that the number of inventors in metropolitan areas scales superlinearly with metropolitan population—that is, inventors disproportionately agglomerate in the large metropolitan areas. While agglomeration of inventors and increased levels of co-patenting relationships both have a positive effect on metropolitan patenting output, inventor agglomeration is a much stronger determinant. Salient structural features of the social

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[1] As an example, much of Silicon Valley's success has been attributed to its informal networks of friendship and collaboration among scientists, engineers, and entrepreneurs in the area (Saxenian 1994).

network of inventors are not correlated with metropolitan scale, and once important socio-economic characteristics of metropolitan areas are controlled for, the structure of metropolitan inventor networks have small effects on patenting.

The paper is organized as follows. The next section describes the U.S. patent data, how it was used to identify metropolitan inventors and networks of inventors, and the details of how it was spatially aggregated and matched with metropolitan population data. Section 3 considers the disproportionate agglomeration of inventors in the largest metropolitan areas and identifies the scaling relationship between metropolitan population size and the size of the metropolitan community of inventors. Section 4 discusses what the connections among inventors in a metropolitan patenting network mean while section 5 describes the features used to characterize and compare these networks. Section 6 introduces the econometric estimation framework used to quantify the effects of inventor agglomeration and network structure on patenting output the results from which are discussed in section 7. Section 8 concludes the discussion.

2. Metropolitan Patents and Inventors

In order to construct measures of metropolitan invention and counts of metropolitan inventors, source data was extracted from the U.S. Patent Office (USPTO) records on all granted U.S. patents from 1980 to 2001 (U.S. Patent Office 2003).[2] Every patent includes all inventors' last names (with varying degrees of first and middle names or initials), each inventor's home town, detailed information about the patent's technology in class and subclass references (over 100,000 subclasses exist), and the discrimination of the owner, or assignee, of the patent (generally a firm, and less often a university, if not owned by the inventor). Patent filings do not, however, provide consistent listings of inventor names or unique identifiers for the authors. Since the USPTO indexes source data by patent number and not by inventor, a variety of conditional matching algorithms were used to identify inventors, each inventor's patents and other inventors with whom the focal inventor has co-authored at least one patent.[3] The final database includes 2,058,823 unique individual inventors and their patent co-authors, and a total of 2,862,967 patents.

By identifying individual inventors, matching inventors with patents, assigning a location to each inventor—specifically a Metropolitan Statistical Area (MSA)[4]—and linking inventors who have co-authored a patent, it is possible to construct *patent co-authorship networks* for 331 MSAs in the continental United States. Every inventor's hometown was matched to a zip code, which was then assigned to an MSA using the *ZIPList5* dataset[5]. County level population data

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[2] We are aware of the criticism that patents are not perfect indicators of inventive activity since not all new inventions are patented, and many economically important types of innovations, such as organizational forms and computer software, cannot be patented at all (Griliches 1979, Pakes and Griliches 1980, Griliches 1990). While these caveats make us cautious about the use of patent data and prudent in the interpretation of how significant our results are, we nevertheless think that patents are the "footprints" of some (by no means all) inventive activity.

[3] The matching procedures are discussed in detail in Fleming, King and Juda (2004).

[4] An MSA includes a core city and surrounding counties, which together form a local labor market area.

[5] Ziplist5 is a commercially available dataset containing every active ZIP code currently defined by the U.S. Postal Service for the entire USA. Every zip code is assigned to an MSA if the zip code lies within a metropolitan county. The MSA county definitions used by ZIPList5 are consistent with the Census Bureau's 2000 MSA definitions. The ZipList5 database is a commercially available database produced by CD Light (www.zipinf.cm)

was extracted from the Bureau of Economic Analysis' "Regional Economic Accounts Tables" (which are available online at www.bea.doc.gov). Counties were assigned to MSAs according to the MSA definitions used to create the metropolitan inventor networks. The analyses presented here relied upon all patents with at least one inventor within a metropolitan area. Thus, if inventors from inside and outside a metropolitan area co-authored the same patent, the patent (and both inventors) would appear in each MSA.[6] Inventors in a given metropolitan area also have collaborative ties with inventors in other MSAs. We will thus distinguish between metropolitan networks and the connections linking inventors within a metropolitan area, referring to the *internal* structure of these networks, and the connections linking inventors across metropolitan boundaries, *inter-metropolitan* connections.

3. Metropolitan Agglomeration of Inventors

A clear finding from examining where patents originate in the United States is that patenting is largely a metropolitan phenomenon. Ullman (1958) found that inventive activity in the Unites States is linked to urban development and agglomeration. Pred (1961) examined U.S. patent data for the mid-nineteenth century and found that patenting activity was significantly greater in the principal cities than the national average. Higgs (1971) found that the number of patents issued in the United States during the period from 1870 to 1920 was positively correlated to urbanization. In a study of inventive activity in the United States during the early stages of the nation's industrialization, Sokoloff (1988) concludes that major urban centers accounted for a disproportionate share of patents. More recently Jaffe, Trajtenberg and Henderson (1993) examined the pattern of citations by new to previously issued patents. They found that new patents are 5 to 10 times more likely to cite previous ones originating in the same metropolitan area. Acs, Anselin and Varga (2002) also find that patenting in the United States is overwhelmingly concentrated in metropolitan counties.[7]

A simple account of the expected agglomerative relationship between the size of a metropolitan population, the size of a metropolitan community of inventors and patenting activity would be the following: larger metropolitan areas have disproportionately larger populations of inventors such that in consequence larger MSAs generate a disproportionate share of patents. Furthermore, the network externalities generated by the agglomerations of inventors in large metropolitan areas make these inventive communities more productive. This plausible story is partially correct, but only partially.

To quantify the relationship between a metropolitan area's inventive output, proxied by the number of new patents granted to inventors residing in the corresponding MSA, and the size

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[6] Patent co-authorship refers to the situation where a patent is either applied for by more than one individual (or organization) or lists more than one individual as a designated inventor. We will use the terms "co-authorship," 'co-patenting," and "co-inventing" interchangeably. As in Kuznets (1962) we understand "inventive activity" to be concerned with technical inventions involving the creation of new knowledge as well as the combination and recombination of existing knowledge.

[7] Examination of data from England (Beggs and Cameron 1988), Sweden (Sjöholm 1996) and the semiconductor industry (Almeida and Kogut 1997) confirms the urban character of patenting for other countries and in specific industries.

of its population, we assume a very simple functional relation between metropolitan population (N) and newly granted metropolitan patents (P):

$$P_{i,t} = cN_{i,t}^{\beta}, \tag{1}$$

where c is a constant, i indexes the metropolitan area and t stands for time (in units of a year). The exponent β determines the scaling relationship between patenting and population. Taking the natural logarithm of equation (1) and assuming the presence of i.i.d. Gaussian noise we have the following basic estimation equation:

$$\ln\left(P_{i,t}\right) = \ln c + \beta \ln\left(N_{i,t}\right) + e_{i,t}. \tag{2}$$

We estimated the exponent β using data for three individual periods (1980, 1990 and 2000) and an OLS estimation procedure (with a correction for heteroskedasticity); the statistically significant (at the 95% confidence level) coefficient values are shown in Figure 1 together with the scatter plots for the dependent and independent variables. Mathematically the scaling relationship between metropolitan population and metropolitan invention is "superlinear," or to use the language of economics, the relationship exhibits increasing returns to scale (*i.e.*, $\beta > 1$).[8]

Not surprisingly, a larger metropolitan population is associated with a greater output of new patents; what is somewhat surprising is the magnitude of the increasing returns to scale. Availing ourselves of the richness of a dataset containing cross-sectional and time-series data, we use a panel data, Feasible Generalized Least Squares (FGLS) framework to estimate the scaling parameter. The estimated pooled coefficient for the effects of metropolitan population on patenting output is 1.29 (with a 95% confidence level of $1.26 \leq \beta \leq 1.32$) with β greater than or equal to 1.25. Regressing the number of metropolitan inventors on metropolitan population (see Figure 2), we find that the relationship is also superlinear, with a pooled coefficient $\beta = 1.24$ (and a 95% confidence interval of $1.22 \leq \beta \leq 1.28$). The relationship between the number of new metropolitan patents and the number of metropolitan inventors (Figure 3) is, however, remarkably linear with a pooled coefficient β very close to unity ($0.97 \leq \beta \leq 0.99$).

The seeming absence of agglomeration or network externalities for patenting productivity is confirmed by the summary statistics for the variable *patents per inventor* (simply defined as the total number of patents granted in a year to metropolitan inventors divided by the total number of MSA-specific inventors). As shown on Table 1, the mean for metropolitan *patents per inventor* is close to or below one, which is possible because very often there are several authors per patent. The small coefficient of variation associated with patents per inventor indicates that this measure of productivity does not vary much across metropolitan areas. Note also that the mean for *patents per inventor* has substantially decreased between 1980 and 2000 (a paired *t*-test performed on the two means indicates that these are indeed statistically different). Although inventors are disproportionately concentrated in the larger metropolitan areas, this concentration

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[8] For a more extensive discussion of the scaling relationship between patenting and metropolitan population see Bettencourt, Lobo and Strumsky (2004).

does not seem to make the communities of inventors in the larger MSAs more productive than those in smaller metropolitan areas. This finding tempers our expectation about the effects of network structure on inventive output.

4. A Social Network as a Graph

In order to render social networks visible sociologists often depict a network as a graph in which the nodes represent social actors (in our case, metropolitan inventors) and the edges linking the nodes represent relationships, exchanges or communication among the agents in the network (Knoke and Kuklinski 1982, Newman 2000, Scott 2000, Wasserman and Faust 1994). The salient features of the graph representing a network are assumed to capture the salient characteristics of the social network being modeled. Using U.S. patent data we have defined *networks of inventors*: that is, we have constructed graphs in which the nodes are inventors and the edges of the graph connect patent co-authors who share the "inventor" title or assignment designation. By matching inventors to metropolitan areas we can distinguish between links among inventors within an MSA and links between inventors across metropolitan boundaries. (We will refer to the connections among inventor within a metropolitan area as the "internal" structure of a network.)

We note that a metropolitan network of inventors includes isolated "nodes" (inventors who are the sole authors of patents), small clusters of inventors connected to each other through shared co-authorship, and larger-sized components grouping many metropolitan inventors together. Individual clusters and components are often linked through key individuals (with high degree of "betweenness") who have connections to multiple inventive communities. The networks are constructed anew for each year on the basis of the new patents granted that year.

The metropolitan networks of inventors we have defined are in essence the connections among individuals who have collaborated in the act of patenting. What do these links actually represent, particularly with regards to information flow and the impact of that flow upon subsequent patenting activity? Singh (2004) reports significant flow of information between patent co-authors, as measured by citations from future patents that are linked by direct, and even indirect, collaborative ties and his results hold even after econometrically controlling for the greater likelihood of a citation arising simply because it refers to work in similar technologies. Singh goes on to demonstrate that almost all of the geographical citation "spillovers" in the United States (*e.g.,* Jaffe *et. al.*, 1993) result from co-authorship networks. (Breschi and Lissoni (2004) find similar results for European inventors.)

Collaborative patenting ties are potentially very effective vehicles for all types of information, especially information that is effectively transmitted only through direct interaction. Thus agglomeration of connections among inventors can be expected to improve and increase inventiveness if connectivity indeed enhances information flows and knowledge spillovers. We are of the view that information that flows along an observed collaborative tie in the years subsequent to its formation varies greatly, from none to a great deal, but that on average, the information flow between former patent co-authors is certainly positive and occasionally substantial. Furthermore, we believe that the distribution of these ties varies from exceptionally weak to extremely strong, such that they support a variety of different types of information flows. Since patent collaboration ties span such a wide spectrum of characteristics and strengths, we have not, in our statistical work, made any assumption regarding the content of information flows or the capability of a tie to transmit information.

Even if we see the links forged by inventors in the act of co-inventing as possible channels for knowledge spillovers, we also echo the cautionary remark made by Hussler (2004), who, using terminology from Hur and Watanabe (2001), views the spillovers evidenced by patents as "intentional spillovers." Inventors, aware that their patents are public documents that make their knowledge accessible, can be very selective in what prior knowledge they chose to cite as relevant to their invention.[9] Here it is important to remember that patents are, in essence, ownership claims, sought by inventors not only for the sake of recording intellectual originality but also in the expectation of financial gain. Thus one would not expect patents to exhaustively record all of the individuals who positively influenced an inventor's thinking.

5. Measuring Network Structure

The formation of connections across nodes in the inventors' network, that is, network agglomeration, should increase subsequent innovativeness because it enhances information flow and knowledge spillovers. Isolates and small clusters will be left without access to new ideas and results. New results and opportunities will remain unexploited, because they will remain unknown outside of the local contexts where the breakthrough occurred. A promising new combination will not occur, because knowledge of a previous combination will not diffuse into a new and potentially fertile context. Increased network connectivity should also enable greater opportunities for technological brokerage between previously disconnected technological communities (Burt 2003, Hargadon 2003). Thus it seems plausible to expect that the agglomeration of connections among inventors will correlate positively with subsequent patenting in a metropolitan area.

We use three variables—a Herfindahl network index, size of the largest component, and network density—to capture salient structural features of a metropolitan inventive network. The *Herfindahl network index* measures the proportion of inventors in a metropolitan network who are grouped into "components" (rather than industries as in the typical use of a Herfindahl index).[10] A high value for the Herfindahl network index (that is, a value close to 1) indicates that most inventors in a metropolitan area are grouped into a few components with few individual inventors as isolated nodes. In the context of patent co-authorships, a high value for the Herfindahl network index implies that most inventors have been working in the same or similar technologies.

The *largest component* of a network is the largest set of inventors that can trace a direct or indirect collaborative connection to one another. The largest component is thus the largest community of co-inventors and collaborators within a metropolitan area. The *size of the largest*

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[9] As discussed by Breschi and Lissoni (2004), Globerman, Kokko and Sjöholm (2000) and Jaffe, Trajtenberg and Henderson (1993) the large majority of "prior art" citations in patent applications are added by patent examiners rather than by the inventors authoring patents.

[10] A "component," or sub-graph, is a set of inventors linked together in such a way that an unbroken path can be traced, via the edges linking inventors, from any given inventor to every other inventor in the component.

component is calculated as the proportion of inventors within each metropolitan area that had a collaborative tie within the largest component in the MSA. The density of links among the nodes is another important indicator of network structure, even more significant in our case given that a dense web of connections among inventors could be interpreted as an instantiation of knowledge spillovers resulting from the flow of information. A straightforward measure of *network density* is the actual number of ties divided by the potential number of ties (calculated as the number of inventors choose 2).

It seems imminently reasonable to assume the existence of interaction effects between a metropolitan area's population size and population density, and the structure of a metropolitan network of inventors. However an examination of the correlations among various such measures indicates otherwise (a set of representative correlations, for the year 2000, is shown on Table 3). There is no significant correlation between the Network Herfindahl index, or the size of the largest component, and either population size or population density. And network density and population density are slightly at cross-purposes. The scaling of inventor agglomeration with metropolitan population has an obvious and direct effect on network size, but it doesn't appear that the structure of inventor networks per se is much related to metropolitan size. Recall that what drives the formation of links in the network under consideration here is patent co-authorship. Possibilities for successful patenting greatly depend on the characteristics of the technologies inventors work on while the ease with which co-patenting occurs is driven by the need to complement skills, the difficulty of the technical problem being tackled, and the organizational and institutional setting in which inventors are embedded. Metropolitan scale does not seem such a relevant feature in this context.

6. Modeling and Estimation Framework

This section provides a formal setting for the econometric estimations of the effects of network size and structure on inventive output. Metropolitan areas will be treated as separate economies that share common pools of capital. Differences among metropolitan economies in both levels and productivity of patent production are then the results of location-specific characteristics and the level of inventive labor. Total metropolitan innovative output (measured by patents) is given by:

$$Y_{i,t} = A_{i,t} N_{i,t}^{\sigma}, \tag{1}$$

where $A_{i,t}$ represents the level of productivity in the *ith* metropolitan area at time *t*, and $N_{i,t}$ denotes metropolitan inventors. Equation (1) can be interpreted either as a metropolitan production function (as in Glaeser *et al.* (1995) and Drennan *et al.* (2002)), or as a regional knowledge production function (as in Acs *et al.* (2002)).

We interpret $A_{i,t}$ broadly, to allow for the possibility that social, technological, cultural and institutional features of a metropolitan area may determine the area's overall innovative capacity. We focus our attention on two broad types of determinants: the characteristics of metropolitan social networks of inventors, and economic characteristics of metropolitan areas, in particular market characteristics such as specialization, diversity, competition and concentration.

We hypothesize that metropolitan innovative productivity is a simple multiplicative function of location-specific characteristics:

$$A_{i,t} = \prod_{j=1}^{J} X_{i,j,t}^{\beta_j}, \tag{2}$$

where j indexes the variables representing the determinants of metropolitan productivity. Inserting equation (3) into equation (2), assuming the presence of an i.i.d. Gaussian productivity shock and taking the natural logarithm of both sides of the resulting equation, we then have the following as our basic econometric estimation equation:

$$\ln Y_{i,t} = c + \sum_{j=1}^{J} \beta_j \ln X_{i,j,t} + (\sigma - 1)\ln N_{i,t} + e_{i,t}. \tag{3}$$

Patent count data is notorious for having a negative binomial distribution (a Poisson distribution with over dispersion). Transforming the patent count data by using the natural logarithmic function has the effect of changing the distribution into a (nearly) normal one—as verified both by visual examination of histograms and by performing the Wilks-Shapiro test (for individual periods and across all periods for all MSAs). Given the presence of both heteroskedasticity and serial correlation in the panel data, we used a pooled fixed effects FGLS framework to estimate the influence of various features of metropolitan inventors networks a time *t* on metropolitan patenting output at time *t+1* (for the estimations we assume a heteroskedastic error structure across cross-sectional units and *AR*(1) autocorrelation within cross-sectional units with cross-section specific autocorrelation coefficients).[11] Metropolitan output refers to new patents granted to metropolitan inventors in a given year. We use a variety of variables measuring metropolitan economic characteristics as control variables (the summary statistics for these metropolitan variables are presented in Table 2). The data covers 331 MSAs in the continental United States and the period 1980 to 2001. All the variables used in the estimations are in natural logarithmic form. Other network variables in addition to the ones discussed above as well as the metropolitan controls used in the regressions are described next.

Metropolitan inventors: The number of inventors in a metropolitan area is highly correlated with metropolitan population (≥ 0.82). The variable measuring the number of inventors in an MSA, that is, the number of nodes in a network, thus also controls for metropolitan size.

Outside collaborators: Inventors within a metropolitan area are linked not only to inventors within the same MSA but also to inventors in other metropolitan areas; these collaborations across MSAs thereby link metropolitan inventive networks. The variable “outside collaborators” is the ratio of the number of inventors outside a given metropolitan area that collaborate with the area’s inventors to the total number of inventors inside the MSA. The mean for this variable is greater than 0.50 for every year covered by our data (see Table 1) and for many metropolitan areas it is greater than 0.60 indicating that many inventors have collaborative links across

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[11] Results obtained from using either a fixed effects linear OLS framework or negative binomial regressions are qualitative and quantitative similar to those obtained using FGLS. These results are available from the corresponding author upon request.

metropolitan boundaries. This variable is included in the regressions so as to test for the importance to metropolitan patenting output of inter-metropolitan links among inventors.

Technology Herfindahl: To control for the technological heterogeneity and diversity within each metropolitan inventors network, we calculated the Herfindahl index of U.S. Patent Office technology classes by patent. The Patent Office divides all patents into approximately 400 technological classes, for example, class 437 (Semiconductor Device Manufacturing: Process) and 935 (Genetic Engineering: Recombinant DNA Technology, Hybrid).[12]

Population density: Innovation might be affected not only by the size of the metropolitan population from which innovators can be drawn, but also by how dense the metropolitan area is– the denser an area, the greater the likelihood of serendipitous interactions thru which knowledge spillovers can result. This variable is defined as metropolitan population divided by metropolitan land area.

PCPI (in thousands of dollars): To control for general economic conditions and changes in wealth, the models include personal income per capita, deflated by Bureau of Labor Statistics' experimental Consumer Price Index for 2002 (Bureau of Economic Analysis 2004). Minimum values of the variable tended to occur early in the time series and within the southwestern states of New Mexico and Arizona. Ideally, our estimation models would include the amount of research and development spending in each region, but this is impossible to measure. Even though R&D data are available for publicly traded firms, the data are not broken down by location (even within firms' internal accounting data), nor are private firm data readily available. Proxies such as technical professional employment would also be desirable, but they are not available as a time series over the years of our study. Given that location patterns of R&D laboratories tend to be stable over time (Acs *et al.* 2002), the fixed effects models should account for much of the variation.

MSA Herfindahl Index: We use a Herfindahl Index (calculated using data for 10 sectors: agriculture; forestry and fisheries; mineral industries; construction; manufacturing; transportation, communications and utilities; wholesale trade; retail trade; finance, insurance and real estate; services; public administration) in order to gauge the extent to which the rate of metropolitan patenting is related to specialization/diversity externalities. (The greater this measure, the more highly specialized, and thus less diverse, is a given MSA).

Market Structure: We needed a measure of a local area's market structure so as to test whether knowledge spillovers, which foster patenting, are greater if an MSA is more competitive. Following Glaeser *et al.* (1992), the total number of firms per worker in an MSA is used as a measure of market structure, *i.e.*, an MSA is taken as locally competitive if it has more firms per worker.

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[12] Since some metropolitan inventive networks work in newer and therefore more fertile technologies, such as biotech or nanotechnology, while other networks revolve around mature technologies, we originally thought it important to control for the technological age or vintage characteristic of an inventive network. To measure of technological vintage we used the average of the sequential "prior art" patent numbers cited by patents within an MSA. As it turns out, this variable displays little variation across MSAs (as indicated by a very low coefficient of variation) so in the end we decided not to include it in the econometric estimations.

Large Firms: Since large firms tend to spend proportionately more on private R&D than do smaller firms, the percentage of an MSA's firms with 1,000 or more employees is included to capture the effects of large firms on patenting activity (Carlino, Chatterjee, and Hunt, 2001).

7. Econometric Results

Given how strongly correlated our two chosen measures of internal network structure are, we estimated two different equations to avoid the problem of multicollinearity. The first equation, model 1 (in Table 4), uses the *network Herfindahl* variable as an indicator of internal network structure while model 2 (in Table 4) uses the variable *size of the largest component*. The two models include the variable *outside collaborators* as a measure of links among inventors across metropolitan areas. The coefficients for the measures of internal network structure and external network connections are positive and significant, and once standardized are very similar in magnitude (0.015 and 0.014 for *network Herfindahl* and *outside collaborators*, respectively, and 0.013 and 0.014 for *size of the largest component* and *outside collaborators*, respectively). These results conform to our expectations. Connections among inventors in a metropolitan area, putatively the channels for information spillovers, foster patenting activity. Connections to inventors outside one's local network infuse an inventive community with novel ideas, and likely serve as channels for new and non-redundant information to flow into an MSA's inventive network.

The coefficient for the variable *metropolitan inventors* (simultaneously a measure of network size and inventor agglomeration) is positive and significant in the two models. It is telling, however, that the standardized coefficient for metropolitan inventors has a magnitude over 70 times that of the standardized coefficients for internal network structure and inter-network connectivity, signifying that inventor agglomeration is a more important determinant of patenting output than network structure.

The results for the variable *network density* hint at an interesting story. The coefficient for *network density* is statistically significant strongly negative, suggesting that while having a community of linked inventors is good for patenting output, too many "internal" linkages is detrimental to inventive productivity (the standardized coefficient for network density is -0.259 in model 1 and -0.258 in model 2). This result brings to mind the dual nature, positive and negative, of the externalities induced by increases in the density of an urban population. The effects of increased density of connections in a network of inventors may have a similarly dual nature. The variables measuring connectivity among inventors (*network Herfindahl*, *size of the largest component*, *outside collaborators*) may be capturing the positive externalities induced by the agglomeration of connections. Conversely, the variable *network density* may be capturing the re-circulation of existing ideas and information flows between inventors within the same companies, universities and research organizations. It may very well be that a metropolitan area with very high network density implies that inventors are likely to be locked into collaborations that can quickly go stale.

While increased network density has a deleterious effect on patenting output, *population density* in a metropolitan area has a positive influence. A high population density represents a higher possibility for the occurrence of fortuitous encounters and unexpected interactions, from which intellectual novelty often emerges. Increased network density represents the opposite: realized interactions which might actually hinder the formation of different links.

Metropolitan areas with more specialized technologies and industries (as measured by the variables Herfindahl index and Techonology herfindahl, respectively) patent more. More often than not patenting involves the refinement of novel techniques rather than the creation of new inventions. In the by now familiar language of inventive search, exploitation rather than exploration makes it easier for inventors to build upon the work of their colleagues and collaborators.

Per capita personal income did not demonstrate a positive effect. This is probably because some very poor regions patent a great deal (for example, in New Mexico, where Pueblo Communities and National Research Labs co-locate) and some very rich regions, such as New York City, do not. Metropolitan areas with fewer and more monolithic firms patented more, which argues against a competitive influence on patenting. This may be driven by corporate towns such as Rochester or upstate New York, where Xerox and IBM patent heavily and provide most of the employment.

8. Conclusion

The totality of our results paint a picture of invention where the agglomeration of inventors in metropolitan areas—the disproportionate concentration of inventors in large MSAs—is of greater consequence for patenting output than whether these metropolitan inventors are intensively linked with one another in a social network. Further evidence against the presence of a significant network effect is the fact that a greater concentration of inventors does not lead to an increase in the productivity of individual inventors.

We should, however, tread cautiously before concluding that networks do not matter for patenting productivity. The existence of collaborative inventive networks in metropolitan areas could play an important, if difficult to measure, role in bringing about the metropolitan agglomeration of inventors. The presence of an established social network of inventors may itself attract out-of-area inventors to an MSA. Contact with a greater population of inventors could boost the productivity of individuals susceptible to becoming inventors, which would eventually result into a larger number of inventors. In this case, however, average inventive productivity—among established inventors—does not increase. In this sense we are unable to distinguish whether any impact of agglomeration on inventiveness arises from attracting disproportionately more inventors to an area or by boosting the inventiveness of nearly-inventors who were already in the area.

Interpretation of the surprisingly disappointing results regarding the effects of network structure on patenting must be tempered by a very important caveat. Our investigation treats all newly granted patents as equal, regardless of their economic potential or technological complexity. It could be the case that the creation of true technological novelty benefits from, or even necessitates, large and densely connected collaborative networks. Thus if we could partition the population of all patents into subpopulations reflecting a viable measure of technological complexity and then use a count of these subpopulations as our dependent variable, our estimation results might be very different. We hope to turn our next research efforts to a solution of this problem.

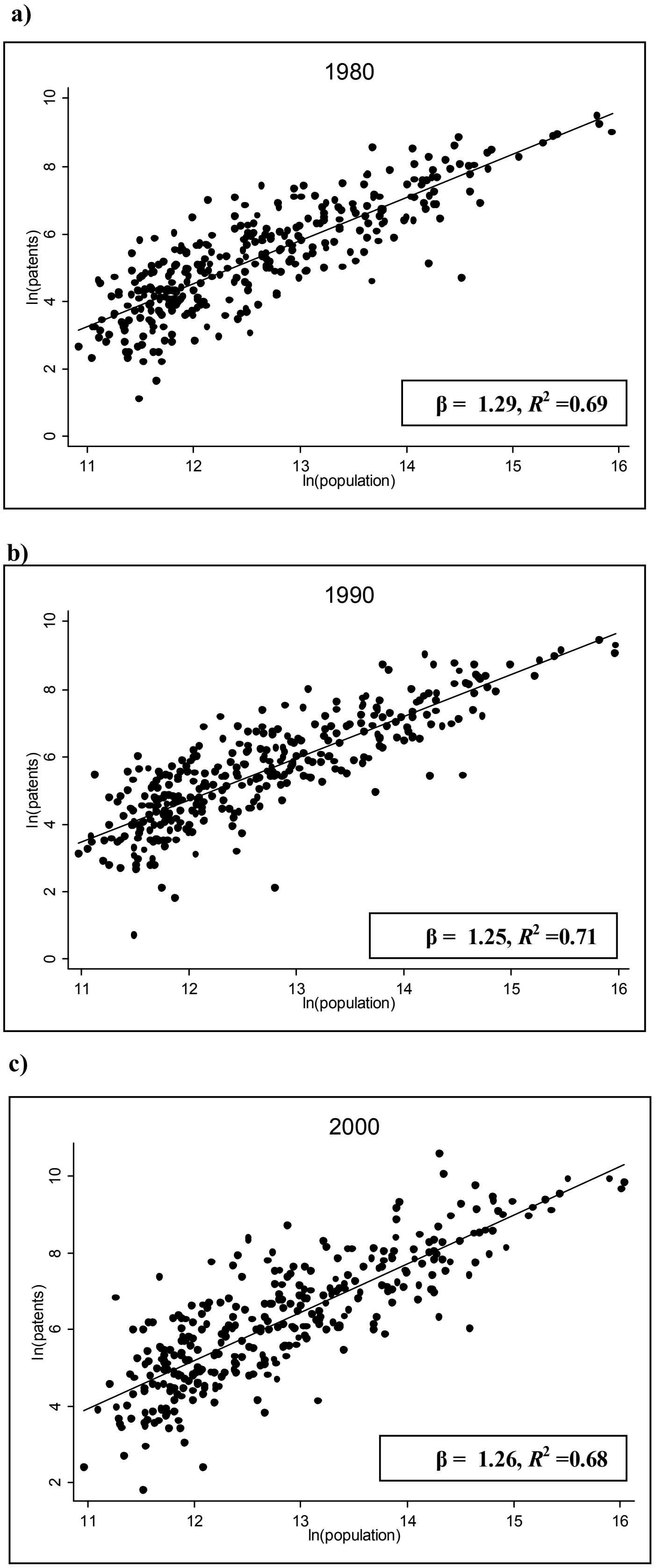

Figure 1. Metropolitan Patents and Population (331 MSAs).

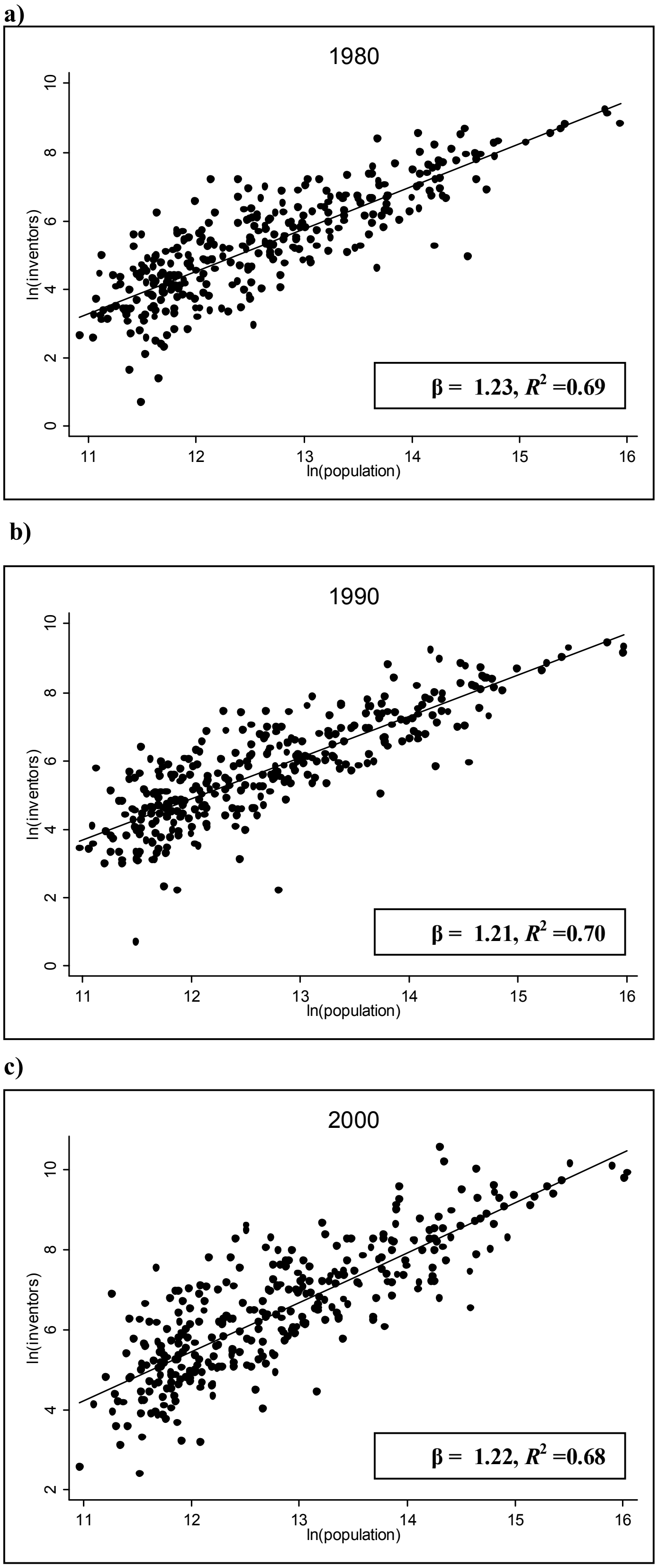

Figure 2. Metropolitan Inventors and Population (331 MSAs).

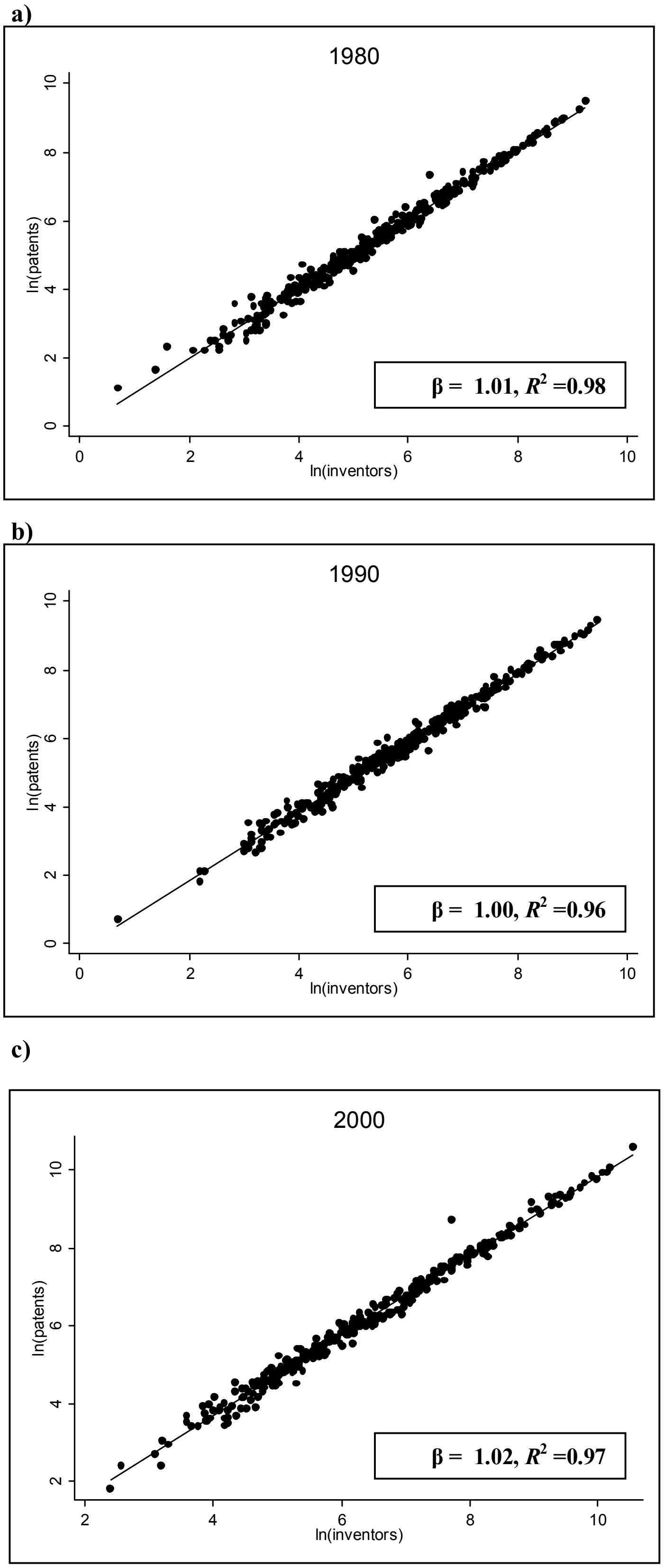

Figure 3. Metropolitan Inventors and Patents (331 MSAs).

Table 1. Summary Statistics for Network Variables.

	Patents	Inventors	Patents per Inventor	Network Herfindahl	LC Size	Network Density	Outside Collaborators	Technology Herfindahl
1980								
Mean	689.83	633.290	1.036	0.048	0.102	0.014	0.717	0.048
Std. Dev.	1471.109	1274.384	0.221	0.071	0.116	0.017	0.127	0.047
CoV*	2.133	2.012	0.213	1.476	1.131	1.214	0.177	1
Max	12990	10220	2.487	0.559	0.651	0.111	1	0.373
Min	3	2	0.571	0.001	0.004	0	0.199	0.009
No. obs.	331	331	331	331	331	331	331	331
1990								
Mean	858.329	955.112	0.876	0.040	0.102	0.016	0.649	0.047
Std. Dev.	1690.914	1823.791	0.162	0.056	0.150	0.026	0.121	0.046
CoV*	1.970	1.910	0.185	1.387	1.471	1.625	0.186	0.979
Max	12882	12594	1.545	0.510	0.680	0.334	1	0.511
Min	2	2	0.474	0.001	0.003	0	0.338	0.011
No. obs.	331	331	331	331	331	331	331	331
2000								
Mean	1724.239	2069.196	0.786	0.054	0.152	0.015	0.521	0.046
Std. Dev.	3753.695	4217.698	0.179	0.072	0.154	0.019	0.114	0.033
CoV*	2.177	2.038	0.228	1.338	1.013	1.267	0.219	0.717
Max	38170	37051	2.931	0.536	0.797	0.157	0.806	0.277
Min	6	10	0.425	0.002	0.003	0.001	0.224	0.012
No. obs.	331	331	331	331	331	331	331	331

*** CoV = coefficient of variation.**

Table 2. Summary Statistics for Metropolitan Socio-Economic Variables.

	Population Density	PCPI	Herfindahl Index	Market Structure	Large Firms
1980					
Mean	352.64	18469.34	0.234	0.067	0.003
Std. Dev.	839.523	3092.739	0.029	0.015	0.001
CoV*	2.381	0.167	0.124	0.224	0.333
Max	12261.87	32785.33	0.358	0.126	0.006
Min	3.434	10363.15	0.163	0.044	0.0
No. obs.	331	331	331	331	331
1990					
Mean	378.303	22128.67	0.226	0.072	0.004
Std. Dev.	847.361	4294.808	0.028	0.014	0.002
CoV*	2.240	0.194	0.124	0.194	0.500
Max	12135.4	41385.1	0.377	0.121	0.007
Min	4.347	11110.183	0.168	0.038	0
No. obs.	331	331	331	331	331
2000					
Mean	419.823	25877.284	0.223	0.079	0.009
Std. Dev.	903.237	5253.508	0.027	0.048	0.002
CoV*	2.151	0.203	0.121	0.608	0.222
Max	12806.3	53738	0.332	0.323	0.008
Min	5.342	13121.6	0.158	0.036	0.004
No. obs.	331	331	331	331	331

***: CoV = coefficient of variation**

Table 3. Correlation Statistics for all Variables (2000)

	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)
1) Patents	1													
2) Inventors	0.9924	1												
3) Patents per Inventor	0.2068	0.1526	1											
4) Network Herfindahl	0.1730	0.1412	0.1364	1										
5) LC Size	0.3192	0.2900	0.3027	0.8392	1									
6) Network Density	-0.2591	-0.2716	-0.2005	0.3192	-0.0209	1								
7) Technology Herfindahl	-0.2232	-0.2437	0.1347	0.3276	0.1660	0.621	1							
8) Outside Collaborators	-0.0899	-0.1261	0.4161	-0.0943	-0.1244	0.0821	0.0792	1						
9) Population	0.7311	0.8535	0.1283	-0.1056	0.0510	-0.2801	-0.2412	-0.0256	1					
10) Population Density	0.3485	0.3641	-0.0104	-0.078	-0.0097	-0.1678	-0.1415	-0.2414	0.4449	1				
11) PCPI	0.4992	0.5287	0.0691	-0.0269	0.1145	-0.3838	-0.2966	-0.3534	0.3808	0.3516	1			
12) MSA Herfindahl	0.0108	0.0169	-0.0145	0.1598	0.0846	0.0227	0.0143	-0.0697	-0.0147	-0.0586	0.0922	1		
13) Market Structure	-0.1934	-0.1964	-0.0513	-0.043	-0.1050	0.1224	0.0216	-0.036	-0.1967	-0.0726	-0.0443	0.0753	1	
14) Large Firms	0.2021	0.2069	-0.0302	0.0250	0.0905	-0.1158	0.0177	-0.0298	0.1697	0.1061	0.0608	0.0031	-0.5018	1

Table 4. Pooled Fixed Effects FGLS model of metropolitan patenting, 1980 - 2001

dependent variable:	Model 1 Patents	Model 2 Patents
Network Herfindahl	0.0214	
	(0.0027)	
LC Size		0.0255
		(0.0025)
Technology Herfindahl	0.0586	0.0608
	(0.0056)	(0.0053)
Network Density	-0.3114	-0.3111
	(0.0077)	(0.0072)
Outside Collaborators	0.0799	0.0802
	(0.0051)	(0.0052)
Inventors	1.0649	1.0612
	(0.0039)	(0.0035)
Population density	0.0483	0.046
	(0.0045)	(0.0042)
PCPI	-0.2038	-0.2034
	(0.0214)	(0.0208)
Herfindahl Index	0.1111	0.1126
	(0.0163)	(0.0159)
Market structure	-0.0756	-0.0743
	(0.0087)	(0.0085)
Large firms	0.0177	0.0166
	(0.0031)	(0.0031)
constant	1.8208	1.8475
Log likelihood	8426	8412
Cross-sectional units	331	331
Years	21	21
Observations	6261	6261

All variables in natural logarithm form. Dependent variable measured in year *t+1*, independent variables in year *t*. Standard errors in parentheses; all coefficients are significant at the at the 99% confidence level.

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