The main objective of this proposal is the construction of a new quantitative and predictive theory of social organization and dynamics in cities. We will assemble and explore new large-scale datasets for urban systems around the world and perform scaling analysis of urban metrics to identify statistical regularities and general underlying principles. These will be synthesized as predictive statistical models inspired by statistical mechanics and non-linear dynamics.

**Scaling analysis as a tool for identifying general principles of urban social dynamics**

The first major component of our proposed research is the use of scaling analysis as a tool for revealing general features in the structure and dynamics of social systems. Systematic variations of the properties of cities with population size establish quantitative regularities across scales that reveal general human and social dynamics at play regardless of time, place or cultural environment. These statistical regularities -to exist- become the touchstones for a general quantitative theory of cities.

It has long been hypothesized in the social sciences that there are general features to human and social dynamics, and that the structure of human interaction changes systematically with the scale (and density) of human contacts. Cities have been central to these considerations from the beginning. Early on Durkheim (1863) and Tönnies (1887) observed that urban agglomeration implies growing functional diversity and specialization of human activity, growing interdependencies and the emergence of social norms that transcend kin. The extent to which these effects are exacerbated by population size remains unknown. Urban sociology traces its origins to early, ecological studies of urban growth and neighborhood change in Chicago (e.g., Park, Burgess, and McKenzie 1925). A related line of work emphasized cities as spaces of intense human interaction that may lead to general behavioral adaptations and cultural change. (Wirth 1938; Fischer 1975, 1995). Wirth, in particular, developed a general framework of how quantitative indicators of social life vary with metrics that define cities, such as population size, density and heterogeneity.

Scaling, as the quantitative analysis of how urban indicators vary systematically with size across a system of cities, goes to the heart of these issues. Scaling has been instrumental in gaining deeper insights into problems across the entire spectrum of science and technology because scaling laws typically reflect underlying generic features and principles that are independent of the details of particular models (Bonner 2006; Carneiro 1967; Barenblatt 2003; Brock 1999). In the physical sciences this has been enormously successful: phase transitions, chaos, unification of fundamental forces, and the discovery of quarks are but a few of the more significant examples, resulting in several Nobel prizes, where scaling has revealed important universal physical principles or structure.

More importantly for cities, scaling has recently been applied to a wide range of biological phenomena leading to a unifying quantitative picture of their organization, structure and
dynamics. Organisms as metabolic engines, characterized by energy consumption rates, growth rates, body size, and behavioral times (West et al. 1997; 1999; Enquist et al. 1998), have a clear counterpart in social systems (Macionis, and Parillo 1998; Levine 1995).

Cities as consumers of energy and resources, and producers of artifacts, information and waste have often been compared to biological entities (Macionis, and Parillo 1998; Levine 1995). Are these just qualitative metaphors, or is there quantitative and predictive substance in the implication that social organizations are extensions of biology, satisfying similar principles and constraints? Are the structures and dynamics that evolved with human socialization fundamentally different from those in biology? Answers to these questions provide a framework for the construction of a quantitative theory of the average city, which would incorporate the roles of innovation and economies of scale, and predictions for growth trajectories, levels of social and economic development and ecological footprints.

**Empirical evidence for scaling laws in urban organization and dynamics**

We recently established (Bettencourt et al 2007; 2008; 2009) that scaling relations characterize many dimensions of social and economic life in the city, including, patterns of human behavior. To be specific, scaling laws are empirical statistical regularities that relate properties of a system Y to its size N (taken to be its population); these are typically simple power laws of the form:

$$Y(N) = Y_0 N^\beta$$

(1)

where $\beta$ is the scaling exponent and $Y_0$ an overall normalization constant. Scaling laws are statements that system properties Y, measured at different scales N, are not a function of the particular system size but only of the relative dimension. The independence of the ratios of Y measured at different system sizes, on N, constitutes the phenomenon of self-similarity.

<table>
<thead>
<tr>
<th>Y</th>
<th>$\beta$</th>
<th>95% CI</th>
<th>Adj-$R^2$</th>
<th>Observations</th>
<th>Country-Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>new patents</td>
<td>1.27</td>
<td>[1.25,1.29]</td>
<td>0.72</td>
<td>331</td>
<td>USA 2001</td>
</tr>
<tr>
<td>inventors</td>
<td>1.25</td>
<td>[1.22,1.27]</td>
<td>0.76</td>
<td>331</td>
<td>USA 2001</td>
</tr>
<tr>
<td>R&amp;D employment</td>
<td>1.34</td>
<td>[1.29,1.39]</td>
<td>0.92</td>
<td>266</td>
<td>USA 2002</td>
</tr>
<tr>
<td>supercreative</td>
<td>1.15</td>
<td>[1.11,1.18]</td>
<td>0.89</td>
<td>287</td>
<td>USA 2003</td>
</tr>
<tr>
<td>R&amp;D establishments</td>
<td>1.19</td>
<td>[1.14,1.22]</td>
<td>0.77</td>
<td>287</td>
<td>USA 1997</td>
</tr>
<tr>
<td>total wages</td>
<td>1.12</td>
<td>[1.09,1.13]</td>
<td>0.96</td>
<td>361</td>
<td>USA 2002</td>
</tr>
<tr>
<td>total bank deposits</td>
<td>1.08</td>
<td>[1.03,1.11]</td>
<td>0.91</td>
<td>267</td>
<td>USA 1996</td>
</tr>
<tr>
<td>cost of housing (p.c)</td>
<td>0.09</td>
<td>[0.07,0.12]</td>
<td>0.21</td>
<td>240</td>
<td>USA 2003</td>
</tr>
<tr>
<td>new AIDS cases</td>
<td>1.23</td>
<td>[1.18,1.29]</td>
<td>0.76</td>
<td>93</td>
<td>USA 2003</td>
</tr>
<tr>
<td>walking speed (p.c.)</td>
<td>0.09</td>
<td>[0.07,0.11]</td>
<td>0.79</td>
<td>21</td>
<td>Several 1979</td>
</tr>
<tr>
<td>violent crime</td>
<td>1.16</td>
<td>[1.12,1.20]</td>
<td>0.82</td>
<td>634</td>
<td>USA 1998</td>
</tr>
<tr>
<td>total housing</td>
<td>1.00</td>
<td>[0.99,1.01]</td>
<td>0.99</td>
<td>316</td>
<td>USA 1990</td>
</tr>
<tr>
<td>total employment</td>
<td>1.01</td>
<td>[0.99,1.02]</td>
<td>0.98</td>
<td>331</td>
<td>USA 2001</td>
</tr>
<tr>
<td>gasoline stations</td>
<td>0.77</td>
<td>[0.74,0.81]</td>
<td>0.93</td>
<td>318</td>
<td>USA 2001</td>
</tr>
<tr>
<td>gasoline sales</td>
<td>0.79</td>
<td>[0.73,0.80]</td>
<td>0.94</td>
<td>318</td>
<td>USA 2001</td>
</tr>
</tbody>
</table>

Table I. A sample set of scaling exponents for the variation of urban indicators Y in American MSAs, with population size N, Eq. (1). Scaling indicates self-similarity in rates of change across the urban system. Data from the Census Bureau, and the Bureau of Economic Analysis’ County Business Patterns.

1
Table I shows a sample of empirical scaling laws across American MSAs. Two examples of explicit data plots are shown in Figure 1. We have found similar scaling laws for every nation for which data aggregated at the MSA level are available, including European countries and China.

These results suggest a general taxonomy of urban properties in terms of their scaling. Scaling exponents fall into three categories, with the value of $\beta$ clustering around the same value in each category: (i) The value $\beta=1$ which corresponds to simple linear scaling with population is usually associated with individual needs (job, house, water consumption); (ii) $\beta<1$ characterizes quantities that display economies of scale, and is associated with material infrastructure, corresponding to analogous quantities in biology. Finally (iii), all interesting social quantities that are intrinsically and uniquely associated with the truly social nature of cities, such as rates of innovation and wealth creation, display $\beta>1$, which signifies increasing returns to population size. This is the quantitative expression of a phenomenon well-known to economists (Acs and Armington 2006; Krugman 1991; Lucas 1988; Glaeser 1994; Romer 1986; Sveikauskas 1975) and other experts in innovation: large cities are disproportionately the sources of new ideas and the engines of economic activity. Table I shows that cities are, by about the same measure, also places with disproportionate higher rates of crime, disease incidence and costs of living.

![Figure 1](image1.png)

**Figure 1**: Examples of scaling relationships for total metropolitan wages and supercreative employment (Florida, 2002) in the USA. Data obtained from County Business Patterns and the Economic Census.

Scaling relations also have important dynamical implications. Most superlinear scaling occurs for quantities that are *rates of change*: including wealth and knowledge creation, and rates of human behavior such as walking speed. Thus, superlinear scaling laws reveal self-similarity in rhythms of human and social behavior (Bornstein and Bornstein 1976), and express quantitatively their systematic acceleration with population size (Simmel 1903; Milgram 1970). Growth fueled by innovation processes manifesting increasing returns to scale (Bettencourt et al 2007) implies cycles of accelerating growth punctuated by major adaptations, Figure 2A. In this regime there is, in principle, no limit to urban growth, contrary to the expectations from classical economic models (Henderson 1988), because accelerating innovation can continue generating new resources and organizational forms. The length of each growth cycle is also predicted to decrease as populations grow, see Figure 2B for New York City.
Figure 2. Successive cycles of increasing returns to scale avoid the singularity (A: left), but require new innovation paradigms. The relative population growth of New York City (B: right) shows super exponential growth cycles (Bettencourt et al. 2007). Inset shows the shortening of growth cycles with population.

Proposed Research

1. Disentangling scaling effects from local factors: Scaling analysis and the method of residues

Despite ubiquitous scaling, each city has its local flavor expressed e.g. as its population makeup, its socioeconomic repertoire and its amenities. How much of these characteristics are to be expected based on population size, and which are truly local factors? The ability to disentangle these two effects is important because: i) it identifies local successes and failures that may more readily respond to local public policy, ii) it corrects a fundamental conceptual error in standard urban rankings, which are defined on a per capita basis, and iii) it makes possible the construction of new ranking metrics, true to the specific local nature of a city.

To achieve the separation between expectations for an average city of a given size and the properties of specific places, we proceed as follows: Once a scaling is identified at the urban system level the residues expressing the deviation of each city’s metric from its size expectation are written as

\[ y_i(t) = \log[Y_i(t)] - \log[Y_S(N_i)] = \log \left[ \frac{Y_i(t)}{Y_S(N_i)} \right] \]  

(3)

where \( Y_i(t) \) is the value of the quantity \( Y \) for city \( i \) at time \( t \), and \( Y_S(N_i) \) is the value of \( Y \) expected from the scaling law at that community size. This is the application of the familiar method of residues (Coleman 1976; Batty and March 1976) to the scaling analysis of urban metrics. This method is designed to separate phenomena at different scales and has had remarkable successes in physics, for example revealing the existence of smaller planets from the perturbations in a larger star’s motion.

The variables \( y \), are independent of population size by construction and can be treated as statistical (dimensionless) fluctuations. Their statistical distribution has zero mean, and its width (e.g. variance) measures the dispersion of fluctuations above and below the average expectation from scaling. Variables \( y \), for different quantities \( Y \), can then be cross-correlated because they are
dimensionless. They can also be correlated to other candidate drivers, thus testing hypotheses about size independent local factors that may e.g. lead to higher crime or economic productivity than expected for a city of a given population size.

In this section we propose to:

1.1. Construct population-size independent urban metrics $y_i$ for all our datasets worldwide, that identify local urban processes responsible for the successes and deficits of a city relative to others in the urban system.

1.2 Display these results via online GIS maps, accessible to other scientists and the general public. A prototype is displayed in Figure 3. We plan to display all our data and analysis in the form of an online worldwide urban observatory.

1.3 Investigate the general properties of the $y_i$ urban indicators in terms of their temporal, spatial and cross quantity correlations. In particular we will investigate in detail possible causal relations between measures of economic productivity, crime and innovation.

1.4 Investigate the stability and robustness of cities exhibiting urban indicators that strongly deviate from their expected averages. Are such deviations a source of growth or decay?

**Figure 3.** A prototype online map of population size independent urban indicators $y_i$ for Gross Metropolitan Product (GMP) of US cities in 2005 (data from US Bureau of Labor Statistics). Red (blue) circles denote cities with a GMP greater (smaller) than expected for their size, according to the scaling law. Circle size denotes the magnitude of $y_i$. These maps reveal spatial and temporal performance trends that reflect true local factors at play in each city.
2. The micro-foundations of urban scaling

Perhaps the most important question raised by our preliminary findings of pervasive urban scaling is to explain the value of the exponents $\beta$ and create a theory capable of predicting them from first principles. We propose to pursue these objectives guided by both theoretical principles from network theory and self-similarity and by datasets where we can span several levels of resolution. Examples of the latter are patents (where filings can be disaggregated by subject class), crime rates (where several types of crime contribute to our reported scaling results) and economic productivity, measured by GMP and wages (which can be decomposed by economic sectors of activity).

We will focus our research plan on two main approaches. First we will decompose a scaling variable into its several available components and test them for scaling behavior. Secondly we are particularly interested in identifying the social network structures that allow on the average new people entering the city to result in greater economic and creative productivity for the system as a whole (Duranton, and Puga 2000). In this way a network theory of social organization can be developed that can account for the micro-foundations of urban scaling behavior and their self-similar iteration through multiple scales.

We propose to explore the nexus between scaling, growth and innovation across scales, from the individual level and via social networks to the whole city, as follows:

2.1. Compile and study networks of patenting, including inventor and patent associations, resulting from: a) co-authorship, b) citation, c) institutional affiliation. We will characterize the resulting networks in terms of graph theory and study their change over time to identify network structures that map the addition of new individuals to the city to increased levels of per capita patenting at the city level.

2.2. We will decompose aggregate measures of crime in terms of subcomponents (murder, assault, vehicle theft, etc) and study their scaling with city size. Wherever possible these measures will be connected with crime maps in specific cities to establish some of the possible causal mechanisms for the multiplicative increases with population size.

2.3. We will decompose total numbers of patents, employment and wages in terms of their component classes and sectors. We will then investigate the scaling relations of these subcomponents and identify which contribute to superlinear scaling. We will also measure and characterize statistically the trend for greater diversification of economic and creative activity with city size and explore the existence of causal connections between diversification and population and economic growth.
3. Multi-scale dynamical models of urban organization and growth

Our preliminary findings suggest that a quantitative, predictive model of the dynamics of an urban system is now possible. Such a model must account simultaneously for dynamics at several levels of organization and for the causal interactions between them.

First, we know for some time that urban systems display a distribution of city sizes that follows a Zipf (power-law) distribution (Zipf 1949). The distribution of city sizes is approximately stationary and is characterized by a single parameter $\alpha$, the Zipf exponent. Second, scaling laws establish how urban indicators vary systematically with population size. These scaling laws are also stationary statistical relations and are characterized by conserved exponents $\beta$, which, as we have seen above, fall in a small number of universality classes. Finally, population size independent fluctuations $y_i$ show characteristic temporal and spatial correlations and categorical cross-correlations. Models for the evolution of these variables must explicitly show how different levels of analysis are connected, and influence each other.

To construct and validate these models we will:

3.1 Measure the turnover rates of people and establishments in cities as a function of their population size. The movement of people, together with demographic data on births and deaths, will lead predictive models of Zipf’s distribution. The differential migration of professions, age groups, income levels, etc will map some of the mechanisms underlying urban scaling laws.

3.2 Build models for the spatial and temporal dynamics for the residues $y_i$ that are consistent with their temporal, spatial and cross correlation properties and capture the role of these deviations in driving urban growth or decay. We expect that models for the evolution of the $y_i$ will be based on sets of weakly coupled stochastic partial differential equations.

4. General research objectives and expected impact

The creation of quantitative and predictive theories of cities and urban systems is an achievement with far reaching scientific and societal consequences. Effective policy, able to guide the development of fast urbanizing nations, is essential to mitigate human strife and irreversible and unsustainable burdens on the natural environment. Success in such an important scientific problem would create new principles and techniques for the general study of social organization and dynamics and for complex systems in general.

Cities are a natural means to achieve the benefits of concentrating people together and accelerate their creative and economic potential, but they are not unique in this respect. With improvements in cyber technologies and transportation much of the social dynamics so conspicuously at play in cities will be realized worldwide and through virtual online organizations. A predictive understanding of human and social dynamics in cities is essential for designing the cyber technologies and transportation networks that can realize the benefits of increasing returns dynamics throughout human societies.
References:


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