Social Cohesion and Embeddedness: A hierarchical conception of social groups*

James Moody
Department of Sociology
The Ohio State University

&

Douglas R. White
Department of Anthropology
University of California-Irvine

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Abstract

While questions about social cohesion lie at the core of our discipline, no clear definition of cohesion exists. We present a definition of social cohesion based on network connectivity that leads to an operationalization of social embeddedness. We define cohesiveness as *the minimum number of actors who, if removed from a group, would disconnect the group*. This definition generates hierarchically nested groups, where highly cohesive groups are embedded within less cohesive groups. We discuss the theoretical implications of this definition and demonstrate the empirical applicability of our conception of nestedness by testing the predicted correlates of our cohesion measure within high school friendship and interlocking directorate networks.

Keywords: Social networks, social theory, social cohesion, connectivity algorithm, embeddedness.

"*...social solidarity is a wholly moral phenomenon which by itself is not amenable to exact observation and especially not to measurement.*"
(Durkheim, (1893 [1984], p.24)

"The social structure [of the dyad] rests immediately on the one and on the other of the two, and the secession of either would destroy the whole. ... As soon, however, as there is a sociation of three, a group continues to exist even in case one of the members drops out."
(Simmel (1908 [1950], p. 123)

Introduction

Questions surrounding social solidarity – understanding how social collectivities are bound together – are foundational for sociologists and have engaged researchers continuously since Durkheim. Analytically, solidarity can be partitioned into an *ideational* component, referring to the psychological identification of members within a collectivity and a *relational* component, referring to the observed connections among members. Durkheim identified the theoretical connection between these two components in *The Division of Labor* by linking changes in the common consciousness to the movement from mechanical to organic societies. Below, we identify an essential feature of the relational component of social solidarity. For clarity and theoretical consistency, we refer to the relational component of social solidarity as *cohesion*, and introduce a new characterization of cohesion that rests on the pattern of multiple connections within a social network. To operationalize this conception, we introduce a methodology of *cohesive blocking* that allows researchers to identify cohesive substructures in a network and simultaneously identify the relative position of such structures within the population.

Methods for identifying "cohesive groups" are not new. Many contemporary researchers, however, have often conflated the two dimensions of solidarity, defining both as cohesion. This has lead to a wide variety of cohesion measurements ranging from individual psychological feelings (Bollen and Hoyle 1990) to global features of relational structures (Freeman 1992; Friedkin 1984), and multiple stages in-between (Carron 1982; McPherson and Smith-Lovin 1986). The resulting "legacy of confusion" (Mudrack, 1989) surrounding issues related to cohesion hampers our ability to resolve theoretical debates about the relation between social organization and individual attitudes and behaviors. Instead, researchers employing widely differing empirical measures talk past one another.

The confusion surrounding cohesion and solidarity is primarily a theoretical problem. In addition to the need to distinguish between ideational and relational components of solidarity, theorists need to be clear about the structure of relations that give rise to social cohesion. The unifying intuition surrounding previous notions of cohesive structures rests on the robustness of the collectivity to disruption. "Cohesive groups" are thought to be those that are held together well and thus difficult to break apart. As we show below, we can identify two features of a relational structure that contribute to its ability to remain connected. On the one hand, collectivities can be united through strong ties to a central leader. On the other hand, collectivities can be united through a diffuse pattern of relations that weave members together through multiple independent connections. We term the collectivities united through a single leader "adhesive" and reserve the term "cohesive" for collectivities united through multiply distributed connections.

This distinction might initially seem trivial if both structures could lead to robust groups.[1] The theoretical expectations for the social qualities of collectivities organized in these two manners, however, differ dramatically. Adhesive groups are dependent on the unilateral action of a single person while cohesive groups will maintain a group status regardless of the unilateral action of *any* actor in the group. As we show below, this distinction (first introduced by Simmel in his discussion of the triad and the dyad) has important theoretical implications for the character of groups structured in this way. While power, resources and information will be concentrated in

[1] Due to the long history of small, face-to-face research on "groups", we would prefer to avoid the use of this term altogether, in favor of broader terms such as "collectivity" or "sub-structure" that carry much less theoretical baggage. Such a substitution, however, results in decidedly awkward writing. We thus maintain the use of "group" at times, but remind readers that our conception is not limited to the small face-to-face structures commonly referred to by the term.

adhesively structured collectivities, power will be diffuse and information widely distributed within cohesively structured settings. Below we show how common operationalizations of 'group cohesion' can confound adhesive and cohesive properties of groups. To the extent that the different types of social organization lead to different group qualities, a methodological inability to distinguish such patterns will lead to ambiguous findings. In our empirical examples, we demonstrate that our conception of cohesion provides unique explanatory power across diverse substantive settings.

Given that one identifies cohesive substructures within a social setting, how are the structures related to each other? Our analysis of cohesion shows that cohesive groups can be related with respect to nesting within the population. Thus, the cohesive blocking of a network provides not only an identification of cohesive substructures, but identifies the position of such groups within the network. Theoretically, we argue that the *nestedness* of a collection of actors captures an important dimension of Granovetter's concept of network embeddedness (Granovetter 1985; Uzzi 1996). While the term "embeddedness" often refers broadly to involvement in multiplex social relations, our conception of nestedness captures the depth of involvement within any set of relations. Widely used in sociology, embeddedness has rarely been given an explicit operational definition. Instead, the term has been used to refer broadly to the importance of social networks for actors, indicating that actors integrated within social networks face different sets of resources and constraints than those who are not embedded in such relations. When contrasted with individualistic theoretical frameworks that deny the importance of relations for behavior, this level of theoretical ambiguity is profitable. Allowing the concept to remain at the level of orienting statement, however, significantly lowers our ability to build on previous work.

In this paper we extend a new relational definition of structural cohesion that rests on the connectivity patterns within social networks and that leads to the concept of nestedness, which is a natural operationalization of one dimension of social embeddedness. Intuitively, cohesive groups are strongly held together. Building on this intuitive idea, we argue that the ability of the group to stick together is captured in how difficult it is to break the group apart. Extending Simmel's insight on the distinction between dyads and triads, we argue that a group so fragile that the removal of a single person would destroy the group is not very cohesive, while a group that can withstand the loss of many members is much more cohesive. This distinction implies a differentiation between two types of relational organization, with differing theoretical implications for the character of the group. After reviewing previous theoretical work, we provide a formal definition of cohesion that can be directly operationalized as network connectivity. We then show that previous methods of identifying cohesive groups ambiguously combine both adhesive and cohesive elements of group structure, and demonstrate the value added of our measure of cohesive group position in two widely differing settings: high school friendship networks and interlocking directorates.

Theory and Background

Research on social cohesion has been plagued with contradictory, vague and difficult to operationalize definitions (for reviews, see Doreian and Fararo 1998; Mizruchi 1992; Mudrack 1989). While often murky, previous definitions of cohesion seem to share a common intuitive core resting on how well a group is "held together." For example, cohesive groups should display "connectedness" (O'Reilly and Roberts 1977), or cohesion is described as a "field of forces that act on members to remain in the group" (Festinger, Schachter, and Back 1950) or as "the resistance of a group to disruptive forces" (Gross and Martin 1952). Dictionary definitions of cohesion rest on similar features, such as "[t]he action or condition of cohering; cleaving or sticking together" (OED, 2000).

In his review of the cohesion literature, Mudrack (1989) identifies a gap between researchers' conceptions and their operationalizations of cohesion.

> "Cohesiveness is a property of 'the group' and yet 'the group' as a distinct entity is beyond the grasp of our understanding and measurement. Consequently, researchers have *perforce* directed their investigations at individuals…" (p.38)

By focusing on individuals, social psychologists and small group researchers often confound outcomes and correlates of cohesion with cohesion itself, commingling the two components of social solidarity. Analytically, we must differentiate the togetherness of a group from the *sense* of togetherness that people express. Using subjective factors, such as a "sense of we-ness" (Owen 1985), "attraction-to-group" (Libo 1953) or the ability of the group to attract and retain members that may result *from* group cohesion unnecessarily limits our ability to ask questions about how group cohesion affects (and is affected by) social psychological factors.[2]

In contrast to Mudrack's claim, one of the primary strengths of a relational conception of cohesion lies in the ability to both conceptualize and operationalize cohesion at the group level. The "forces" and "bonds" that hold the group together are the social relations among members of the group, and cohesion is an emergent property of the relational pattern. Unfortunately, much of the previous work on relational cohesion has been focused on very small groups, using cliques — collections of people with direct relations to every other person in the group — as the archetype of cohesion. Since few people have the resources required to form cliques of thousands of people, this conceptual frame necessarily limits our ability to explore cohesive patterns in large social settings. By identifying topographical aspects of cohesion that do not depend on size, we are able to measure cohesion directly, something Mudrack says is impossible, and scale cohesion to groups of any size.

From the proceeding, a preliminary and intuitive definition of cohesion follows:

> *Def. 1.1. A collectivity is cohesive to the extent that the social relations of its members hold it together.*

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[2] Definitions that rest on the ability to attract or retain members are theoretically insufficient. Consider a situation such as the stock market in the late 1990s. The large profits to be gained in the market *attracted* many and the continuing profits *retained* those same actors. Certainly all investors do not constitute a cohesive group, and if they were to form meaningful relations, the Securities and Exchange Commission would cite them for conflict of interest. On the other hand, examples of clear social units, such as dysfunctional families, where people are strongly linked (through blood) but unhappy to be members provide an example of cohesion where the psychological sense of "attraction" to the group might be quite low.

There are five important features of this preliminary definition. First, it focuses on what appears constant in intuitive notions of cohesion: a property describing how a collection of actors is united. Second, it is expressed as a property of the group. Individuals may be more or less embedded within a cohesive group, but the group itself maintains a unique level of cohesion. Third, this conception is continuous. Some groups will be weakly cohesive (not held together well) while others will be strongly cohesive. Fourth, the cohesion rests on observable social relations among actors. Finally, network size is irrelevant.

Based on this definition, one is immediately drawn to identifying the relational features that hold collectivities together. Clearly, a collection of individuals with no relations among themselves is not cohesive. If we imagine relations forming among a collection of isolates,[3] we could identify a point where each person in the group is connected to at least one other person in such a way that we could trace only a single path from each to the other. A weak form of social cohesion starts to emerge as these islands become connected through relations.[4] As additional relations form among previously unconnected pairs, we can trace multiple paths through the group. Intuitively, the ability of the group to "hold together" increases as the number of ways we can link group members increases. This intuition is captured well by Markovsky and Lawler (1994) when they identify "reachability" as an essential feature of cohesion.

That cohesion seems to increase as we add relations among disconnected pairs has lead many researchers to focus on the *volume* (or density) of relations within and between groups as their defining characteristic (Alba 1973, Fershtman 1997; Frank 1995; Richards 1995). There are two problems with using volume to capture cohesion in a network. First, consider again our group with one node-independent path connecting all members. We can imagine moving a single relation from one pair to another. In so doing, the ability to trace a path between actors is lost,

[3] Actors without any relations to others in the population.
[4] See Hage and Harary (1996) for a discussion of this process among islands in Oceania. We recognize that social groups can form from the dissolution of past groups; the above discussion is useful only in understanding the essential character of relational cohesion.

but the number of relations remains the same. That is, since the number of ties remains constant when one rearranges ties in a network, simply moving one relation will not change volume, but will change reachability and the number of independent paths, implying that volume alone cannot account for cohesion.

Second, continuing the example from above, the initial (and weakest) moment of cohesion occurs when we can trace only a single path from each actor in the network to every other actor in the network.[5] Imagine that additional relations form, but our ability to trace a chain from one person to another still depends on a *single* person. This would occur, for example, if relations revolved around a charismatic leader. In such cases, each person has ties to the leader, and through the leader is connected to every other member of the group. While connected, such groups are notoriously fragile. As Weber (1978: 1114) pointed out, the loss of a charismatic leader will destroy a group whose structure is based on an all-to-one relational pattern. Thus, increasing relational volume but focusing it through a single individual does not necessarily increase the ability of the group to hold together.

The robust nature of collectivities such as cults and terrorist networks, however, suggests that robust collectivities can exist with an all-to-one structure. To distinguish collectivities that depend on a single actor[6] from those that are mutually interconnected, we suggest distinguishing *co*hesive groups from *ad*hesive groups. This terminology rests on the types of actors commonly found in all-to-one groups. In the case of a charismatic group, for example, we often refer to members of the group as “adherents.” Similar structures appear in formal bureaucracies when subordinates are linked to supervisors, but not to each other. *Co*hesion implies mutual connection and equality that is less centralized than *ad*hesion (consider words within similar prefixes, such as

[5] Technically, this is known as a *spanning tree* of the graph.
[6] Or a series of single actors, each of which is a separator of the graph.

*co*lleague, *co*mpanion, or *co*hort, compared to words such as *ad*junct, *ad*versary, or *ad*ministrator), which is exactly the meaning we wish to convey.[7]

Markovsky and Lawler (1994, Markovsky 1998) make a similar point when they argue that a uniform distribution of ties is needed to prevent a network from splitting into multiple subgroups.

> "… the organization of [cohesive] group ties should be distributed throughout the group in a relatively uniform manner. This implies the absence of any substructures that might be vulnerable, such as via a small number of 'cut-points' to calving away from the rest of the structure." (Markovsky, 1998 p. 245).

Such vulnerable substructures form when network relations are focused through a small number of actors. If pairs of actors are linked to each other through multiple others, the structure as a whole is less vulnerable to this type of split.

The substantive character of groups dependent upon a single actor differs significantly from those expected of groups with multiple independent connections, and often in ways that do not connote group solidarity. First, the group as a whole will reflect the will and unilateral activities of the group leader. Dramatic instances of this characteristic are evident in cults, but are clear in any group where an individual's actions can determine the social standing of the collectivity. Second, while such structures *may* be stable and robust to disruption, such stability often rests more on the qualities of the links than on the overall relational *structure*.[8] Consider a terrorist network, which often has a spoke-and-hub configuration in which each spoke cell knows nothing of the other cells. Because any randomly captured member is unlikely to be at the hub, this person cannot know enough of the structure to put the entire group at risk, and the network as a whole can be maintained in the face of a concerted effort to exterminate it. If the hub is

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[7] The OED describes adhesion as the "Union of organs by confluence of normally unlike parts, …; in opposition to cohesion, the coalescence of like parts…" In general, the prefix co-/com- or con- means together, with, or to similar degree; while ac-/ad-/af-/ag-/al-/ap-/as- or at- means toward, near, or adjacent to.

[8] Formally, this is a reflection of edge connectivity, which is discussed in detail in White and Harary, n.d.

identified, however, the organization as a whole will be destroyed or disconnected. Thus, the stability of the structure depends on the ability to keep the hub hidden.[9] Finally, as implied by Markovsky and Lawler, adhesive organization promotes segmentation and fractionalization. Rarely will a purely adhesive structure succeed in isolating lateral ties among members. Instead, isolated local connections will form, embedding mutually connected groups within an adhesive global structure. Such substructures will likely develop norms and values that are distinct, which will lead to schism and fractionalization, as is suggested by the literature on the progression between church and sect.

In contrast to adhesive groups, however, collectivities linked through multiple independent paths are less easily segmented and unilateral action on the part of any actor is insufficient to disconnect the group. The presence of multiple connecting paths implies that if any one actor is removed or changes relations, alternative linkages among members exist. Information and resources can flow through multiple paths, making minority control of resources within the group difficult. As such, the inequality of power implicit in an adhesive structure is not present in a cohesive structure. We discuss the theoretical implications of multiple connectivity in detail below, after having formally qualified the topography of such groups.

Given these examples, we amend our preliminary definition of cohesion to make explicit the importance of multiple mutual connectivity.

> *Def. 1.2. A group is cohesive (rather than adhesive) to the extent that multiple independent social relations among multiple members of the group hold it together.*

This addition defines a metric for a continuous conception of cohesion. Groups that rest on connections through a single actor are weakly cohesive, those that rest on connections through

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[9] One measurement of the inequality of the dependence of a network on a single member is the pairwise dependency of the graph; see Freeman (1980) and White and Borgatti (1994).

two actors are somewhat more cohesive, and those that rest on connections through many actors are more cohesive yet.

This nicely captures Simmel's (1950:135) distinction between a dyad and a triad. In a dyad, the existence of the group rests entirely in the actions of each member. Either member, acting unilaterally, could destroy the group by leaving. Much of the power and intimacy associated with purely dyadic relations rests on this fact. However, once we have an association of three, each person is connected to every other through two paths (one directly, one through an intermediary), and thus a weakly cohesive group will remain even if one of the members decides to leave. The social unit is no longer dependent upon a single individual, and thus the actions of the social unit take on new (and uniquely social) characteristics.

From this perspective, *a group's cohesion depends on the minimum number of people who unite the group*. This continuous conception of cohesion implies two types of sub-groups that can characterize the position of groups in the population. First, we might have groups that "calve away" from the rest of the population, such as those implied by Markovsky and Lawler. In such cases, cohesive groups rest "side-by-side" in the social structure, one distinct from the other. Second, cohesive groups could be structured like Russian dolls -- with increasingly cohesive groups nested within them. For example, a group may have a highly cohesive core, surrounded by a somewhat less cohesive periphery. Such group structures are common, and have been described in areas as widely ranging as cults (Lofland and Stark 1965) and the trade among nations (Smith and White 1992).

A nested conception of cohesion provides a unique linkage between social cohesion and network embeddedness (Granovetter 1985). The concept of embeddedness, originally introduced in the early 1950s (see Portes and Sensenbrenner 1993 for a discussion) and reintroduced by Granovetter (1985), has had a significant influence within current sociological research and theory.[10] While used most often in research on economic activity (Baum and Oliver 1992; Portes and Sensenbrenner 1993; Uzzi 1996; Uzzi 1999) or stratification (Brinton 1988), embeddedness has been employed to describe social support (Pescosolido 1992), processes in health and health policy (Healy 1999; Ruef 1999), family demography (Astone et al. 1999) and the analysis of criminal networks (Baker and Faulkner 1993; McCarthy et al. 1998). While embeddedness often mixes murkily with the even broader concept of social capital, most treatments of embeddedness refer to some notion of an actor's relative depth of involvement in social relations (multiplex ties, multiple social worlds). If cohesive groups are nested within one another, then each successive group is more deeply embedded within the network. As such, one aspect of embeddedness — the depth of involvement in a relational structure — is captured by the extent to which a group is nested within the relational structure. To distinguish this aspect of embeddedness, we refer to it as nestedness.

In the next section, we introduce a set of formal terms needed to exactly characterize our conception of social cohesion. We then provide a final definition of group cohesion that allows a direct operationalization of the concept through network connectivity (White 1998). After showing that previous network conceptions of subgroup cohesion cannot uniquely distinguish cohesive from adhesive groups, we clarify the relation between cohesion and nestedness by way of a method for identifying connectivity sets and identify theoretically significant aspects of connectivity for large scale social processes.

Connectivity and Social Cohesion

To formalize this conception of group cohesion, we treat the relations among actors as a graph, and use the graph theoretic properties of connectivity sets to identify cohesive groups. Formally, we refer to a graph $\mathbf{G}(\mathbf{V},\mathbf{E})$, where the vertices, $\mathbf{V}$, represent our set of $|\mathbf{v}|$ actors and the edges, $\mathbf{E}$, represent the relations among actors defined as a set of pairs (v_i, v_j). Actor i is *adjacent*

[10] At last count, the social citation index lists close to 900 citations to Granovetter's 1985 article on social

to actor *j* if $(v_i, v_j) \in \mathbf{E}$.[11] The cohesion of a group depends on how pairs of actors can be linked through chains of relations. A *path* in the network is defined as an alternating sequence of distinct nodes and edges, beginning and ending with nodes, in which each edge is incident with its preceding and following nodes. We say that actor *i* can *reach* actor *j* if there is a path in the graph starting with *i* and ending with *j*. Two paths from *i* to *j* are *independent* if they only have nodes *i* and *j* in common. We can characterize the network based on whether or not every pair of actors is reachable. If there is a path linking every pair of actors in the network then the graph is *connected*. In general, a set is *maximal* with respect to a given property if it has the property but no proper superset does. A *component* of a graph **G** is a maximal connected subgraph of **G**.

Components are defined as maximal sets in which each actor is reachable from every other actor. In any component, the paths that link two non-adjacent vertices must pass through a given sub-set of other nodes. These nodes, if removed, would disconnect the two actors. Any such set of nodes, **S**, is called an (*i*,*j*) cut-set if every path connecting *i* and *j* passes through at least one node of **S**. If there is only one node in **S**, it is called a cut-node. Let **N**(*i*,*j*) be the smallest size of an (*i*,*j*) cut-set. **N**(*i*,*j*) will be the maximum number of independent paths connecting *i* and *j* (see Harary 1969 for proof). The *node connectivity*, *k*, of **G** is the smallest N(*i*, *j*) node separator in the graph. Thus, a graph is *k-connected* if there are at least *k* independent paths connecting every pair of actors in the graph. A component is 1-connected and in common terminology a 2-connected component is called a *bicomponent* and a 3-connected component a *tricomponent*. In general, the sub-graphs of **G** defined by various levels of *k*-connectivity are called the *connectivity sets* or *k-components* of the graph.[12]

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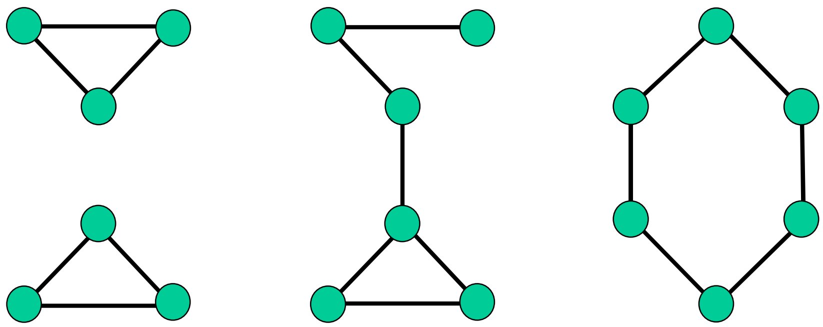
embeddedness.

[11] For the purposes of connectivity, we assume that actors do not relate to themselves and thus $(v_i, v_i) \notin \mathbf{E}$.

[12] Harary et al. (1965:25) were the first to note that "in general, if the sociometric structure of a group forms a block [in his context: is 2-connected], then the group is very cohesive." Beyond this reference to the cohesiveness of bicomponents, White (1998) is the first to define cohesion more generally in terms of k-connectivity, insofar as we are aware, and the first to show that groups formed through procedures based on generalizing the clique concept are not necessarily cohesive. White and Harary (n.d.) formalize these definitions and critiques and go on to discuss the relation between connectivity and density.

Figure 1 presents examples of differing levels of graph connectivity. Note that in each of the three graphs the *number* of relations is held constant, but the edges are *arranged* such that total group cohesion increases from left to right.

Figure 1. Examples of Connectivity Levels



Disconnected (0-connected) | A component (1-connected) | A bicomponent (2-connected)

Our interest is in identifying a *cohesive* property of the topography of social relations. Based on the intuitive notions captured in definition 1.2 and the formal graph properties presented above, we can now provide a final definition of group cohesion.

> Def 1.3a. *A group's cohesion is equal to the minimum number of actors who, if removed from the group, would disconnect the group.*

This definition can be operationalized directly with node connectivity. Because of the necessary relation between the number of cut-nodes and the number of independent paths, the disconnect version of definition 1.3 can be restated *without any loss of meaning* in held together terms as follows.

> Def 1.3b. *A group's cohesion is equal to the number of independent paths linking each pair of actors in the group.*

This definition of social cohesion retains all five aspects of our original intuitive notion of group cohesion, while providing a direct operationalization in graph connectivity that can be applied to groups of widely ranging sizes. Based on Simmel's discussion of the dyad, we argue that a minimum connectivity of 2 (a bicomponent) is the basis for moderate or stronger forms of *cohesion*, since any group with connectivity less than 2 is at the mercy of unilateral action by a single actor, which due to the different theoretical implications of the structure is best identified as an adhesive structure.[13]

Relation to Previous Conceptions of Cohesive Groups

Cohesion as connectivity differs from previous network approaches in many ways (see White 1998). Other common conceptions of social cohesion are based on relative relational volume within and between groups (Fershtman 1997; Frank 1995; Alba 1973), relational distance between actors (Seidman and Foster 1978, Luce 1950, Markovsky and Lawler, 1994), or the number of common connections among actors (Seidman 1983). While such features *may* identify strongly cohesive groups, it is possible for groups to meet one of these definitions and yet depend on a single actor to remain connected. Table 1 gives the results of applying 6 common methods for identifying cohesive subgroups to the network of 14 actors in figure 2 below, which has two cut-nodes ({4} and {11}) that if removed would break the graph into three pieces.

[13]Given our formal definition of cohesion as node connectivity, the distinction between adhesion and cohesion marks the ends of a continuum within the relational dimension of solidarity. Given that the graph is connected, the greater the dependence on a small number of actors the more adhesive will be the character of the group. As the connectivity of graph increases, vulnerability to unilateral action decreases, and the collectivity takes on the characteristics associated with structural cohesion.

Figure 2. Conflicting Conceptions of Cohesion

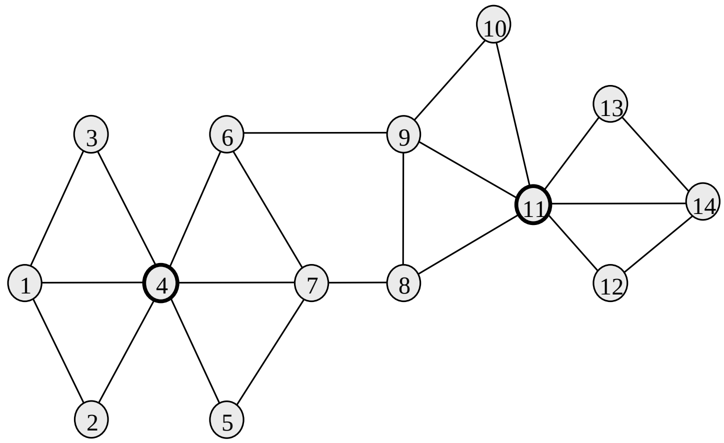


Table 1. Alternative partitions of actors into "cohesive" subgroups.

Sub-group detection method	Resulting Sub-group assignments
K-Core	All members are in a 2-core
K-plex	
k=1, minsize=3 (cliques)	{1 2 4} {1 3 4} {4 5 7}{4 6 7}{8 9 11}{9 10 11}{11 12 14} {11 13 14}
k=2, minsize=4	{1 2 3 4}{4 5 6 7}{6 7 8 9}{8 9 10 11}{11 12 13 14}
k=3, minsize=5	{1 2 4 5 7}{1 2 4 6 7}{1 3 4 5 7}{1 3 4 6 7}{4 6 7 8 9} {6 7 8 9 11}{8 9 11 12 14}{8 9 11 13 14}{9 10 11 12 14} {9 10 11 13 14}
N-clique / N-Clans*	
2-Cliques	{1 - 7}{4 - 8}{4 6 - 9}{6 - 9 11}{6 8 - 11}{8 - 14}
3-Cliques	{1 - 9}{4 - 9 11}{4 6 - 11}{6 -14}
4-Cliques	{1-11}{4-14}
Lambda Sets λ=3	{1 4 6 7}{8 11 9 14}
Factions (UCI-NET)	
(correlation criteria) 2-groups	{1-7}{8-14}
3-groups	{1-4}{5-9}{10-14}
4-groups	{1-3}{4-7}{8-11}{12-14}
Negopy (Richards, 1981)	{1-7}{8-14}
Bicomponents	{1-4}{4-11}{11-14}

* It is not necessary for n-cliques and n-clans to be identical, but in this instance they are.

Most previous methods of social cohesion have focused on the fully connected clique as the archetype of a cohesive group. Since each person in a clique is connected to every other person in the group, a natural generalization of the clique was to look at the number of people each person was tied to in a group. In a *k*-core (Seidman 1983), each member of the group is connected to at least *k* other people in the group.[14] In figure 2, the strongest *k*-core is a 2-core containing the entire graph. Since the 2-core contains cut nodes 4 and 11 it is 1-connected and thus has the weakest possible level of cohesion. Note that k-cores and connectivity sets are related: every *k*-component must be a *k*-core, but not every *k*-core will be a *k*-component.

A related group definition based on the minimum number of contacts is the *k*-plex, where each member of a group of *n* actors is connected to at least *n*-*k* other actors. If *k*=1, then every person needs to be directly connected to at least *n*-1 other people and the group is a clique (in figure 2, all cliques have 3 members). Looking at 2-plexes in our example, we find 5 groups with at least 4 members (there are an additional 25 2-plexes with 3 members). The 4-person *k*-plexes are internally strongly cohesive (they are 2-connected), but they are not maximal. All the 3-plexes are 1-connected and therefore only weakly cohesive.

Two common distance based measures are the n-clique (Luce 1950) and the n-clan (Mokken 1979), in which each member of the group needs to be within n-steps of every other member.[15] Depending on how distant we allow pairs of actors within the group to be (2 steps, three steps, etc.), the identified groups become increasingly inclusive. However, even in the closest linked groups (the 2-cliques), 3 of the 6 identified 2-cliques contains a single cut-node. A distance-based method is often at risk to such separating points because cut-nodes necessarily fall on the shortest path between pairs of points. A single cut-node will bring two sides of a graph into the distance-based group, but the group will depend on the single cut-node to remain connected, and hence lack even moderate cohesion.

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[14] Formally, a k-core is a connected, maximal induced subgraph that has minimum degree greater than or equal to k.

[15] Markovsky (1998, p.360) suggests a distance measure scaled by the maximum possible distance in a given referent network, of which the observed network is a member. This measure can be conditioned by direction and tie strength, resulting in a measure of groups based on the potential flow between actors. As with clans, such a distinction is at risk to separation by a single cut node. This is why they must introduce a second criterion of homogeneity to account for potential cuts in the network.

A third commonly used measure induces groups based on the edge connectivity. A *lambda set* is a maximal subset of nodes with the property that the minimum number of edges that would have to be removed to disconnect any pair of vertices within the subset is strictly greater than that for any pair of vertices, one of which is in the subset and one of which is not in the subgroup. In figure 2, two sets of high degree nodes are identified ({1 4 6 7} and {8 11 9 14}. In each case, there is a single cut-node (node 4 in the first group, node 11 in the second) that if removed would disconnect the sub-group. Like distance measures, edge-connectivity approaches often depend on high-degree nodes that lie on many of the paths that connect a pair of actors.[16]

Another popular approach relies on relative relational volume, attempting to maximize within group density while minimizing between group density (Alba 1973; Fershtman 1997; Frank 1995; 1996). This maximization is achieved using an iterative search procedure. Groups identified by programs such as NEGOPY or UCI-NET's *factions* routine (as well as similar algorithms; c.f. Fershtman 1997; Frank 1996) assign each node to mutually exclusive groups. We applied NEGOPY and UCI-NET 's factions program to the graph in figure 2. NEGOPY automatically determines the number of groups in the network, and in this example found two groups, each of which contain a single cut-node. When FACTIONS was prompted for a 2-group solution, it returned the same two groups. However, when prompted for more groups, the program often identified smaller groups that were also characterized by cut-nodes. Density is related to cohesion, in that minimal and maximal bounds of density can be identified at every cohesion level (see White and Harary, n.d.), but the *arrangement* of relations is more important than the *volume* of relations for identifying cohesive groups.[17]

[16] For a more complete treatment of the relation between edge and node connectivity, see White and Harary n.d..

[17] This factor is evident in current implementations of the relative density approach. Consider, for example, Frank and Yasumoto's decision to 'disaffiliate' members of the 'cohesive' groups they identified, to better capture group membership (Frank and Yasumoto 1998, fn10). It is important to note that density and k-connectivity are related, since k-connectivity implies a k-core that implies density of at least $k/(n-1)$.

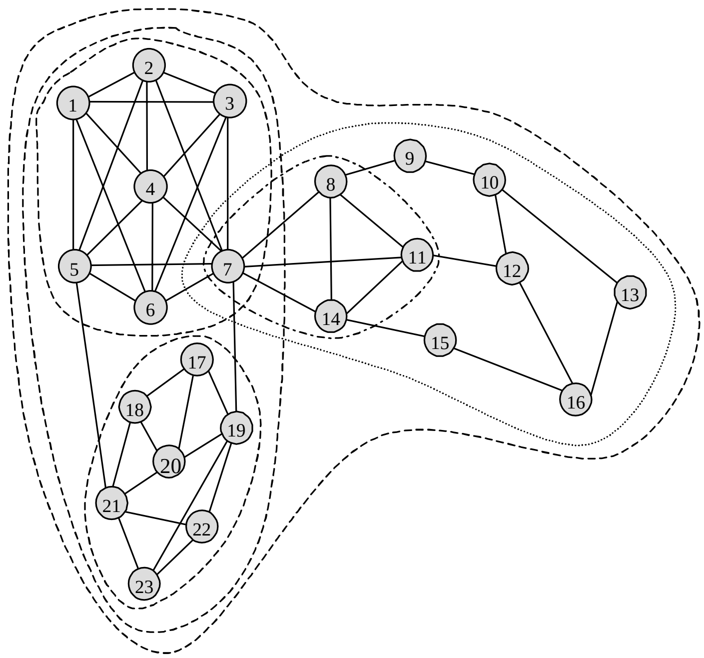
Group identification methods based on number of interaction partners, minimum within group distance or relative density, may be cohesive, but they are not necessarily so. In every case presented above, the method used to identify groups cannot distinguish an adhesive group from a cohesive group. As such, any empirical application of these methods to a theoretical problem of *cohesion* risks ambiguous findings. Because each measure optimizes a topographical feature that correlates with node connectivity, but optimizes a feature other than node connectivity, observed empirical results *may* follow from the cohesive properties of the identified group, or they may be uniquely due to the particular features optimized by each technique. By distinguishing *cohesion* from factors such as density or distance, we can isolate the relative importance of cohesion in social relations from these other factors. Distance between members, the number of common ties and so forth might affect outcomes of interest, but our ability to extend social theory in formal network terms depends on our ability to *unambiguously* attribute social mechanisms to such topographical features. A cohesive blocking of a network provides researchers with the ability to disentangle the effects of cohesion from other topographical features of the network.

Hierarchical Nesting of Connectivity Sets

Social cohesion is rarely evenly distributed throughout a social network. Some members of the group are core members who are more deeply integrated in the group, while others are less strongly connected, occupying positions on the edge of the social structure. Consider for example the network in figure 3.

The network has a single component inclusive of all the nodes. This network contains two minimally cohesive subgroups (biconnected components), namely nodes {1 –7, 17-23} and {7-16}, with node {7} involved in both bicomponents. Within the first bicomponent, however, members {1-7} form a 5-component and members {17-23} form a 3-component. Similarly, nodes {7,8,11, and 14} form another 3-component (a four-person clique) within the second bicomponent, while the remainder of the group contains no sets more strongly connected than the bicomponent. Thus, the group structure of this network is hierarchically ordered, with two large overlapping bicomponents, and internal to each one or two more cohesive subgroups. This nesting of groups within the overall structure reflects a differential level of social embeddedness within the overall network.

Figure 3. Nested Connectivity Sets



How might one identify nested cut-sets empirically? By combining well known algorithms from computer science (Even and Tarjan 1975, Ball and Provan 1983, Kanevsky 1993, 1990), we can identify cut-sets in a network as follows:

Table 2. Cohesive Blocking Procedure for Identifying Connectivity Sets in a Graph

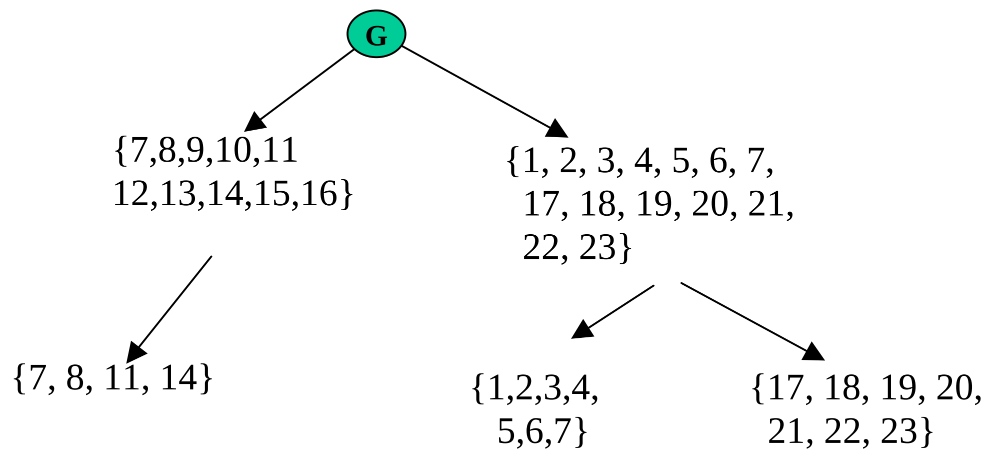
1. Identify the connectivity, *k*, of the input graph.
2. Identify all *k*-cut-sets at the current level of connectivity.
3. Generate new graph components based on the removal of these cut-sets (nodes in the cut-set belong to both sides of the induced cut.)
4. If the graph is neither complete nor trivial, return to (1), else end.

This procedure is repeated until all nested connectivity sets have been enumerated. [18] Walking through the example in figure 3, we would first identify the bicomponents (**step 1**), and identify the cut-node {7} (**step-2**).[19] Separating the two subgraphs at node 7 (**Step 3**) induces two new components: {7-16} and {1-7,17-23} which are neither complete nor trivial. Within each induced sub-graph we repeat the process, starting by identifying the sub-graph connectivity. Within the {7-16} bicomponent, we identify {8,10}, {10,16}, and {14,16} as the 2-cuts for this sub-graph, each of which leads to a single minimum degree cut (we call these types of cuts *singleton cuts*, e.g., of 9, 13, or 15). The graph remaining after the singleton cuts have been removed is {7, 8, 11, 14, 10, 12, and 16}, which is 1- connected, with {7 8 11 and 14 the largest included tricomponent). Because {7,8,11, and 14} form a completely connected clique, we stop here and return to the other graph induced by removing node {7}, ({1-7,17-23}). Again, this graph is a bicomponent. Cut-sets {5,7} and {21,19}, {21,7} and {5,19} induce two graphs of higher cohesion: {1-7} and {17-23} that are of maximal connectivity, as further cuts will induce only singleton partitions.

One can represent the hierarchical nesting of connectivity groups as a directed tree, with the total graph as the root, and each sub-graph that derives from it a new node. A *cohesive blocking* of a network consists of identifying all cohesive substructures within the network and relating them to each other in terms of the nested branching of the subgroups. The blocking for the example above is given in figure 4 (with singleton cuts not represented).

[18] SAS IML programs for identifying the full connectivity sets of a network are available (Moody 1999).
[19] An efficient algorithm for doing so can be found in Gibbons (1985).

Figure 4. Cohesive blocking for the network in Figure 3



Testing for *k*-connectivity (**Step 1**) can be accomplished with a network maximum flow algorithm developed by Even and Tarjan (1975).[20] An algorithm for identifying all *k*-cut-sets of the graph (**Step 2**) has been developed by Ball and Provan (1983), extended to finding all minimum-size separating vertex sets by Kanevsky (1993, 1990).[21] One must apply this pair of procedures for every induced sub-graph, and thus the total running time of the algorithm can be high. Two steps can be taken to reduce the computation time. First, there are linear time algorithms for identifying *k*-connected components for $k \leq 3$, and one can start searching with these algorithms, limiting the number of levels at which one has to run the full connectivity algorithms (Hopcroft and Tarjan 1973, Fussell et al. 1993). Second, in many empirical networks the most common cut-set occurs for singleton cuts. Because the procedure is nested, one can search for nodes with degree less than or equal to the connectivity of the parent graph,[22] remove them from the network, and thus apply the network flow search only after the singleton cuts have been removed.[23]

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[20] This is an extension of Dinic's algorithm and runs in $O(V^{1/2} * E^2)$ time.
[21] Which runs in $O(2^k V^3)$ time.
[22] The graph from which the current graph was derived.
[23] Additionally, there are approximation approaches (Auletta et al. 1999; Khuller and Raghavachari 1995) that could be used to identify graph connectivity within a certain amount of error, which would be faster.

Because cut-sets induce nested groups, one can describe connectivity sets not only by the resulting connectivity of the induced subgroup, but also by the level at which the group appears. That is, each group was uncovered only after a search process that broke the network at its most vulnerable points (identified the minimum cut-sets). Subgroups that appear at lower depths in the hierarchy are more deeply nested within the overall network, providing a direct measure of embeddedness within a collectivity.

Sets of actors who remain connected to each other through multiple cuts are in some sense insulated from perturbations at the edge of the social structure. They are deeply connected, and re-connected, through the multiple independent paths that define their cohesive group. This deep connectivity nicely captures the intuitive sense of being involved in relations that are, in direct contrast to "arms-length" relations, embedded in a social network (Uzzi, 1996). Another aspect of the embeddedness concept that relates to connectivity and hence to our measured of nestedness in cohesive blocks is that of multiplexity. For Granovetter (1985), multiplex ties are more deeply embedded in multiple types of networks or social worlds. Cohesive nestedness captures one type of multiplexity, namely that in a k-connected block, every pair of directly connected nodes has at least a multiplexity of k ties if we include as well the indirect ties that are the result of indirect connections. Nodes can also belong to multiple small-worlds as defined by cohesive blocks and their potential for overlap. We define an actor's nestedness in a social network as *the deepest cut-set level within which a given actor resides*. An actor's deepest point in a given network follows from the number of cuts needed to reach a point where no further cuts can be made.

How does nestedness relate to other common network measures? First, as a property related to position within a network, we would expect nestedness to be related to measures of centrality and degree (Freeman 1977). To the extent that our measure captures the general location of actors and differentiates prominent actors, an actor's nestedness level can be thought of as a type of centrality (Wasserman and Faust 1994, p.169, see Harary et al, 1965 for a previous discussion along these lines). However, depth in the network is the outcome of a particular group configuration, and as such is not attributable to the individual alone. Second, because connectivity is related to degree (each member of a *k*-component must have at least *k* relations), nestedness is necessarily correlated with degree. Third, as being on a shortest path between two actors tends to correlate with degree, betweeness centrality and nestedness will correlate as well. As we show in the empirical examples below, however, connectivity level is not *equivalent* to such measures.

As a dimension of network embeddedness, nestedness has some desirable properties. Many of the previous discussions of embeddedness focus on an actor's multiplex involvement with other actors, highlighting that such actors are not free to form relations with others at will. Intuitively, embedded actors are tightly bound to others and deeply involved in the social network. Since our intuitive understanding of embeddedness refers to how difficult it would be to extract a set of actors from the wider network, our conception of nestedness, or depth in the connectivity structure, provides a direct link between this dimension of embeddedness and the measure we use.

From the formal properties of connectivity sets it is clear that the analysis of the cohesion structure of a network provides not only a method for identifying groups, but a method for positioning groups relative to each other with respect to depth of involvement in the social network. Because this method provides the ability to both identify cohesive groups and identify the position of each group in the overall structure, we term this method *cohesive blocking*. It is important to note the flexibility of this approach. The concept of cohesion presented here is not limited to nested groups, but instead provides a way of ordering groups within hierarchical trees, with traditional segmented groups occupying separate branches of the cohesion structure (recall figure 4). The ability to accommodate both nested and segmented structures within a common frame is a strength of our model. This method will allow researchers to distinguish the effect of topographic features related to connectivity from the volume or distance characteristics most commonly measured. In the next section, we explore some of the substantive theoretical implications of collectivities organized through multiple independent connections.

Theoretical Implications of Network Connectivity

The fact that information, resources and risks flow through social relations is an important reason why networks matter for social life. If resources flow through networks then the structure of the network largely determines the distribution of resources, access to information, and allocation of power in a social system. Previous work on power exchange structures (Blau 1964; Cook et al. 1983; Lawler and Yoon 1993) and structural holes (Burt 1992) all suggest that the ability to control resource flow is an important source of power. A connectivity conception of social cohesion provides a direct link between the distribution of resources and the distribution of connections through which resources flow.

The ability of any person to monopolize resource flow, and thus have power within the setting, is limited as connectivity increases. One of the defining properties of a *k*-component is that every pair of actors in the collectivity is connected by at least *k* independent paths. This implies that information and resources can be exchanged within the group without any individual being able to control the resources. Substantively, such networks are characterized by a reduction in the power provided by structural holes (Burt 1992; White et al. 1976): even in networks where actors are not linked directly, no single actor can control the flow of information or goods between a pair of actors in the *k*-component.

Industrial developments of "just-in-time" inventory systems, provide an interesting example. When viewed as a network of resource flows, the most efficient systems resemble spanning trees — and thus the production of the entire product depends on each element producing a required good at the right time. Under such a structure, labor has accentuated power since strikes at one plant can disable the entire system. In fact, recent trends toward "just-in-time" production processes are not new, but were used extensively early in the auto industry. It became clear, however, that the structure of production flow gave labor power. To counter, management expanded the production network to include alternative sources (other factories and storehouses), building redundancy into the system and removing labor's power (Schwartz, 2001).

The length of a path is often considered critical for the flow of goods through the network, as the flow may degrade with relational distance. That is, the probability that a resource flows from node *i* to node *j* along path *p* is equal to the product of each dyadic transition probabilities for that path. When multiplied over long paths the efficacy of the information diminishes even if the pairwise transmission probability is high. For example, the probability that a message will arrive intact over a 6-step chain when each dyadic transmission probability is 0.9 will be 0.53. For a strongly cohesive group, however, information degradation decreases with each additional independent path in the network. The fragility of long-distance communication rests on the fact that at any step in the communication chain, one person's failure to pass the information will disrupt the flow. The comparable probability of a 6-step communication arriving intact over two independent paths is 0.78.[24] As the number of independent paths increases, the likelihood of the information being interrupted drops.

When the flow is not subject to degradation, but only to interruption, increasing connectivity will provide faster and more reliable transmission throughout the network.[25] In a high-connectivity network, even if many people stop transmission (effectively removing themselves from the network), alternate paths provide an opportunity for spread. On the other hand, when the object of flow is subject to mutation, such as that observed in information and biologic flows, connectivity allows simultaneous rapid transmission (since flow is not degraded) and multiple exposure. In the information case, multiple exposures allow individuals to combine similar information, increasing overall accuracy as the same piece of information arrives through

[24] We calculate this as the product of the dyadic probabilities for each path, minus the probability of transmission through both paths. Thus, for two paths the formula is $2(p_{ij})^d - (p_{ij})^{2d}$, where d is the distance.
[25] Computer viruses are an excellent example of such flows, as recent outbreaks such as "Melissa" and the "Love Bug" show.

multiple channels. If virus are flowing through the network, mutations make prevention that much more difficult, increasing the hazard of the network for disease distribution. Similar examples can be found with respect to the flow of normative and other cultural goods. A theory of norms based on socialization and transmission would suggest that the higher the level of cohesion, the greater the consistency in normative behavior.

The structure of subgroups within the network is important as well. It is always possible for two k-cohesive groups to overlap by k-1 nodes, however, we usually observe distinct, non-overlapping ($k+j$)-cohesive subgroups within the larger k-connected population. This structure has important implications for the carrying capacity of a network over long distances. Local pockets of high connectivity can act as amplifying substations for information (or resource, or viral) flow that comes into the more highly connected group, boosting the signal's strength, and sending it back out into the wider population. The observed patterns typical in small world graphs (Milgram 1969; Watts 1999; Watts and Strogatz 1998) are a natural result of local relational action nested within a larger network setting. Thus processes based on the formal properties of connectivity may account for many of the observed substantive features of small world networks.

Since connectivity sets can overlap, group members can belong to multiple groups. While observed overlaps at high levels of connectivity may be rare, any observed overlaps are likely substantively significant.[26] If an individual belongs to more than one maximal k-cohesive group, that individual is part of a unique subset of k-1 individuals whose removal will disconnect the two groups. Members of such bridging sets are *structurally equivalent* with respect to the larger cohesive sets that they bridge. As such, a *positional* and *relational* structure comes out of

[26] Some researchers consider overlapping subgroups too empirically vexing to provide useful analysis. It is important to point out that (1) k-components are strictly limited in the size of such overlaps, making the substantive number of such intermediate positions small -- especially compared to cliques, (2) that each such position, because of its known relation to the potential flow paths and cycle structure of the network, can be theoretically articulated in ways that are impossible for clique overlaps, and (3) even when they are empirically difficult to handle, may well be an accurate description of relationship patterns.

the analysis of cohesive groups, groups that are much larger, fewer, and easier to distinguish than by our traditional notions of sociological cliques, providing some of the same theoretical purchase blockmodels were designed to provide (Burt 1990; Lorrain and White 1971; White et al. 1976), but focusing on subgraphs that may overlap rather than partitions of nodes.

Finally, this relational conception of cohesion provides sociology with a tool useful for understanding large-scale network integration processes related to the formation of social classes, ethnicity, and social institutions. While a longstanding promise of network research (Rapoport and Horvath 1961; Emirbayer 1997; White et al. 1976), the specific conceptual tools needed to identify the empirical traces of such processes have been sorely lacking. In contrast, Brudner and White (1997) and White et al. (1999) identified sociologically important cohesive groups in two large (n = 2332 & 1458 respectively), sparse networks using the concept of strong or structural cohesion. The first of these studies showed that membership in a structurally cohesive group, defined by marital ties among households in an Austrian farming village, was correlated with stratified class membership, defined by single-heir succession to ownership of the productive resources of farmsteads and farmlands. In the second study, they found that the structurally cohesive group defined by marital ties of Mexican villagers was restricted to a core that included families with several generations of residence and excluded recent immigrants and families in adjacent villages. The structurally cohesive group defined by *compadrazgo*,[27] on the other hand, crosscut this village nucleus and integrated recent immigrants. In contrast to the first study, the Mexican case established a network basis for the observed cross-village egalitarian class structure.

Identifying an empirical connection between the core members of a structurally cohesive subgroup and those institutions that provide formal access to power and decision-making suggests a new approach to the study of social stratification and the state. White et al. (1999), for example,

[27] Ritual kinship established between parents and godparents.

identify an informally organized “invisible state” created by the intersections of structurally cohesive groups found at the local and regional levels, the governing bodies of local municipalities, and the relations among members of different local municipalities. They show further that those who share administrative offices during overlapping time spans build dense clique-like social ties within a political nucleus while maintaining sparse tree-like ties with structurally cohesive groups in the larger region and community. These sparse ties act as amplifiers for the feedback relations between larger cohesive groups and their government representatives.

Social network researchers have traditionally focused on small, highly connected groups. Identifying connectivity as a central element of cohesion frees us from focusing on these small groups by identifying patterns through which influence or information can travel long distances. This focus on larger group connectivity might be especially important in epidemiological networks and explanations of norm formation and maintenance. The rise of electronic communication and information flows suggests that distance will become less salient as information can travel through channels that are remarkably robust to degradation. By extending our vision of cohesion from small local groups to large, extended relations, we are able to capture essential elements of large-scale social organization that have only been hinted at by previous social network research, providing an empirical tool for understanding realistically sized lived communities.

Two Empirical Examples: High Schools and Interlocking Directorates

To demonstrate the importance of cohesive blocking for behavior in empirical settings we use data from two different types of networks. First, we use data on friendships among high school students taken from the National Longitudinal Survey of Adolescent Health (*Add Health*). This example illustrates how cohesive groups can be identified in large settings based on friendship, one of the most commonly studied network relations. The second example uses data on the interlocking directorate networks of 57 large firms in the United States (Mizruchi 1992). Since business solidarity has been an important topic of research on interlocks, we apply our method to this network and show how our conception of cohesion relates to similarity of political activity. Of course, there is not space here to treat the subtle theoretical issues surrounding each of these substantive areas. Instead, the analyses below are designed to highlight how our concept of cohesion can add to empirical research in widely differing research settings.

Cohesion in Adolescent Friendship Networks

Add Health is a school-based study of adolescents in grades 7-12. A stratified nationally representative sample of all public and private high schools (defined as schools with an 11th grade) in the United States with a minimum enrollment of 30 students was drawn from the Quality Education Database (QED) in April, 1994.[28] Network data were collected by providing each student with a copy of the roster of all students for their school. Students identified up to five male and five female (10 total) friends from this roster.[29] For this paper, we use data on over 4000 students taken from a dozen schools with between 200 and 500 students (mean=349) providing a diverse collection of public (83%) and private schools from across the United States.[30]

Nestedness and School Attachment

For each school, we employed the cohesive blocking procedure described in table 2 to identify all connectivity sets for each school friendship networks. At the first level, we have the entire graph, which is usually unconnected (due to the presence of a small number of isolates).

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[28] For details of the Add Health design, see P.S. Bearman, Jo Jones, and J.R. Udry, "Connections Count: The Add Health Design" (1996), http://www.cpc.unc.edu/projects/addhealth/design.html

[29] The maximum number of nominations allowed was 10, but this restriction affected few students (3.1%). Mean out-degree is 4.15 with a standard deviation of 3.02. For purposes of identifying connectivity sets, we treat the graph as undirected, the algorithms needed for identifying connectivity can be modified to handle asymmetric ties. It was for directed graphs that Harary et al (1965) developed their concept of cohesion as connectivity, although they offered no computational algorithms.

Most of the students in every school are contained within the largest bicomponent, and often within the largest tricomponent. As the procedure continues, smaller and more tightly connected groups are identified. At high levels of connectivity (k>5), identified subgroups rarely overlap. On the whole, schools tend to be strongly connected, with a small number of students on the fringe of the social structure, the bulk of the students linked within a wide moderately cohesive (k=3 to 5) school network, and a small number of very tightly cohesive, deeply embedded groups (k=6 to 7), forming multiple cores within the wider school network.

When no further cuts can be made within a group, we have reached the end of the nesting process for that particular set of nodes. The level at which this cutting ceases describes the nestedness for each member of that group. An example of the nesting sequence for a school network of 401 nodes is given in figure 5. Here we see that most of the nodes are multiply connected at level 3, but a small 4-component occupies a separate stream of the structure. The highest levels of connectivity in this particular school are made up of a 6-component and a 7-component found at levels 55 and 56 respectively.

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Figure 5 about here
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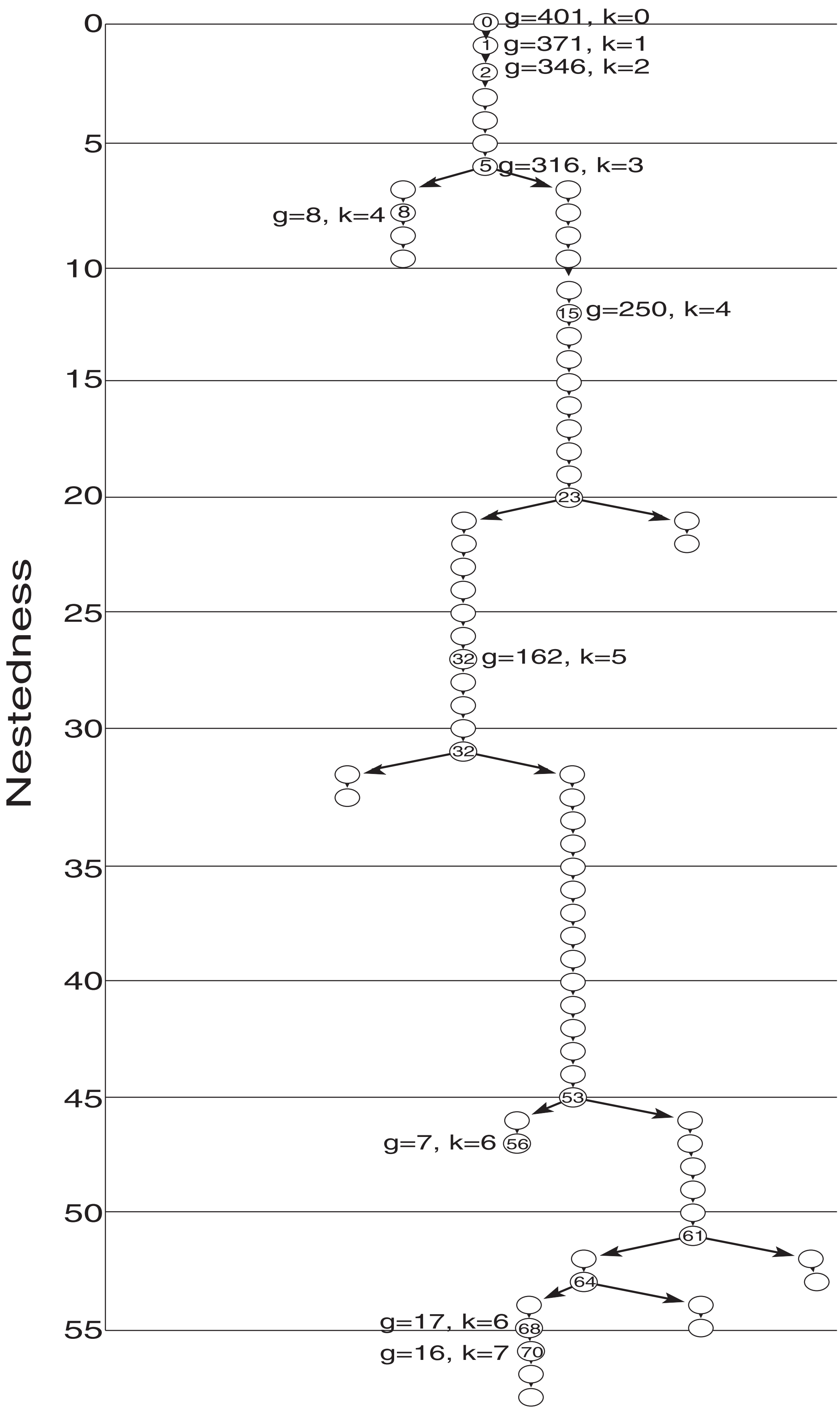
Nestedness within the community should be reflected in a student's perception of his or her place in the school. The Add Health in-school survey asks students to report on how much they like their school, how close they feel to others in the school and how much they feel a part of the school.[31] Here, we use the mean of the three items as a measure of school attachment. If students correctly perceived their place in the overall school network, and if the hierarchical nesting captures this embeddedness, then there ought to be a significant positive relation between

[30] This represents all schools in the dataset of this size. The selected size provides a nice balance between computational complexity and social complexity, as the schools are large enough to be socially differentiated and small enough for group identification to be carried out in a reasonable amount of time.
[31] These are three items from the Perceived Cohesion Scale (Bollen and Hoyle 1990). The other three items used for Bollen and Hoyle's scale were not included in the Add Health school survey.

Figure 5. Cohesive Blocking of a Network



nestedness in the network and school attachment, net of any other factor that might be associated with school attachment.

Other variables that might affect a student's attachment to school include individual demographic and behavior characteristics and other features of the school friendship network. Since gender differences in school performance and school climate are well known (Stockard and Mayberry 1992), we would expect female students to have lower attachment to school than male students. As students age we would expect the school to become a less salient focus of their activities, and grade in school is also controlled.[32] Students who perform well in school or who are involved in many extra-curricular activities should feel more comfortable in schools, and students from small schools should feel more embedded than students from large schools.

A significant value of our approach is that we can differentiate the unique effects of network topographical features that are often conflated in standard network measures. First, the number of contacts a person has (degree centrality) reflects their level of involvement in a network. Substantively, we expect that those people with many friends in school are more likely to feel an integrated part of the school. Second, we might argue that having friends who are all friends with each other is an important feature of network involvement. As such, the density of one's local network is tested. Due to the relation between centrality and nestedness, we test for a possible spurious relation between centrality and nestedness. We expect that those people who are most central in the network should have a greater sense of school attachment. Finally, it may be the case that the lived community of interest for any student is that set of students with whom they interact most often. As such, membership in a relative density based friendship group could account for one's sense of attachment to school. We used NEGOPY to identify density based interaction groups within the school. If our conception of nestedness captures a unique dimension of network embeddedness, as our discussion above implies, then controlling for each of these features, we would expect to find a unique independent effect of nestedness on school attachment.

Table 3 presents the regression coefficients for models of school attachment on the nestedness level, school activity, demographic, and other network factors. Model 1 presents a baseline model containing only attribute variables. As expected, females, students in higher grades, and students from larger schools tend to have lower school attachment, while students who are involved in many extracurricular activities or who get good grades feel more attached to the school. In model 2, our measure of network nestedness is added to the model.[33] We see that there is a strong positive relation between nestedness and school attachment (note that the size of the standardized coefficient for nestedness is the largest in all model specifications). In models 3 - 6, we test the specification including our measure and each of the alternative network measures. While each network feature shows a significant relation to school attachment, in each case nestedness remains positive, significant and strong. In models 7 and 8, we include all potentially confounding network variables, and the relation between nestedness and attachment remains. The largest change in the coefficient for nestedness comes with the addition of degree. This is likely due to collinearity, as every member of a k-component must have degree $\geq$ k.

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[32] Since school friendships tend to form within grade, controlling for grade in schools captures an important focal feature of the in-school network.

[33] In addition to the nestedness level, we also tested a model using the largest *k*-connectivity value for each student. The results are very similar. Students involved in high-cohesion groups had higher levels of school attachment. All statistical significance patterns were the same for both models, and the relative values (as measured by the standardized coefficients) were similar. Further models were tested using a 2-level hierarchical specification (students within schools) that shows no substantive difference in the models.

Table 3. School Attachment and Network Embeddedness.
OLS Regression Coefficients, Std. Errors, Standardized Regression Coefficients

	Mod 1	Mod 2	Mod 3	Mod 4	Mod 5	Mod 6	Mod 7	Mod 8
Intercept	3.76***	3.57***	3.51***	3.52***	3.39***	3.65***	3.48***	3.31***
	(.157)	(.153)	(.154)	.155	.15	.184	.188	.159
Female	-.179***	-.142***	-.146***	-.142***	-.147***	-.139***	-.147***	-.152***
	(.031)	(.030)	(.030)	(.030)	.030	.031	.031	.030
	-.093	-.073	-.075	-.074	-.076	-.072	-.076	-.079
Grade in school	-.076***	-.062***	-.060***	-.061***	-.062***	-.057***	-.056***	-.060***
	(.013)	(.012)	(.012)	(.012)	.012	.016	.016	.012
	-.095	-.077	-.075	-.077	-.078	-.072	-.070	.075
Grade point	.143***	.117***	.112***	.119***	.120***	.103***	.102***	.114***
Average	(.022)	(.021)	(.021)	(.021)	.021	.022	.022	.021
	.111	.091	.087	.093	.093	.080	.079	.088
Extracurricular	.112***	.082***	.082***	.080***	.078***	.068***	.068***	.079***
activities	(.008)	(.008)	(.008)	(.008)	.008	.009	.009	.008
	.226	.164	.165	.162	.158	.139	.137	.158
School size	-.011	-.062**	-.061**	-.052**	-.036			-.033
	(.023)	(.023)	(.023)	(.023)	.023			.023
	-.007	-.043	-.042	-.036	-.025			-.023
Nestedness		.014***	.015***	.014***	.009***	.016***	.011***	.007***
		(.001)	(.001)	(.001)	.002	.001	.002	.002
		.255	.253	.241	.157	.284	.197	.136
Local density			.025**				.020*	.027**
			(.008)				.009	.008
			.048				.040	.053
Betweenness				.048*			-.016	-.025
centrality				(.022)			.035	.034
				.037			-.012	-.019
Number of					.029***		.025*	.037***
friends (degree)					.006		.010	.009
					.122		.104	.154
Density groups[a]						-.325***	-.303***	
						.452	.444	
Adj. R^2	.09	.147	.149	.148	.152	.187	.188	.155
N=3606								

* $p \le .05$, ** $p \le .01$, *** $p \le .001$

[a]Cell values represents mean coefficient value of group-specific dummies followed by the standard deviation. Significance judges by the joint F-test (df=128,3472). Since every member of a school is assigned to a density based group, $n_i \sum_i g_i$ = the school size, and thus size cannot be included in the models that also include density based groups.

These findings suggest that the school as a whole is united through social cohesion. The importance of nestedness in the school network for school attachment follows theoretically from the insular character of connectivity depth. This finding holds net of particular sub-group differences in school attachment, the number of friends people have, their betweenness centrality level and the interaction density among their friends. That most of these other factors continue to contribute to school attachment implies *unique* effects to each of these dimensions of network structure, which would be confounded if any one of these measures was used alone.

Cohesion among Large American Businesses

A long-standing research tradition in inter firm networks has focused on the interlocking directorates of large firms (Mizruchi 1982; Mizruchi 1992; Palmer et al. 1986; Roy 1983; Roy and Bonacich 1988; Useem 1984). An important question in this literature is whether (and to what extent) business in the United States is unified. Business unity is important, theoretically, because it "is at the core of the debate over the extent to which American society is democratic." (Mizruchi, 1992 p.32). If businesses collude in the political sphere, then populist democracy is threatened. Yet, much of the literature has been vague in defining exactly what constitutes business unity, and thus empirical determination of the extent and effect of business unity is hard to identify.

We approach the question of business unity as a problem of social cohesion. If the business community is unified, one indication of such unity ought to be a group of organizations ordered as a cohesive set. If influence and information flow within such groups, coordinated action, and thus political activity, ought to be more similar among pairs of firms that are cohesively linked. Mizruchi (1992) makes this argument well, and highlights the importance of financial institutions for unifying business activity. He identifies the number of indirect interlocks between two firms as "...the number of banks and insurance companies that have direct interlocks with both manufacturing firms in the dyad" (p.108). Using data on large manufacturing firms, we identify the cohesive group structure based on indirect interlocks and relate this structure to political action similarity.

The sample Mizruchi constructed consists of 57 of the largest manufacturing firms drawn from "the twenty major manufacturing industries in the U.S. Census Bureau's Standard Industrial Classification Scheme" in 1980 (Mizruchi 1992, p.91). In addition to data on directorship structure, he collected data on industry involvement, common stockholding, governmental regulations and political activity. The question of interest is whether the structure of relations among firms affects the similarity of their behavior. To explore whether firms that are similarly embedded also make similar political contributions, Mizruchi constructs a dyad-level political contribution similarity score.[34] Me models this pair-level similarity as a function of geographic proximity, industry, financial interdependence, government regulations and interlock structure.

Following Mizruchi, we focus on the pattern of cohesion based on interlocks created by banks and financial firms. Most firms in the network are involved in a strongly cohesive group, with 51 of the 57 firms members of the largest bicomponent. The strongest connectivity set consists of 28 firms in a single 14-component. The nestedness structure of the network is 19 layers deep, and at the lowest level (at which no further minimum cuts can be made which would not isolate all nodes) 16 firms are members of a 14-connected component.

Does joint membership in a nested subset lead to greater political action similarity? To answer this question, we add an indicator for the deepest layer within which both firms in a dyad are nested. Thus, if firm *i* is a member of the 2nd layer but not the third, and firm *j* is a member of the 4th layer but not the fifth, the dyad is coded as being nested in the 2nd layer. As with the prior school example, we control for other network features. Table 4 presents the results of this model.

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34 The score is calculated as $S_{ij} = n_{ij} / \sqrt{n_i n_j}$, where S_{ij} = the similarity score, n_{ij} equals the number of common campaign contributions, and n_i and n_j equal the number of contributions firm *i* and *j* make respectively. The dyad level analysis is based on 1596 firm dyads.

Table 4. QAP Regression of political action similarity on dyad attributes
(Standardized Coefficients in parentheses)

Variable	Variable Description	1	2	3	4	5
Proximity	Headquarters located in same state	.017 (.043)	.013 (.032)	.013 (.034)	.015 (.039)	.015 (.039)
Same primary industry	Same primary two-digit industry	.012 (.024)	.017 (.034)	.017 (.036)	.016 (.034)	.016 (.034)
Common industry	Number of common two-digit industries	-.004 (-.008)	-.007 (-.015)	-.007 (-.015)	-.003 (-.006)	-.003 (-.006)
Market constraint	Interdependence based on transactions and concentration	.011+ (.098)	.009+ (.080)	.009+ (.082)	.009+ (.083)	.009+ (.083)
Common stockholders	Financial institutions that hold stock in both firms	.034* (.213)	.029* (.182)	.028* (.174)	.029* (.181)	.029* (.181)
Direct interlocks	Board of directors overlaps between firms	.021+ (.047)	.016 (.036)	.017+ (.037)	.018+ (.041)	.018+ (.041)
Indirect interlocks	Financial institutions with which firms interlock	.026** (.178)	.010** (.070)	.009 (.060)	.007 (.050)	.007 (.051)
Regulated industries	Primary membership in regulated industry	.036+ (.115)	.034+ (.107)	.032 (.102)	.030 (.096)	.030 (.096)
Defense contracts	Common recipient of defense contracts	.084** (.170)	.083** (.166)	.082* (.165)	.082* (.166)	.082 (.165)
Nestedness level	Level of embeddedness in the indirect interlock network		.004* (.201)	.005* (.257)	.004* (.203)	.004+ (.202)
Degree difference	Absolute difference in degree			.001 (.087)		-0.00 (-0.00)
Centrality difference	Absolute difference in betweenness centrality				.010+ (.111)	.010 (.111)
Constant		.171**	.156**	.137**	.144**	.144**
R-square		.195	.217	.222	.229	.228

+ p ≤.10, * p ≤.05, ** p ≤.01

In column one, we replicate the analysis presented in Mizruchi (1992), and in the remaining models we present findings with additional network indicators.[35] In the baseline model, we find that the more financial stockholders two firms have in common the greater the similarity of their political contributions. Additionally, being directly connected through financial institutions or jointly receiving defense contracts leads to similarity of political action. In model 2, we add the nestedness measure.[36] Net of the effects identified in model 1, we find a strong positive impact of cohesion within the indirect interlock network (based on the standardized coefficient values, nestedness has the strongest effect in the model). As in the school networks, we test for the potentially confounding effects of degree and centrality.[37] No effect of simple network volume is evident, but betweenness centrality does evidence a moderate association with political similarity. When both variables are entered into the model, the statistical significance of nestedness drops slightly, but the magnitude of the effect remains constant.

The more deeply nested a given dyad is in the overall network structure, the more similar their political contributions. It is important to point out that this holds net of the adjacency of two firms. In addition to the direct adjacency created through a financial interlock, we find the global network position, as indicated by the nestedness level, is a significant predictor of political similarity.

Mizruchi identifies two potential explanations for the importance of financial interlocking on political behavior. Following Mintz and Schwartz (1985), banks and financial institutions may exercise control of firms by sitting on their boards. As such, two firms that share many such financial ties face many of the same influencing pressures and therefore behave similarly. A

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[35] Following Mizruchi (1992, p.121) we use the nonparametric quadratic assignment procedure (QAP) to assess the significance level of the regression coefficients. See Mizruchi for measurement details.

[36] If instead of the joint nestedness level, we use the connectivity level (*k*) for the highest *k* both members are involved in, we find substantively similar results. The statistical significance of the connectivity level is slightly lower than the embeddedness level.

[37] We cannot test for density-based subgroup effects, because NEGOPY assigns all members to the same group. This is a result of the high average degree within this network
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second argument, building on the debate surrounding structural equivalence and cohesion (Burt 1978), is that actors in similar network positions (i.e. with similar patterns of ties to similar third parties) ought to behave similarly. If the multiple, independent paths which link pairs of actors help transfer information among firms, then the finding that joint nestedness leads to political similarity may be evidence for the indirect cohesion argument made by Friedkin (1984), who argues that influence travels through multiple steps, and thus has an effect beyond the direct link between two actors.

Conclusion and Discussion

Social solidarity is a central concept in sociology. We have argued that solidarity can be analytically divided into two components, an ideational component and a relational component. We can further distinguish adhesive from cohesive relational patterns. Understanding how collections of actors are linked together, how the interconnections among these actors change, and what influence involvement in cohesive groups plays in the lives of actors and organizations depends on a clear conceptualization of cohesion.

When does cohesion start? Following authors such as Markovsky and Lawler, we argue that cohesion starts when every actor can reach every other actor through at least a single path. The paths that link actors are the relational glue holding them together. We then show that cohesion *scales* in that it is weakest when there is one path connecting actors, stronger when there are two, stronger yet with three, and so forth reaching a maximum when every person is directly connected to every other person in a complete clique. As such, we have identified an essential dimension upon which cohesion rests. Namely, that for a group to be cohesive it cannot be easily separated. Thus, the essential substantive feature of a strongly cohesive group is that it has a status beyond any individual group member. We operationalize this conception of a social cohesion through the graph theoretic property of connectivity (Harary et al. 1965, White 1998), showing that cohesion increases with each additional independent path in a network.

This operationalization of cohesion differs from previous methods for identifying cohesive groups in social networks. Many methods designed to identify cohesive groups rest on properties of the network that do not necessarily correspond to a network pattern that is free from disconnection due to unilateral action on the part of any actor in the group. Conceptualizations based on relational distance, number of interaction partners or relative in-group density thus cannot distinguish between adhesive and cohesive social structures. While such groups may have social significance, the operative feature of such groups may not be strong cohesion, but instead a result of social distance, number of partners, local density or centralization with strong adherence to leadership. By identifying a unique dimension upon which cohesion rests, we provide the ability to empirically separate the relative effect of *cohesion* from the other properties of networks that may be socially significant. Our empirical results in schools and corporate networks suggest that each of these dimensions may play a unique role in understanding behavior, and thus any method that conflates these effects may lead to ambiguous results.

Our conceptualization of cohesion simultaneously provides an operationalization of one dimension of network embeddedness. Cohesive sets in a network are nested, such that highly cohesive groups are nested within less cohesive groups. Since the process for identifying the nested connectivity sets is based on identifying the most fragile points in a network, those actors who are involved in the most highly connected portions of the network are often deeply insulated from perturbations in the overall network. Given the importance of the generalized concept of embeddedness in sociological work, a direct measure of this dimension of embeddedness is an important asset that will help provide clear empirical examinations of the relation of embeddedness to social outcomes.

Our presentation of cohesion has focused on the basic topographical features of social cohesion, without respect to the particular features that might be relevant in any given case. We suspect that researchers may modify aspects of our conception of cohesion as theory dictates. Thus, in settings where flows that degrade quickly are of primary interest, one could account for the level of cohesion by incorporating a measure of the length of the paths or the strength of the ties. We caution, however, that much of the theoretical power of our conception of cohesion rests on the idea that multiple, indirect paths, routed through strongly cohesive subgroup, can magnify signals such that long distances can be united through social connections. Additionally, while we expect that cohesive groups will also be stable groups, we argue that this is an empirical question that can only be answered in particular settings. Finally, in a companion paper, White and Harary (n.d.) integrate network density and edge-connectivity with node-connectivity to provide a ratio level measure of cohesion that further distinguishes between levels of cohesion *within* given levels of *k*-connectivity.

In the present paper, we have applied our method to two very different groups in an effort to show how cohesion might be profitably used in very different empirical settings. Due to space limitations, subtle theoretical questions related to school attachment and political action have been ignored. Many potential questions, concerning interactions between embeddedness and type of actor for example, would be profitable to explore. The current settings are clearly only a limited subset of the types of relations through which cohesion effects might be important, and further research is required to understand how type and strength of relations affect the importance of cohesive structures for substantive outcome. Clearly, one could identify many more such applications in areas such as personal friendship networks (Cohen 1979-1980; Fischer 1982; Giordano et al. 1998), international networks (Rossem 1996; Smith and White 1992), kinship networks (Hage and Harary 1983; Schweizer and White 1998; Houseman and White 1998), community and regional integration (White et al. 1999a), social class integration (Brudner and White 1997), the state (White, Schnegg, Brudner and Nutini, 1999b), disease and information diffusion (Anderson and May 1991; Klovdahl 1985; Morris 1993), or social influence (Friedkin 1998), just to name a few. Our hope is that by providing a clear and concise definition and operationalization of cohesion, researchers in all such fields will better be able to conduct their work.

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