Inequality and Collective Action.*

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Abstract

We analyze the effect of inequality in the distribution of endowment of private inputs (e.g., land, wealth) that are complementary in production with collective inputs (e.g., contribution to public goods such as irrigation and extraction from common-property resources) on efficiency in a simple class of collective action problems. In an environment where transaction costs prevent the efficient allocation of private inputs across individuals, and the collective inputs are provided in a decentralized manner, we characterize the optimal second-best distribution of the private input. We show that while efficiency increases with greater equality *within* the group of contributors and non-contributors, in some situations there is an optimal degree of inequality *between* the groups.

1 Introduction

How does the distribution of private endowments across individuals affects efficiency in collective action problems? While the literature in political science and economics on collective action is large, its interrelationship with economic inequality is a relatively underresearched area.

The assumption of decreasing returns, a standard one in most economic contexts, implies that the more scarce an input in a given production unit, the higher is its marginal return. As a result, one would expect a more equal distribution of this

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input across production units to improve efficiency. If the market for this input operated well, then the forces of arbitrage would make sure it is allocated equally to maximize efficiency. However there is considerable evidence that suggest that the market for inputs such as land or capital does not operate frictionlessly and the private endowment of an individual determines how much of that input she can use in her production unit.[1] There is a large literature showing that small farms are more efficient than large farms in agricultural sector of developing countries. This is typically advanced as one of the main arguments for land reform in terms of efficiency. Some authors (e.g., Bardhan, 1984, Boyce, 1987) have gone one step further and argued that a more egalitarian agrarian structure is also more likely to solve collective action problems, especially those related to irrigation.[2]

However, from the theoretical point of view the effect of inequality of private endowments such as land or wealth on efficiency is not obvious in the presence of collective action problems. Indeed, in his pioneering work on the subject, Olson (1965) makes the following case in favor of inequality:

> "In smaller groups marked by considerable degrees of inequality – that is, in groups of members of unequal "size" or extent of interest in the collective good – there is the greatest likelihood that a collective good will be provided; for the greater the interest in the collective good of any single member, the greater the likelihood that the member will get such a significant proportion of the total benefit from the collective good that he will gain from seeing that the good is provided, even if he has to pay all of the cost himself." (*The Logic of Collective Action*, p. 34).

We can interpret the "size" of a player with her endowment of the private input if it is complementary with the collective good in production. The above argument seems to assume some kind of an indivisibility associated with the provision of the collective good: the public good is viable only at some minimum scale. If everyone is to share the benefits equally, then the benefit received by any given player may not exceed the cost of the collective good, but if one player is given a high enough share, she will provide it unilaterally which would benefit others. As is well known, in the presence of indivisibilities, equality is not usually efficient. This raises the question that would some degree of inequality be efficient even when perfect equality would lead to a positive amount of the public good being provided? The answer is not obvious particularly since inequality in the allocation of the private input across

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[1] Evans and Jovanovic (1989) analyzed panel data from the National Longitudinal Survey of Young Men (NLS), which surveyed a sample of 4000 men in the US between the ages of 14-24 in 1966 almost every year between 1966-81, and found that entrepreneurs are limited to a capital stock no more than one and one-half times their wealth when starting a new venture.

[2] Indeed, the limited evidence that is available on the effect of land tenure reform suggests that the productivity gains can be large (Banerjee, Gertler and Ghatak, 2001).

production units involves some loss of efficiency due to decreasing returns. Also, if more than one player is involved in the provision of the public good, how would inequality *within* the class of contributors affect efficiency?

The public economics literature has addressed the question of inequality among contributing players in some detail. A key finding is the surprising "distribution-neutrality" result for a particular class of collective action problems, namely the provision of *pure* public goods.[3] These are public goods where individual contributions are perfect substitutes in the production of the public good and everyone gets the same benefit from the public good irrespective of the level of their contributions. Then in a Nash equilibrium wealth distribution within the set of contributors does not matter for the amount of public goods provision. The intuition behind this result is very clearly stated by Bergstorm, Blume and Varian (1986). Suppose after the redistribution every player adjusts his contribution to the public good by exactly the same amount as his change in wealth and leaves the consumption of the private good unchanged. In that case the size of the public good is the same as before and so the initial allocation is still available to all players. Those who have lower wealth because of redistribution have a restricted budget set and would clearly prefer the previous allocation if it is still available. The budget set of those who have higher wealth because of redistribution expands, but given that those who have lower wealth have reduced their contribution to the public good, these players are better off using their extra wealth to make up for the deficit. If they choose to do otherwise which results in a lower level of the public good, they could not have been at an optimum before the redistribution since to a limited extent that option was available to them before.

Subsequent work has shown that the neutrality result breaks down when individual contributions are complements in the production of the public good instead of being perfect substitutes (Cornes and Sandler, 1996, pp. 184-190) or, if the public good is impure, namely, the benefit received by a player depends not only on the total level of contributions, but also her own contribution (Bergstorm, Blume and Varian, 1986, Cornes and Sandler, 1994). In this paper we consider a different point of departure from the distribution-neutrality framework. The distribution-neutrality result assumes that the contribution towards the public good and the private input are fully convertible.[4] In practice, particularly in the building of rural infrastructure in developing countries, the contribution towards the public good often takes the form of labor. To fix ideas, let us think of the private input as capital. Then this

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[3] Some of the contributions to the theoretical literature related to this result are Warr (1983), Cornes and Sandler (1984), Bergstrom, Blume and Varian (1986), Bernheim (1986) and Itaya, de Meza and Myles (1997).

[4] The distribution neutrality literature is couched in the framework of a consumer choosing to allocate a given level of income between her private consumption and contribution to a pure public good. We adopt the framework of a firm using a private input and a public input to produce some good. While not exactly equivalent, formally these frameworks are very similar and what we call the private input is similar to the private consumption good in the distribution neutrality literature.

assumption bypasses an important issue of economic inequality: labor is not freely convertible into capital. Typically, labor and capital are not perfect substitutes in the production technology, and because of credit market imperfections capital does not flow freely from the rich to the poor to equate marginal returns. We take this more plausible scenario as our starting point and examine the effect of distribution of wealth among members of a given community on allocative efficiency in various types of collective action problems (involving public goods as well as common property resources or CPR) in the presence of missing and imperfect capital markets. This is particularly important in less developed countries where the life and livelihood of the vast masses of the poor crucially depend on the provision of public goods (roads, canal irrigation, public health and sanitation) and the local CPR (forestry, fishery, grazing lands), and where markets for land and credit are often highly imperfect or non-existent.

Our work is also motivated by the growing empirical literature on the relationship between inequality and collective good provision. For example, in an econometric study of 48 irrigation communities in south India Bardhan (2000) finds that the Gini coefficient for inequality of landholding among the irrigators has in general a significant negative effect on cooperation on water allocation and field channel maintenance but there is some weak evidence for a U-shaped relationship. Similar results have been reported by Dayton-Johnson (2000) from his econometric analysis of 54 farmer-managed surface irrigation systems in central Mexico.

Our work is also partly motivated by the literature on the relationship between economic development and the initial inequality of the distribution of wealth. While the empirical evidence on this question is mixed (see Benabou, 1996 and Banerjee and Duflo, 2000) there have been several theoretical models which suggest inequality is likely to have a negative effect on economic development. The main reasons that are advanced for this connection are, first, inequality increases agency costs in the labor and credit markets (e.g., Banerjee and Newman 1993) and second, inequality encourages redistributive policies that discourage capital accumulation (e.g., Alesina and Rodrik, 1994).[5] Our paper suggests a possible link that has been largely neglected in this literature: inequality may accentuate certain types of collective action problems. That this sort of a link in explaining differential growth performances across countries is not without empirical basis is suggested by recent work by Knack and Keefer (1997). They find that measures of social cohesion (measured by among other things the propensity of people to join voluntary organizations), which by their very nature are outcomes of a various types of collective action problems, positively affect per capita income growth rates across countries. They also found that the level of social cohesion is strongly and negatively associated with economic inequality.[6] Putnam's

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[5] Agency costs refers to all costs that arise from monitoring, screening and contract enforcement when the owner of an asset (land, capital, a machine) and its user are not the same person.

[6] See also Temple and Johnson (1998).

(1993) well known study of regional disparities in Italy has also emphasized the importance of social capital and how "horizontal" social networks (i.e., those involving people of similar status and power) are more effective in generating trust and norms of reciprocity than "vertical" ones.

The plan of the paper is as follows. In the next section we provide a brief non-technical discussion of our framework and the main results. In sections 3-5 we provide a formal analysis. In section 6 we discuss the implications of relaxing some of our main assumptions. In section 7 we provide some concluding observations. The appendix contains all formal proofs.

2 The Main Argument

We consider a simplified setting where the producers use as inputs one private good (say, land) and one collective good (say, irrigation water) to produce a private good (say, rice). The private and collective inputs are complementary in the production function. This collective good may be a public good like a public irrigation canal, or a CPR, like a community pond or forest. Formally speaking, we allow the actions of individual players regarding this collective input to involve positive or negative externalities.

Since the private and collective inputs are complementary, the marginal return from contributing to the public good is increasing in the wealth of a player. As a result, there will be a threshold level of wealth such that only those who have a higher level of wealth than this threshold will participate in providing the collective good and those with a lower level of wealth will free ride on the former group.[7] This means that redistributions that increase the wealth of the richer players at the expense of non-contributing poorer players would achieve a greater amount of the public good, and *other things being constant*, this should increase joint surplus. In our framework this is how Olson's original argument shows up. However, this argument focuses only on the total amount of the public good and not on joint surplus. In particular, the gain from increasing the size of the collective input has to be measured against the cost arising from worsening the allocation of the private input in the presence of decreasing returns.

We show that the amount of contribution towards the collective input is a concave function of the endowment of the private input of the player for most well-known production functions (e.g., the Cobb-Douglas and the CES) and also the equilibrium level of joint surplus (of both contributing and non-contributing players) is a concave function of the wealth distribution and hence displays inequality aversion. In

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[7] Baland and Platteau (1997) provide some very interesting examples where richer agents tend to play a leading role in collective action in a decentralized setting. For example, in rural Mexico the richer members of the populaton take the initiative in mobilizing labor to manage common lands and undertake conservation measures such as erosion control.

addition, the total amount of the collective input is a concave function of the wealth distribution *among contributing players*. We provide a precise characterization of what the optimal distribution of wealth that maximizes joint profits is in the case of imperfect convertibility between the private input and contribution to the collective input. We show that the joint profit maximizing wealth distribution under private provision of the public good involves equalizing the wealth level within the group of all non-contributing players (at some positive level) and also within the group of all contributing players. The contrast with the conclusions of both Olson and the distribution neutrality literature is quite sharp. The key assumptions leading to the result are, market imperfections that prevent the efficient allocation of the private input across production units, and some technical properties of the production function that are shared by widely used functional forms such as Cobb-Douglas and CES under decreasing returns to scale. With constant returns to scale, the joint profits within the group of contributors are independent of the distribution of wealth as in the distribution neutrality theorem.

The above result takes the number of contributors to the collective input as given. It turns out to be difficult to characterize the optimal distribution of wealth when the number of contributors can be chosen. A key question of interest here is: does perfect equality among all players maximize joint surplus? We provide a limited answer to this question. It turns out that perfect equality among all players (i.e., inter-group inequality in addition to intra-group inequality) is not always optimal. If wealth was equally distributed among all players, the average wealth of contributing players is low and this could reduce the level of the public good. In contrast concentrating all wealth in the hand of one player will maximize the average wealth of contributors, but will involve significant losses due to decreasing returns in the individual profit function with respect to wealth which is a standard assumption in most economic applications. The optimal distribution of wealth characterized above achieves a compromise between these two different forces.

We show that in the case of pure public goods, it is always optimal to have some degree of inequality between contributors and non-contributors. In contrast in the case of pure private goods (i.e., there are no externalities of any sort) perfect equality is optimal. The second result is obvious - it follows from the fact that the only effect of redistribution on joint profits is via the direct effect on the allocation of the private input across production units. Any indirect effect via the other input (we cannot call it the collective input now that there are no externalities) cancels out due to the envelope theorem (i.e. this effect is already taken into account by the agent in her maximization process) . The first result has an interesting intuition. A special property of pure public goods is that even if the difference in the wealth between the richest player and the second richest player is arbitrarily small, the former provides the entire amount of the public good with everyone else free riding on her. This property is the key to explain why perfect equality is not joint profit maximizing in this case. Start with a situation where all players except for one have the same wealth level, and this one player has a wealth level which is higher than that of others by an arbitrarily small amount. As a result this player is the single contributor to the public good. A small redistribution of wealth from other players to this player, keeping the average wealth of the other players constant, will have three effects on joint profits: the effect due to the worsening of the allocation of the private input, the effect of the increase in total contributions on the payoff of the non-contributing players, and the effect of the increase in total contributions on the payoff of the single contributing player. The result follows from the fact that the first effect is negligible since by assumption the extent of wealth inequality is very small, the second effect is positive, and the third effect can be ignored by the envelope theorem. As a result, the net effect of increasing inequality on joint surplus is positive.

In the polar opposite case of pure private goods (i.e., where there are no externalities) we show that perfect equality is always joint profit maximizing. As there are no externalities by assumption, there are two effects from an increase in the wealth of the richer player: the effect on her own profits via her choice of the other input (which is no longer collective) which can be ignored by the envelope theorem, and a negative effect due to diminishing returns to the private input. With decreasing returns to scale, therefore perfect equality is strictly joint profit maximizing. However, with constant returns to scale, the second effect can be ignored as well and so a wealth distribution involving some inequality can achieve the same level of joint surplus as perfect equality.

Now let us consider impure public goods which have aspects of both pure public and pure private goods. In particular, while a player benefits from the total level of contribution of all players (this is the pure public good aspect), her benefits are higher the higher is the level of her own level of contribution. With constant returns to scale, since the negative effect of inequality due to the diminishing returns to the private input is absent, one would expect that joint profit maximization would involve some degree of inequality even for impure public goods using a logic similar to that of pure public goods. We show that this is indeed the case. However, with the general case of impure public goods and decreasing returns, there are two opposing forces and whether perfect equality or some degree of inequality will maximize joint surplus would depend on their relative strengths. We show this using some numerical examples.

We finish this section with a brief summary of our main argument on inter-group inequality. Since the marginal valuation for the public good is increasing in the wealth of a player, in a decentralized equilibrium the richest players provide the entire amount of the public good and the other members of the group free ride on them. This means that redistributions that increase the average wealth of the richer players at the expense of the non-contributing poorer players would achieve a greater amount of the public good, and *other things being constant*, this should increase joint surplus.

This is the intuition for there being some optimal degree of inter group inequality. We can interpret Olson's (1965) original argument highlighting the positive role of inequality in alleviating inefficiency in public good problems in this way - to the extent inequality increases the wealth of the richer players, it also increases the equilibrium provision level of the public good. However, under diminishing returns with respect to the private input, this does not typically mean that all wealth should be given to the contributing players. Under some circumstances, such as in the case of pure public goods, or with impure public goods and constant returns to scale, the two effects balance out at some positive degree of inter-group inequality.

3 The Model

There are $n > 1$ players. Each player uses two inputs, k_i and z_i, to produce a final good. The input k_i is a purely *private* good, such as land, capital, managerial inputs. We assume that there is no market for this input and so a player is restricted to choose $k_i \leq w_i$ where w_i is the exogenously given endowment of this input of player i. While we will focus on this interpretation, there is an alternative one which views w_i as capturing some characteristic of a player, a skill or a taste parameter.[8] In contrast, z_i is a *collective* good in the sense that it involves some externalities, positive or negative. We assume that each player chooses some action $x_i \in [0, \overline{x}]$ which can be thought of as the effort by player i that goes into using a common property resource (henceforth, CPR) or contributing towards a public good. Let $X \equiv \sum_{i=1}^{n} x_i$ be the sum total of the actions chosen by the players. How individual actions aggregate into the collective input is modeled in the following simple way: $z_i = bx_i + cX$. The input x_i is an input that appears twice in the profit function, once on its own as a private input, and once in combination with the quantities used by other firms. The input X can be a good (e.g., R&D, education) or a bad (e.g., any case of congestion or pollution). This formulation allows each player to receive a different amount of benefit from the collective input which depends on the level action they choose. In contrast, for pure public goods every player receives the same benefits irrespective of their level of contribution. This case, as well as many others (involving both positive and negative externalities) appear as special cases of this formulation as we will see shortly. We assume that the cost of supplying x_i units of the input that has spillovers, is simply

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[8] The assumption that the market for the private input does not exist at all, while stark, is not crucial for our results. All that is needed is that the amount a person can borrow or the amount of land she can lease in depends positively on how wealthy she is. Various models of market imperfections, such as adverse selection, moral hazard, costly state verification or imperfect enforcement of contracts will lead to this property. There is also considerable evidence for this. Evans and Jovanovic (1989), using a panel data set from the US found that entrepreneurs are limited to a capital stock no more than one and one-half times their wealth when starting a new venture.

x_i.We assume that the production function is subject to non-increasing returns with respect to the private and the collective good.

Let $\mathbf{w} = (w_1, w_2, .., w_n)$ denote the wealth levels of the players. These are arranged in descending order of magnitude, i.e., $w_1 > w_2 > .. > w_n > 0$. Let $W \equiv \sum_{i=1}^{n} w_i$ denote the total amount of wealth of the n players. Let the payoff function of player i be denoted by $f(w_i, z_i) - x_i$. We make the following assumption about the production function:

> **Assumption 1:** $f(w, z)$ is a strictly increasing, strictly concave function that is twice continuously differentiable with respect to both arguments, $f_{12} \geq 0$ for all (w, z) , $\lim\limits_{w \to 0} f_2(w, z) = 0$ and it satisfies the Inada endpoint conditions.

We make the following assumption about the parameters b and c:

> **Assumption 2**
>
> $$b \geq 0 \text{ and } b + cn > 0 \tag{A2}$$

We allow c to be positive, negative or zero. Only when $c < 0$ the last condition becomes relevant which ensures that $|c|$ is not too large. This implies that if a social planner chose the level of the collective input, she would choose a positive level of x_i for at least one player. If this is violated, under the first-best $x_i = 0$ for all i. When $c = 0$ we have the case of a pure private good - there are no externalities. For $b = 0$ and $c > 0$ we have the case of pure public goods, one that the existing literature has mostly focused on. For $b > 0$ and $c > 0$ we have the case of impure public goods as defined by Cornes and Sandler (1996). For $b > 0$ and $c < 0$ we have a version of the commons problem: an individual privately gains by increasing her action relative to that of the others.

We begin our analysis by the following result which shows how the choice of x_i by player i depends on how much wealth she has:

> **Lemma 1:** $\gamma(w) \equiv \arg\max\limits_{z \geq 0}\{f(w, z) - z\}$ *is strictly positive for* $w > 0$ *and is an increasing function.*

This property follows directly from the complementarity between w_i and z_i and diminishing returns to z. An increase in w_i raises the marginal return of z_i relative to its marginal cost which is assumed to be constant and equal to 1. To restore equilibrium at the individual level, the amount of the collective input must increase. This property is not satisfied if w and z_i are substitutes in the production function. We will discuss this case in section 4. Several widely used functional forms in economics that display complementarity among the inputs satisfy the conditions stipulated in this lemma. These include the following production functions:

(a) the Cobb-Douglas, $f(w,z) = w^{\alpha} z^{\beta}$ with $\alpha + \beta \leq 1$. In this case $\gamma(w) = \beta^{\frac{1}{1-\beta}} w^{\frac{\alpha}{1-\beta}}$ which is clearly an increasing function;

(b) the generalized CES production function, namely, $f(w,z) = (\delta w^{\rho} + (1-\delta) z^{\rho})^{\frac{k}{\rho}}$ under parameter restrictions that ensure non-increasing returns to scale ($k \leq 1$), and complementarity between w and z ($\rho < k$). In the appendix we work out the details of this case. For $k = 1$ (constant returns to scale) it is possible to solve for $\gamma(w)$ explicitly, which turns out to be $\left(\frac{\delta}{\{1-(1-\delta)^{\frac{1}{1-\rho}}\}} \right)^{\frac{1}{\rho}} (1-\delta)^{\frac{1}{1-\rho}} w$.

From the economic point of view an important question is while both a rich individual and a poor individual would increase their contribution to the collective input if their wealth increases by the same amount, who would want to expand their contribution to the collective input more? This is crucial for determining the effect of the distribution of wealth on X and joint profits. This would depend on the curvature of γ, which in turn would depend on the third-order derivatives of the production function. Intuitively, the question is whether diminishing returns with respect to z would kick in at a faster or a slower rate at a higher wealth level. This would determine whether for a richer person a relatively small or large increase in z would restore her individual optimum compared to a poorer person. It turns out that all widely used functional forms in economics where the inputs are complements to each other have the property that diminishing returns kick in at the same rate or a faster rate the higher is the wealth level. This implies γ is linear or strictly concave. We show that the following technical assumption about the production function is equivalent to γ having this property.

Assumption 3

$$h(w,z) \equiv \frac{\partial f(w,z)}{\partial z} \text{ is quasi-concave.}$$

The following lemma proves that h being (weakly) quasi-concave is equivalent to γ being (weakly) concave:

Lemma 2: *Suppose Assumption 1 holds.* $\gamma(w)$ *is concave if and only if* $h(w,z) \equiv \frac{\partial f(w,z)}{\partial z}$ *is quasi-concave.*

For the Cobb-Douglas and CES production functions, $\gamma(w)$ is strictly concave if returns to scale are decreasing ($\alpha + \beta < 1$ for Cobb-Douglas and $k < 1$ for CES) and linear if there are constant returns to scale ($k = 1$). The following lemma provides additional characterization of the class of production functions for which γ is linear:

> **Lemma 3**: *Suppose Assumption 1 holds. If $f(w, z)$ is homogeneous of degree 1 then $\gamma(w) = Aw$ where A is a positive constant.*

This result follows from the fact that if a production function is homogeneous of degree 1 then if all inputs are increased proportionally, the marginal return of each input is unaffected. As a result, the first order condition of a player is unchanged if both her wealth level and the amount of the collective input received by her (i.e., z_i) are expanded proportionally. The Cobb-Douglas production with $\alpha + \beta = 1$ and the CES production function with $k = 1$ are examples of production functions that are homogeneous of degree 1 and we have already verified that $\gamma(w)$ is linear in these cases.

4 The Decentralized Equilibrium

We characterize the decentralized equilibrium in the following two steps. First, for a given distribution of the private input $\mathbf{w} = \{w_i\}_{i=1..n}$, we solve for the optimal contributions of each agent, $\hat{x}_i$, the total contribution X, and the joint surplus, Π. Second, we are looking for the distributions of w_i, which maximize the total contribution and joint profits to be able to analyze the effects of inequality on these two variables.

4.1 Characterization of the Nash Equilibrium for a Given Distribution of the Private Input

Let us consider the decentralized allocation. Player i takes the contribution of other players, X_{-i}, as given and solves:

$$\max_{x_i \geq 0} \pi^i = f(w_i, bx_i + cX) - x_i.$$

The first-order conditions are

$$f_2(w_i, bx_i + cX)(b + c) - 1 \leq 0$$

$$x_i \geq 0,$$

together with the standard Kuhn-Tucker complementary slackness condition. Let the function $g(w_i) > 0$ denote the solution to $f_2(w_i, g(w_i))(b + c) = 1$.[9] Holding the other players' contribution as fixed, we write $f(w_i, (b+c)x_i + cX_{-i}) = f(w_i, z_i)$. By Lemma

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[9] This function would be identical to $\gamma(.)$ defined in the previous section if $b + c = 1$.

1, $g(w_i)$ is increasing in w_i. In addition, if the condition in Lemma 2 is satisfied, g will be also concave.

Suppose $m \leq n$ players contribute in equilibrium. By the assumed complementarity between w_i and z_i in the production function, for a given value of z_i, $f_2(w_i, z_i)$ is increasing in w_i. Therefore, $g(w_i)$ is increasing. Also, by Assumption 1, $\frac{\partial z_i}{\partial x_i} = b + c > 0$. Therefore irrespective of the value of c, for a given level of contribution of other players, X_{-i}, the richest player has the greatest marginal profit from contributing, followed by the second richest and so on. As a result the set of contributing players will consist of the richest m ones, where $n \geq m \geq 1$. For the case of pure public goods, i.e., $b = 0$, only the richest player will contribute, i.e., $m = 1$.

Let us denote by $\hat{x}_i$ the optimal contribution of player i and let $X = \sum_{i=1}^{m} \hat{x}_i$. We assume that the wealth of the richest player exceeds some threshold level so that $x_1 > 0$. Given $z_1 = g(w_1) > 0$ and Assumption 2, we will have $x_1 = \dfrac{g(w_1)}{b + c} > 0$.

The above inequality implies that $m \geq 1$. Then, the equilibrium conditions for the optimal individual contributions are as follows:

$$\hat{x}_i = \begin{cases} \dfrac{g(w_i) - cX}{b}, \ i = 1, .., m \\ 0, \ i = m + 1, .., n. \end{cases} \tag{1}$$

$$X(m) = \frac{\sum_{i=1}^{m} g(w_i)}{b + mc}. \tag{2}$$

$$g(w_m) \geq \frac{c \sum_{i=1}^{m} g(w_i)}{b + mc} > g(w_{m+1}). \tag{3}$$

The first condition (1) states that, for all contributing agents, the first order condition must hold as equality. The second condition (2) is equivalent to $X(m) = \sum_{i=1}^{m} \hat{x}_i$. It states that the total contribution must have a value that is consistent with the individual maximization problems of all m contributors, and is obtained by adding up the m first-order conditions from (1). The third condition (3) is the most interesting one, as it determines the size of the contributing group, m. To see why it should hold note that the $m+1$-th agent would not contribute if $f_2(w_{m+1}, cX(m))(b + c) < 1$, since any further contribution has a marginal benefit that is lower than marginal cost. This condition is equivalent to $cX(m) > g(w_{m+1})$, which is exactly what the second inequality in (3) states. By the same logic, the m-th agent would not be contributing if $g(w_m) < cX(m-1) = \frac{c \sum_{i=1}^{m-1} g(w_i)}{b + (m-1)c}$, which can be rearranged as the condition $g(w_m) < \frac{c \sum_{i=1}^{m} g(w_i)}{b + mc}$. Thus, if she is to contribute, the condition $g(w_m) \geq \frac{c \sum_{i=1}^{m} g(w_i)}{b + mc}$ must be satisfied. The number of contributing agents, m is the smallest integer for which (3) is satisfied. If this inequality holds for all integers $m = 1, 2..n$, then all agents contribute.

The following lemma, together with the fact that g is increasing, ensures the existence of a unique value of m that solves (3):

> **Lemma 4**: *If* $k+1 \leq m$, *then the function* $s(k) = \dfrac{c\sum_{i=1}^{k} g(w_i)}{b+kc}$ *is increasing in* k. *If* $k > m$ *the function* $s(k)$ *is decreasing in* k.

Several useful observations follow directly from (1)-(3):

Observation 1. For the case of a pure public good ($b = 0$, $c \geq 0$) (3) cannot hold for $m > 1$ as that would imply

$$g(w_m) \geq \frac{\sum_{i=1}^{m} g(w_i)}{m}$$

which is impossible given that $w_1 > w_2 > .. > w_n$ by convention and Lemma 1 shows $g(.)$ is increasing. For pure public goods only the richest player contributes. This has the implication that even when the difference in the wealth between the riches player and second riches player is arbitrarily small, the former provides the entire amount of the public good.

Observation 2. For the case of pure private goods ($c = 0$), there is no interdependence across players and all of them will contribute.

Observation 3. The contributions of players are strategic complements for $c < 0$ and strategic substitutes for $c > 0$. Formally, this follows from the fact that $\frac{\partial^2 \pi^i}{\partial x_i \partial X_{-i}} = cf_{22}(w_i, bx_i + cX)(b+c)$. Intuitively, the reason is that the contributions of various players are perfect substitutes in the payoff function, and in the case of public goods (commons) an increase in the contribution of others is similar to an increase (decrease) in the player's own contribution, which reduces (increases) the marginal return of her contribution due to diminishing returns in the collective input (i.e., z).

4.2 Effect of Wealth Inequality on Total Contributions and Joint Profits

From (2),

$$X = \frac{\sum_{i=1}^{m} g(w_i)}{b+mc} \equiv \tilde{g}(\mathbf{w}).$$

If $g(w_i)$ is concave then $\tilde{g}(\mathbf{w})$ being the sum of m concave functions is a concave function. If X is concave in $\mathbf{w}$ then, holding the number of contributors constant, the total contribution is maximized when all contributing agents have equal amounts of the private input.

Proposition 1: *Suppose Assumptions 1-3 are satisfied. If $g(w_i)$ is strictly concave in w_i then X is strictly concave in w. If $g(w_i)$ is linear in w_i then X is linear in w.*

For the intuition behind this result, recall that given our assumption that diminishing returns with respect to the collective input used by the i-th individual sets in at a faster rate at a higher wealth level. Therefore the optimal level of the collective input is a concave function of the wealth level (Lemma 2). The above result follows from this, and the fact that according to our specification the collective input used by the i-th individual is a linear function of the individual's own contribution and the total level of contributions. Then it immediately follows that the total amount of the collective input used by all contributors is a linear function of total contributions (namely, $\sum_{i=1}^{m} z_i = (b + mc)X$). As a result, it is concave in the wealth level of the contributors.

The effect of wealth inequality on X has positive implications. Our analysis shows that greater equality *among those who contribute towards the collective good* will increase the value of X. Therefore a more equal wealth distribution among contributors will increase the equilibrium level of the collective input. In contrast, any redistribution of wealth from non-contributors to contributors that does not affect the set of contributors will increase X.[10] In terms of a two-player example, this implies that so long as both players contribute, any inequality in the distribution of wealth reduces X. But with sufficient inequality if one player stops contributing then any further increases in inequality will increase X.

Let us now turn to the normative implications of changes in the distribution of wealth. Under the first-best, which can be thought of a centralized equilibrium where players choose their contributions to maximize joint surplus, the first-order condition for player i is:

$$f_2(w_i, bx_i + cX)(b + nc) \leq 1.$$

The difference with the decentralized equilibrium is that now individuals look at the social marginal product of their contribution to the collective input, i.e., $f_2(w_i, bx_i + cX)(b + nc)$ as opposed to the private marginal product, i.e., $f_2(w_i, bx_i + cX)(b + c)$. Then it directly follows that those who will contribute will contribute more or less than in the decentralized equilibrium according as $c > 0$ or $c < 0$. Also, the number of contributors will be more or less than in the decentralized equilibrium according as $c > 0$ or $c < 0$.

Therefore, for the case of positive externalities, the total contribution in a decentralized equilibrium is less than the efficient (i.e., joint surplus maximizing) level. Conversely, for the case $c < 0$, total contributions exceed the socially efficient level.

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[10] In the above formula for X, holding m constant a redistribution from non-contributors to contributors will increase w_i $(i = 1, 2, .., m)$ with the increase being strict for some i.

From this one might want to conclude that greater inequality among contributors increases efficiency in the presence of negative externalities and reduces efficiency if there are positive externalities.[11] Indeed, the literature on the effect of wealth (or income) distribution on collective action problems have typically focussed on the size of total contributions. However, that is inappropriate as the correct welfare measure is joint profits.

In the presence of decreasing returns to scale the distribution of the private input across firms will have a direct effect on joint profits irrespective of its effect on the size of the collective input. In particular, greater inequality will reduce efficiency by increasing the discrepancy between the marginal returns to the private input across different production units. In the case of negative externalities, these two effects of changes in the distribution of the private input work in different directions, while in the case of positive externalities, they work in the same direction. Now we proceed to formally analyze this issue.

Using the conditions (1)-(3) agent i's surplus will be:

$$\pi_i(w_i, x_i, X) = f(w_i, g(w_i)) - \frac{g(w_i) - cX}{b},\ i = 1..m$$

$$\pi_i(w_i, x_i, X) = f(w_i, cX),\ i = m+1..n$$

Joint surplus is given by:

$$\Pi = \sum_{i=m+1}^{n} f(w_i, cX) + \sum_{i=1}^{m} f(w_i, g(w_i)) - \frac{\sum_{i=1}^{m} g(w_i)}{b + mc}.$$

Let us denote the joint profits of contributing players by $\Pi^c \equiv \sum_{i=1}^{m} f(w_i, g(w_i)) - \frac{\sum_{i=1}^{m} g(w_i)}{b+mc}$ and that of non-contributors as $\Pi^n \equiv \sum_{i=m+1}^{n} f(w_i, cX)$.

First consider the effect of distribution of wealth among non-contributors. This is trivial, since $f(w_i, cX)$ is concave, so is $\sum_{i=m+1}^{n} f(w_i, cX)$. Therefore perfect equality of wealth among non-contributors will maximize their joint profits. Note that even if $f(w, z)$ is homogeneous of degree 1, this is true.

Next, let us consider the effect of distribution of wealth among contributors. Let

$$\tilde{\pi}(w) \equiv f(w, g(w)) - \frac{g(w)}{b + mc}.$$

Notice that $\Pi^c = \sum_{i=1}^{m} \tilde{\pi}(w_i)$. The following lemma helps characterize the effect of wealth inequality on Π^c :

> **Lemma 5**: *Suppose Assumptions 1-3 hold and* $c \geq 0$, *or* $c < 0$ *but* $|c|$ *small. If* $g(w)$ *is concave then* $\tilde{\pi}(w) \equiv f(w, g(w)) - \frac{g(w)}{b+mc}$ *is concave. If* $f(w, z)$ *is homogeneous of degree one then* $\tilde{\pi}(w)$ *is linear in* w.

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[11] Note however that a sufficiently large degree of inequality among contributors may reduce X below the first-best level in the $c < 0$ case.

The intuition behind this result is the following. In the absence of externalities (i.e., $c = 0$) to find the effect of a change in w on the profit of a player, we can focus only on the direct effect and ignore the indirect effect via the envelope theorem. As a result, the second derivative of the profit function also depends only on the direct effect through w. In the presence of externalities, we must take into account the indirect effect of w on X that affects other players. This residual term, which is a fraction of X (namely, $\frac{1}{b+c} - \frac{1}{b+mc} = \frac{(m-1)c}{(b+c)(b+mc)}$) increases joint profits for $c > 0$ and reduces it for $c < 0$ compared to the case where $c = 0$. Since we already know X is concave, in the former case this reinforces the concavity of the joint profit function and in the latter case it goes the other way. As a result, for $c < 0$ a sufficient condition to ensure concavity of $\tilde{\pi}(w)$ is $|c|$ to be small.

Lemma 5 immediately implies that for $c \geq 0$ and for $c < 0$ but $|c|$ small, $\Pi^c = \sum_{i=1}^{m} \tilde{\pi}(w_i)$ is concave in the wealth of contributors so that greater equality will increase joint profits. As a result, perfect equality of wealth among contributors maximizes their joint surplus. For the special case where $f(w, z)$ is homogeneous of degree one Π^c is linear in the wealth of the contributors. In this case a redistribution of wealth among contributors will not affect joint surplus. However, equalizing wealth among non-contributors will maximize Π^n. For $c < 0$ but $|c|$ large (while continuing to satisfy Assumption 1) we cannot determine the curvature of Π^c in general.

It turns out that for the Cobb-Douglas case, with decreasing returns, $\tilde{\pi}(w)$ is strictly concave even if $c < 0$ but $|c|$ not necessarily very small. In this case, $\tilde{\pi}(w) = w^{\frac{\alpha}{1-\beta}}[(b+c)\beta]^{\frac{1}{1-\beta}} \left(\frac{1}{\beta(b+c)} - \frac{1}{b+mc} \right)$. In order for total contribution to be positive, we need $\tilde{\pi}(w_1) > 0$. This inequality holds if and only if $b(1-\beta) > (\beta - m)c$, which holds for $c \geq 0$ or $c < 0$ and $|c| < \frac{b(1-\beta)}{m-\beta}$. We assume this to be true - otherwise no player ever contributes. Hence $\tilde{\pi}(w)$ is concave in w for $\alpha + \beta < 1$. For case of constant returns, i.e., $\alpha + \beta = 1$, $\tilde{\pi}(w)$ is linear.

Given the initial wealth distribution $\mathbf{w}$, there is some $m \geq 1$ such that players with wealth $w \geq w_m$ contribute and those with $w \leq w_{m+1}$ do not. For this given value of m, it is clear by the concavity of Π, that the wealth distribution maximizing joint surplus should have $w_i = \widehat{w}$ for all $i = 1, .., m$ and $w_j = \tilde{w} < \widehat{w}$ for all $j = m+1, .., n$. subject to the following two conditions:

$$(n-m)\tilde{w} + m\widehat{w} = W$$

$$g(\widehat{w}) \geq \frac{cmg(\widehat{w})}{b+mc} > g(\tilde{w}).$$

The first of the above conditions can be rewritten as:

$$\tilde{w} = \frac{W - m\widehat{w}}{n-m}$$

Using this, the expression for joint profits becomes:

$$\Pi = (n-m)\ f(\frac{W - m\widehat{w}}{n-m}, \frac{cmg(\widehat{w})}{b+mc}) + m\left[f(\widehat{w}, g(\widehat{w})) - \frac{g(\widehat{w})}{b+mc}\right]. \tag{4}$$

Also, the total contribution is $X = m\frac{g(\widehat{w})}{b+mc}$. The following result characterizes the joint-surplus maximizing wealth distribution for a given m.

> **Proposition 2**: *Suppose Assumptions 1-3 are satisfied, $c \geq 0$ and if $c < 0$, $|c|$ is small. For a given m the joint profit maximizing wealth distribution under private provision of the public good involves equalizing the wealths of all non-contributing players to $\tilde{w} > 0$ and also those of all contributing players to $\widehat{w} > \tilde{w}$.*

This result shows that maximum joint surplus is achieved for both contributors and non-contributors, if there is no intra-group inequality. This is a direct consequence of joint profit of each group being concave in the wealth levels of the group members. The contrast with the conclusions of both Olson and the distribution neutrality literature is quite sharp. The key assumptions leading to the result are, market imperfections that prevent the efficient allocation of the private input across production units, and some technical properties of the production function that are shared by widely used functional forms such as Cobb-Douglas and CES under decreasing returns to scale. With constant returns to scale, the joint profits within the group of contributors are independent of the distribution of wealth as in the distribution neutrality theorem.

In the above result we did not talk about inter-group inequality. Formally, we took m as given while considering alternative wealth distributions. An obvious question to ask is, what is the joint-profit maximizing distribution of wealth when we can also choose the number of contributors, m. For example, does perfect equality among all players maximize joint surplus? This turns out to be a difficult question. Below we provide a partial answer to this question for the case of positive externalities, i.e., $c > 0$.

Suppose all players are contributing when wealth is equally distributed. Then from Proposition 2 we know that limited redistribution that does not change the number of contributors cannot improve efficiency. This immediately suggests the following result:

> **Corollary to Proposition 2**: *Suppose all players contribute under perfect equality. Then after a redistribution if all players continue to contribute then joint profits cannot increase.*

But suppose we redistribute wealth from one player to the other $n-1$ players up to the point where this player stops contributing. Recall that when the group size is $m < n$, $X = m\frac{g(\widehat{w})}{b+mc}$. It is obvious that an increase in the average wealth of contributing players keeping the number of contributors fixed will increase X. It turns out that an increase in m holding the average wealth of contributors constant will always increase X.[12] However, if we simultaneously decrease m from n to $n-1$ and increase the average wealth of contributors, it is not clear whether X will go up or not. If X goes down then we can unambiguously say that joint profits are lower due to this policy (for $c > 0$) since the effect of this policy on the efficiency of allocation of the private input across production units is definitely negative. However, if X goes up then there is a trade off: the increase in X benefits all players (since $c > 0$), including the player who is too poor contribute now, but this has to be balanced against the greater inefficiency in the allocation of the private input. From Proposition 2 we can find the optimal degree of inequality that balances these two effects when $m = n - 1$ but we cannot say in general whether the joint profits in this case exceeds or is less than joint profits with perfect equality.

It turns out that for the two polar cases, one where the collective input is a pure public good (i.e., $b = 0$) and the other, where there are no externalities (i.e. $c = 0$) we can provide a definite answer to this question:

> **Proposition 3**: *For pure public goods ($b = 0$ and $c > 0$) perfect equality (i.e. $m = n$ and $\widehat{w} = \frac{W}{n}$) is never joint profit maximizing for both constant and decreasing returns to scale. For pure private goods ($b > 0$ and $c = 0$) perfect equality is always joint profit maximizing but with constant returns to scale the same level of joint surplus can be achieved by a wealth distribution displaying some inequality.*

We noted a special property of pure public goods in the previous section (Observation 1), namely, even if the difference in the wealth between the richest player and the second richest player is arbitrarily small, the former provides the entire amount of the public good with everyone else free riding on her. This property is the key to explain why perfect equality is not joint profit maximizing in this case. Start with a situation where all players except for one have the same wealth level, and this one player has a wealth level which is higher than that of others by an arbitrarily small amount. As a result this player is the single contributor to the public good. A small redistribution of wealth from other players to this player, keeping the average wealth of the other players constant, will have three effects on joint profits: the effect due to the worsening of the allocation of the private input, the effect of the increase in X on the payoff of the non-contributing players, and the effect of the increase in X on

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[12] Formally, this is because $\frac{m}{b+mc}$ is increasing in m. The intuition is, the new entrant to the group of contributor will contribute a positive amount, which would reduce the incentive of existing contributors to contribute due to diminishing returns. However, in the new equilibrium X must go up, as otherwise the original situation could not have been an equilbrium.

the payoff of the single contributing player. The result follows from the fact that the first effect is negligible since by assumption the extent of wealth inequality is very small, the second effect is positive, and the third effect can be ignored by the envelope theorem. It should also be noted that this result goes through for both constant and decreasing returns to scale.

The second part of the result follows from the fact that when $c = 0$ a player will always choose $x_i > 0$ however small her wealth level. Then all players are contributors so long as they have non-zero wealth and it follows directly from Proposition 2 that perfect equality will maximize joint profits.

Under constant returns to scale, from Proposition 2 we know that joint profits are linear in the total wealth of contributors. Given that joint profits are higher under some degree of inequality for pure public goods compared with joint profits under perfect equality, in this particular case one would expect this property to be true for $c > 0$ and $b > 0$. This conjecture turns out to be correct:

> **Proposition 4**: *If the production function displays constant returns to scale then perfect equality is never joint profit maximizing for impure public goods (i.e., $c > 0$).*

The logic of this result is similar to that of the pure public goods case. The difference is, in that case the richest person is the only contributor even when the wealth difference between him and the second richest player is very small and therefore a small amount of inequality does not result in large losses due to the inefficient allocation of the private input. With constant returns to scale and for impure public goods, the difference between the wealth level of the contributors and non-contributors need not be very small. But, joint profits of contributors depend only their total wealth and not how it is distributed. As a result creating some inequality from the point where only player is exactly indifferent between contributing and not, to the point where she strictly prefers not to contribute, the loss due the inefficient allocation of the private input is small. However, unlike the pure public goods case, this could involve a significant amount of inequality with respect to the perfectly equal wealth distribution.

While we do not have general results for the case of decreasing returns, where there are two opposing forces, we can provide some illustrative examples using the Cobb-Douglas production function $f(w, z) = w^{\alpha} z^{\beta}$ for a two player game. If Figures 1, 2 and 3 we plot how the difference between joint profits under perfect equality and under inequality (where the degree of inequality is chosen to maximize joint profits given than only one player contributes) vary with b and c for three alternative sets of values of α and β such that $\alpha + \beta = 0.6$. As we can see that for a given set of values of α and β, the higher is b/c the more likely that perfect equality maximizes joint profits. Formally, the condition for a player not to contribute in a two player game is $g(\overline{w}-\varepsilon) < cX = cg(\overline{w}+\varepsilon)$ where $\overline{w}$ is the average level of wealth and ε denotes the extent of inequality. In the Cobb-Douglas case this condition is the same as:

$$\left(\frac{\overline{w}+\varepsilon}{\overline{w}-\varepsilon}\right)^{\frac{\alpha}{1-\beta}} > \frac{b}{c}+1.$$

The reason is, the higher is b/c, the greater will have to be the extent of inequality to lead one player to stop contributing. This will involve greater and greater losses due to the inefficient allocation of the private input. The extent of the loss would be higher, the greater is the extent of diminishing returns with respect to the private input (α).

4.3 Convertibility between the private input and contribution to the collective input

It is important for our result that x_i and w_i are different types of goods and one cannot be freely converted into the other. Suppose the individual can freely allocate a fixed amount of wealth between two uses, namely, as a private input and as her contribution to the collective input. This is the formulation chosen by the literature on distribution-neutrality (e.g., Warr (1983), Bergstrom, Varian and Blume (1986), Cornes and Sandler (1996) and Itaya et al (1997)). This literature focuses on pure public goods, i.e., where $z_i = cX$. For ease of comparability, let us consider this case first. Let k_i denote the amount of the private input chosen by player i. Then player i's decision problems is to maximize $f(k_i, cX)$ with respect to k_i and x_i subject to the budget constraint $k_i + x_i \leq w_i$.The first-order condition of an individual who contributes a positive amount in equilibrium is

$$f_1(k_i, cX) = cf_2(k_i, cX), \qquad i = 1, 2, .., m.$$

As $k_i = w_i - x_i$ from the budget constraint of the individual, and $x_i + \overline{X}_{-i} = X$ for all $i = 1, 2, .., m$, this condition implicitly defines the following function:

$$w_i - x_i = h(X).$$

Summing across all players who contribute in equilibrium, we get $X + mh(X) = W$. This equation can be solved for X which therefore depends only on total wealth, W and not on its distribution. Joint profits will also be independent of the distribution of wealth.

The above formulation is similar to that of a consumer allocating a fixed amount of money to alternative goods in order to maximize utility. An alternative formulation to capture free convertibility between k_i and x_i is to pose the problem as that of a firm maximizing profits by choosing inputs which can be sold or purchased from the market at a given price. One could think of k_i as capital which has a given price r such that a firm that has an excess of it (relative to its endowment w_i) can sell it to other firms, and a firm that has a shortage of it can buy it at the same price, say r. Similarly, one can think of x_i as labor that can be used to contribute towards the collective input, or sold in the labor market at price w.[13] Now the first order condition of a contributing player, i, is

$$f_1(k_i, cX) = \frac{r}{w} c f_2(k_i, cX), \qquad i = 1, 2, .., m.$$

This condition is the same as in the previous formulation, except for the multiplicative constant $\frac{r}{w}$ and so the distribution neutrality result goes through.

Turning now to impure public goods, i.e., where $b > 0$, the first order condition for player i according to the first formulation is:

$$f_1(k_i, bx_i + cX) = (b + c) f_2(k_i, bx_i + cX), \qquad i = 1, 2, .., m.$$

It is clear that in general the distribution neutrality result will not go through now. It will go through for some special cases, such as the case where $f(w, z)$ is homothetic. In this case, the values of k_i and z_i at a point of individual optimum satisfies the condition

$$\frac{k_i}{bx_i + cX} = A$$

where A is a positive constant. It is readily verified that the distribution neutrality result holds in this case. Our analysis shows that in this case, relaxing the assumption of perfect convertibility of the private input and the contribution to the collective input implies that the distribution neutrality result no longer holds. Specifically, greater equality among contributors always improves efficiency for impure public goods (i.e., $c > 0$) while for collective inputs subject to negative externalities, the effect of inequality on efficiency is ambiguous. In the latter case, we characterize conditions under which we can sign the effect of inequality on efficiency. Our results do not depend on the production functions being homothetic, but in the general case even with free convertibility, distribution neutrality can break down if the collective input is not a pure public good, as is well recognized in the literature (see for example, Bergstrom, Varian and Blume (1986) and Cornes and Sandler (1996)).

4.4 The private input and the collective input are substitutes

Above, we assumed that the private input and the public good are complements in the production function. In this section we examine the implications of these

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[13] In our framework labor is not directly used in production. We can think of another sector which uses labor. Alternatively, we can extend the basic model by adding labor as a third input. The distribution neutrality result will go through.

being substitutes. For simplicity, we examine the case where w and z are perfect substitutes: $\pi(w_i, x_i, X) = f(w + bx_i + cX) - x_i$, where f is increasing and strictly concave and b and c satisfy Assumption 2. The first order conditions for the agent's problem:

$$(b+c)f'(w_i, bx_i + cX) \leq 1$$

with the strict equality holding when $x_i > 0$. Let us denote by w^* the solution to $f'(w) = \frac{1}{b+c}$, which exists and is unique given the above assumptions. In contrast to the complements case, it is now the poorest player who has the highest marginal product of contributing. In the pure public good case ($b = 0$) the poorest player will be the only contributor if $w_n < w^*$ and if $w_n \geq w^*$ the public good will not be provided at all.

As before, joint surplus goes up if wealth is equally distributed among non-contributors. Also, we cannot say for sure whether the optimal distribution of wealth involves perfect equality, or some inequality among the contributor (the poorest agent) and the rest. This is clearly seen for the case of the pure public good ($b = 0$). For simplicity, suppose there are two players with wealth levels $w_1 = \overline{w} + \varepsilon$ and $w_2 = \overline{w} - \varepsilon$ and, in addition assume for simplicity that $c = 1$. Now joint profits are: $\Pi(\varepsilon) = f(w^*) - \{w^* - (\overline{w} - \varepsilon)\} + f(\overline{w} + \varepsilon + w^* - (\overline{w} - \varepsilon)) = f(w^*) - w^* + \overline{w} + f(2\varepsilon + w^*) - \varepsilon$ and so $\Pi'(\varepsilon) = 2f'(2\varepsilon + w^*) - 1$. We know that $f'(2\varepsilon + w^*) - 1 \leq 0$ since by definition $f'(w^*) - 1 = 0$ but whether $2f'(2\varepsilon + w^*) - 1 \leq 0$ or > 0 cannot be determined *a priori*. For the intuition behind this, notice that, those who choose $x_i > 0$, i.e., the poorest players, use the efficient amount of the input. Other players have more than the efficient level of the input in their production units. Any redistribution from the poor to the rich players do not affect the profit of the former as they exactly compensate for this by increasing their contribution. Since rich players have more than the efficient level of the input in their firms, normally a transfer of an additional unit of wealth would reduce joint profits since the marginal gain to the rich player is less than the marginal cost to the poor player. But every extra unit of wealth received by the rich player increases the input received by her firm by *twice* the amount because of the increase in the effort by the poor player and as a result it is not clear whether joint profits increase or decrease.

4.5 The individual contribution to the collective input and the total contribution are complements

In the paper we studied the case where an individual player's contribution to the collective input and the total contribution of all players are perfect substitutes. Consider an alternative formulation where they are complements:

$$z_i = \left(\frac{x_i}{X}\right)^{\theta} X^{\gamma}$$

where $0 \leq \theta \leq 1$ and $0 \leq \gamma \leq 1$. According to this specification, not only each player positively gains from the total contribution of all players, their gains are greater, the larger is their contribution is relative to the total. This induces people to choose a higher level of x_i which benefits others through the term X^{γ}. But it also reduces how much others can enjoy the collective good by a congestion effect capture by the term $\left(\frac{x_i}{X}\right)^{\theta}$. If the latter effect is unimportant compared to the former, then we have a public good and indeed for $\theta = 0$ we have the textbook case of a pure public good. But if it is the other way round then the congestion effect dominates the beneficial externality effect and in the limit, for $\theta = 1$ we have the textbook case of the commons. When these two effects exactly balance each other out ($\theta = \gamma$), we have the case of the a pure private good.

Analytically, this case turns out to be quite hard even when we take a specific form of the production function, namely Cobb-Douglas, and consider a two player game. We can show that if we compare allocations under perfect equality (both players have the same level of wealth) and perfect inequality (one player has all the wealth and the other player has nothing) joint surplus is always higher under perfect equality for non-negative externalities (i.e., $\theta \geq \gamma$). However, if there are substantial negative externalities then under some parameter values joint surplus will be higher under perfect inequality. The intuition for this result lies in the fact that when the negative externality problem is very severe then under perfect equality the players choose their actions related to the collective input at too high a level relative to the joint surplus maximizing solution. Perfect inequality converts the game to a one player game and hence gets rid of this problem. On the other hand due to joint diminishing returns to the private input and the collective input, joint surplus is lower under perfect inequality compared to perfect equality if there were no externalities. What this result tells us is that perfect inequality is desirable only when the negative externality problem is severe and when the extent of diminishing returns is not too high.

Instead of comparing the allocations under perfect equality and perfect inequality, if we consider the effects of a continuous change in inequality on total contributions and joint profits, the results are not clear cut. We can prove that in the case of commons its total use (X) decreases with increasing wealth inequality and joint profits per unit of total contributions (i.e., Π/X), or what one may call the average rate of return on the collective input, increases with inequality. But the absolute level of joint profits may increase or decrease with inequality. Numerical simulations suggest that this absolute amount of surplus in general decreases with inequality, except for the presence of substantial negative externalities. In the case of public goods (pure and impure), we prove that the average rate of return on the public good input decreases with inequality. But as the extent of positive externalities become large (approaching the pure public goods case) the total amount of public good provision (and the absolute amount of the joint profits) may increase with inequality. However there exists a range of moderate presence of positive externalities such that total contributions as well as the absolute amount of joint profits decreases with inequality.

5 Concluding Remarks

In this paper we analyze the effect of inequality in the distribution of endowment of private inputs that are complementary in production with collective inputs (e.g., contribution to public goods such as irrigation and extraction from common-property resources) on efficiency in a simple class of collective action problems. In an environment where transaction costs prevent the efficient allocation of private inputs across individuals, and the collective inputs are provided in a decentralized manner, we characterize the optimal second-best distribution of the private input. We show that while efficiency increases with greater equality *within* the group of contributors and non-contributors, in some situations there is an optimal degree of inequality *between* the groups.

The limitations of our model suggest several directions of fruitful research. The model is static. It is important to extend to the case where both wealth distribution and efficiency of collective action are endogenous. For example, it is possible to have multiple stationary states with high (low) wealth inequality leading to low (high) incomes to the poor due to low (high) level of provision of public goods, which via low (high) mobility can sustain an unequal (equal) distribution of wealth. Also, in the intertemporal case it will be interesting to analyze the effects of inequality on the sustainability of cooperation in a situation of repeated games. Second, non-convexities of technology and differential availability of exit options seriously affect collective action in the real world, and our model ignores them.[14] For example, the public good may not be generated if the total amount of contribution is below a certain threshold. This is the case for renewable resources like forests or fishery where a minimum stock is necessary for regeneration, or in the case of fencing a common pasture. Third, the empirical literature suggests that even when the link between inequality and collective action in that literature is consistent with the results in our model, the mechanisms involved may in some cases be quite different. For example, transaction costs in conflict management, and costs of negotiation may be higher in situations of inequality. Finally, the kind of collective action problem we focus on in this paper is the free rider problem following the public economics literature. Here, the problem arises in the sharing of the costs of collective action. But there is another problem, often called the bargaining problem, whereby collective action breaks down

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[14] The model of Dayton-Johnson and Bardhan (1999) examines the effect of inequality on resource conservation with two periods and differential exit options for the rich and the poor in the case when technology is linear. Baland and Platteau (1997) discuss the effect of non-convexities of technology in a static model.

because the parties involved cannot agree on the sharing of the benefits.[15] Inequality matters in this problem as well. For example, bargaining can break down when one party feels that the other party is being unfair in sharing the benefits (there is ample evidence for this in the experimental literature on ultimatum games). More generally, social norms of cooperation and group identification may be difficult to achieve in highly unequal environments.

6 Appendix

Proof of Lemma 1: Consider the first order condition, $f_2(w,z) - 1 = 0$. By Assumption 1, $f_2(w,z) > 0$ for all $w > 0$ and $\lim_{w \to 0} f_2(w,z) = 0$. Therefore $\gamma(w) > 0$ for all $w > 0$. By concavity, a global maximum exists and $f_{22}(w,z) < 0$. By definition, $f_2(w,\gamma(w)) - 1 = 0$. Notice that under our assumptions $\gamma(w)$ is differentiable, and hence continuous. In particular, $\frac{dg(w)}{dw} = -\frac{f_{12}}{f_{22}} > 0$.■

Proof of Lemma 2: By the definition of $h(w,z), h(w,\gamma(w)) = 1$. Totally differentiating with respect to w we get $h_1 + h_2\frac{dg(w)}{dw} = 0$, or, $\frac{dg(w)}{dw} = -\frac{h_1}{h_2}$. Notice that $h_1 = \frac{\partial^2 f(w,z)}{\partial z \partial w} > 0$ (as w and z are complements) and $h_2 = \frac{\partial^2 f(w,z)}{\partial z^2} < 0$ (by strict concavity). Differentiating once again with respect to w we get:

$$\frac{d^2\gamma(w)}{dw^2} = -\frac{h_1^2 h_{22} + h_2^2 h_{11} - 2h_1 h_2 h_{12}}{h_2^3}.$$

The condition $\frac{d^2\gamma(w)}{dw^2} \leq 0$ is equivalent to the determinant $\begin{vmatrix} 0 & h_1 & h_2 \\ h_1 & h_{11} & h_{12} \\ h_2 & h_{12} & h_{22} \end{vmatrix}$ being ≤ 0 which in turn is equivalent to $h(w,z)$ being quasi-concave (see Theorem 21.20 of Simon and Blume (1994)).■

Proof of Lemma 3: Since $f(w,z)$ is homogeneous of degree 1, $f_2(w,z)$ is homogeneous of degree 0. If $\lambda > 0$, $f_2(\lambda w, \lambda\gamma(w)) = f_2(w,\gamma(w))$. Since by definition $f_2(w,\gamma(w)) = 1$, so $f_2(\lambda w, \lambda\gamma(w)) = f_2(w,\gamma(w)) = 1$. It must be then $\gamma(\lambda w) = \lambda\gamma(w)$ which means $\gamma(w) = Aw$ where $A > 0$ is a constant.■

Proof of Lemma 4: Since agent $k+1$ contributes a positive amount by assumption, $g(w_{k+1}) > \dfrac{c\sum_{i=1}^{k} g(w_i)}{b+kc}$. Straightforward algebra shows that this is equivalent to the inequality $\dfrac{c\sum_{i=1}^{k+1} g(w_i)}{b+(k+1)c} > \dfrac{c\sum_{i=1}^{k} g(w_i)}{b+kc}$. The second part of the lemma can proved in the same way.■

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[15] See for example, Elster (1989).

Proof of Lemma 5: Totally differentiating with respect to w we get:

$$\frac{\partial \tilde{\pi}(w)}{\partial w} \equiv f_1(w, g(w)) + \left(f_2(w, g(w)) - \frac{1}{b+mc}\right) g'(w).$$

From the definition of $g(w)$ and the first-order condition of a contributing player, $f_2(w, g(w)) \equiv \frac{1}{b+c}$. Substituting in we get

$$\frac{\partial \tilde{\pi}(w)}{\partial w} \equiv f_1(w, g(w)) + \frac{(m-1)c}{(b+c)(b+mc)} g'(w).$$

Totally differentiating once again with respect to w :

$$\frac{\partial^2 \tilde{\pi}(w)}{\partial w^2} \equiv f_{11}(w, g(w)) + f_{12}(w, g(w)) g'(w) + \frac{(m-1)c}{(b+c)(b+mc)} g''(w).$$

From the proof of Lemma 1, $g'(w) = -\frac{f_{12}}{f_{22}}$. Therefore f_{11} $f_{11} + f_{12} g'(w) = \frac{f_{11} f_{22} - f_{12}^2}{f_{22}} < 0$ since $f(w,z)$ is concave. Therefore $\frac{\partial^2 \tilde{\pi}(w)}{\partial w^2}$ is concave if (i) $g(w)$ is concave and $c > 0$;(ii) $c = 0$ and (iii) $c < 0$ but $|c|$ small. The second part of the lemma follows from the fact that if $f(w,z)$ is homogeneous of degree one then $g(.)$ is linear and $\tilde{\pi}(w) = f(\lambda w, \lambda g(w)) - \frac{g(\lambda w)}{b+mc} = \lambda \left[f(w, g(w)) - \frac{g(w)}{b+mc} \right]$ is linear as well.■

Proof of Proposition 2: For a given value of m it follows from the concavity of the profit functions of both contributors and non-contributors that there should not be any intra-group heterogeneity. Also, $\widehat{w} > \tilde{w}$ given that contributors must be richer than non-contributors (see (1)-(3)). It is never optimal to set $\tilde{w}$ at a very low level given the Inada endpoint conditions, namely, $\lim_{\tilde{w} \to 0} f_1(\tilde{w}, cX) = \infty$. Since $\widehat{w} > \tilde{w}$ by assumption, it would never be optimal to make $\widehat{w}$ arbitrarily small, since that would mean $\tilde{w}$ would be even smaller and almost all of W would be left unused.■

Proof of Proposition 3: Let $\overline{w} \equiv \frac{W}{n}$. Let ε be a positive number, however small and suppose that the initial wealth distribution is such that one player has wealth $\overline{w} + \varepsilon$ and the wealth level of the other $n-1$ players is $\overline{w} - \frac{\varepsilon}{n-1}$. Then the richest player is the only contributor and $X = \frac{g(\overline{w}+\varepsilon)}{c}$. The first-order condition of the richest player is $f_2(\overline{w} + \varepsilon, cX) = \frac{1}{c}$. The joint profit of non-contributing players is $(n-1) f\left(\overline{w} - \frac{\varepsilon}{n-1}, g(\overline{w} + \varepsilon)\right)$ and the profit of the single contributing player is $f\left(\overline{w} + \varepsilon, g(\overline{w} + \varepsilon)\right) - \frac{g(\overline{w}+\varepsilon)}{c}$. Consider the effect of a small increase in ε on joint profits. It is straightforward to show that $\frac{d\Pi}{d\varepsilon} = \left[f_1\left(\overline{w} + \varepsilon, g(\overline{w} + \varepsilon)\right) - f_1\left(\overline{w} - \frac{\varepsilon}{n-1}, g(\overline{w} + \varepsilon)\right)\right] + (n-1) f_2\left(\overline{w} - \frac{\varepsilon}{n-1}, g(\overline{w} + \varepsilon)\right) g'(\overline{w} + \varepsilon) + \left[f_2\left(\overline{w} + \varepsilon, g(\overline{w} + \varepsilon)\right) - \frac{1}{c}\right] g'(\overline{w} + \varepsilon)$. Using the first-order condition of the richest player, the effect of a change in ε on her profits via the change in X drops out. Also, for ε close enough to 0 the first term is arbitrarily small. This captures the loss of surplus due to decreasing returns to the private input and is a second-order effect. However, the effect of an increase in ε that leads to an increase in X has a first-order effect on the payoffs of the $n-1$ non-contributing players which does not become arbitrarily small as ε goes to 0. As a result it is not optimal to set ε arbitrarily close to 0.

For the second part of the proposition, consider without loss of generality redistribution between any two players holding the wealth of other players constant. Since in this case the left inequality in (3) is satisfied for all agents, as a result of the redistribution both players continue to choose $x_i > 0$ and then it follows directly from Proposition 2 that greater inequality reduces joint profits unless the production function exhibits constant returns to scale, in which case joint profits remain the same (see Lemma 5).

Proof of Proposition 4: We want to compare joint surplus under perfect equality (i.e., all players have wealth $\overline{w} \equiv \frac{W}{n}$) and the wealth distribution that is obtained by a redistribution that leads to k players contributing and the others not contributing. From the discussion above, we know that all players contribute under perfect equality. From Proposition 2, for a given value of k, with constant returns to scale the level of joint surplus within the group of contributors does not depend on the distribution of wealth. Therefore, without loss of generality we assume that all k contributors have wealth $\overline{w} + \frac{\varepsilon}{k}$ and all $n-k$ non-contributors have wealth $\overline{w} - \frac{\varepsilon}{n-k}$. Let us denote by Π^E the joint surplus under perfect equality and with $\Pi^I(\varepsilon)$ the one under inequality such that there are k contributing players. Since we assume constant returns to scale, it follows from Lemma 3 that $g(w) = Aw$, where A is a positive constant. Let total wealth be normalized to $n\overline{w}$. A player stops contributing if

$$A(\overline{w} - \frac{\varepsilon}{n-k}) < cX = \frac{kcA(\overline{w} + \frac{\varepsilon}{k})}{b+kc}$$

or, $\varepsilon > \frac{b(n-k)}{b+nc}\overline{w}$. Start with $\varepsilon = \frac{b(n-k)}{b+nc}\overline{w}$ and consider a small increase in ε.

$$\begin{aligned}\frac{d\Pi^I(\varepsilon)}{d\varepsilon} &= \left[f_1\left(\overline{w} + \frac{\varepsilon}{k}, g(\overline{w} + \frac{\varepsilon}{k})\right) - f_1\left(\overline{w} - \frac{\varepsilon}{n-k}, \frac{kc}{b+kc} g(\overline{w} + \frac{\varepsilon}{k})\right)\right] + \\ &+ f_2\left(\overline{w} - \frac{\varepsilon}{n-k}, \frac{kc}{b+kc} g(\overline{w} + \frac{\varepsilon}{k})\right) \frac{(n-k)c}{b+kc} g'(\overline{w} + \frac{\varepsilon}{k}) + \\ &+ \left[f_2\left(\overline{w} + \frac{\varepsilon}{k}, g(\overline{w} + \frac{\varepsilon}{k})\right) - \frac{1}{b+c}\right]\end{aligned}$$

The last term drops out because of the envelope theorem and the second term is positive. Because of constant returns to scale $f_1(\lambda w, \lambda z) = f_1(w, z)$ for $\lambda > 0$. Notice also that for $\varepsilon = \varepsilon^* = \frac{b(n-k)}{b+nc}\overline{w}$

$$\frac{kc}{b+kc}\left(\overline{w} + \frac{\varepsilon^*}{k}\right) = \overline{w} - \frac{\varepsilon^*}{n-k}.$$

But then the first term is:

$$
\begin{aligned}
& f_1\left(\overline{w}+\frac{\varepsilon^*}{k}, g(\overline{w}+\frac{\varepsilon^*}{k})\right)-f_1\left(\overline{w}-\frac{\varepsilon^*}{n-k}, \frac{kc}{b+kc}g(\overline{w}+\frac{\varepsilon^*}{k})\right)= \\
= \;& f_1\left(\overline{w}+\frac{\varepsilon^*}{k}, g(\overline{w}+\frac{\varepsilon^*}{k})\right)-f_1\left(\frac{kc}{b+kc}\left(\overline{w}+\frac{\varepsilon^*}{k}\right), \frac{kc}{b+kc}g(\overline{w}+\frac{\varepsilon^*}{k})\right)=0
\end{aligned}
$$

This implies that $\Pi^I(\varepsilon)$ achieves a maximum for some $\bar{\varepsilon}>\varepsilon^*$, i.e. $\max_{\varepsilon\in[\varepsilon^*,1]}\Pi^I(\varepsilon)>\Pi^I(\varepsilon^*)$.

Now let us prove that inequality is always joint profit maximizing. To show it, notice that under constant returns to scale we have $\Pi^E=\Pi^I(\varepsilon^*)$. Notice that at $w-\varepsilon^*$ the poorer agents are just indifferent between contributing and not contributing. Then, by Proposition 1, we know that since joint profits are linear in total wealth, it must be true that:

$$
\Pi^E(\bar{w})=\Pi^C(\bar{w}+\frac{\varepsilon^*}{k}, \bar{w}-\frac{\varepsilon^*}{n-k}),
$$

where $\Pi^C(\bar{w}+\frac{\varepsilon^*}{k}, \bar{w}-\frac{\varepsilon^*}{n-k})$ denotes the joint profits when there are k agents with wealths $\bar{w}+\varepsilon^*$ and $n-k$ agents with wealths $\bar{w}-\varepsilon^*$, all of whom contribute. The above equality can be re-written as:

$$
\begin{aligned}
n[f(\bar{w},g(\bar{w}))-\frac{g(\bar{w})}{b+nc}] &= k[f(\bar{w}+\frac{\varepsilon^*}{k}, g(\bar{w}+\frac{\varepsilon^*}{k}))-\frac{g(\bar{w}+\frac{\varepsilon^*}{k})}{b+nc}]+ \\
& +(n-k)[f(\bar{w}-\frac{\varepsilon^*}{n-k}, g(\bar{w}-\frac{\varepsilon^*}{n-k}))-\frac{g(\bar{w}-\frac{\varepsilon^*}{n-k})}{b+nc}].
\end{aligned}
$$

Also:

$$
\Pi^I(\varepsilon^*)=k[f(\bar{w}+\frac{\varepsilon^*}{k}, g(\bar{w}+\frac{\varepsilon^*}{k}))-\frac{g(\bar{w}+\frac{\varepsilon^*}{k})}{b+kc}]+(n-k)f(\bar{w}-\frac{\varepsilon^*}{n-k}, \frac{kc}{b+kc}g(\bar{w}+\frac{\varepsilon^*}{k})).
$$

Using the above expressions:

$$
\begin{aligned}
\Pi^C(\bar{w}+\frac{\varepsilon^*}{k}, \bar{w}-\frac{\varepsilon^*}{n-k})-\Pi^I(\varepsilon^*) &= \\
k\frac{g(\bar{w}+\frac{\varepsilon^*}{k})}{b+kc}-k\frac{g(\bar{w}+\frac{\varepsilon^*}{k})}{b+nc}-(n-k)\frac{g(\bar{w}-\frac{\varepsilon^*}{n-k})}{b+nc} &= \\
kg(\bar{w}+\frac{\varepsilon^*}{k})[\frac{1}{b+kc}-\frac{1}{b+nc}-\frac{(n-k)c}{(b+kc)(b+nc)}] &= 0,
\end{aligned}
$$

where we used the fact that $\varepsilon^*=\frac{b(n-k)}{b+nc}\overline{w}$. Combining the above results we obtain that $\max\Pi^I(\varepsilon)>\Pi^I(\varepsilon^*)=\Pi^E$ and thus some degree of inequality (with $\varepsilon>\frac{b(n-k)}{b+nc}\overline{w}$) is joint profit maximizing indeed.■

The CES Example

For the CES production function:

$$f(w,z) = (\delta w^\rho + (1-\delta)z^\rho)^{\frac{k}{\rho}}$$

we show that if $0 < \rho < k \leq 1$ then $\gamma(w)$ is increasing and concave. First we need to ensure that f is concave and w and z are complements. The condition for non-increasing returns is $k \leq 1$, since $f(\lambda w, \lambda z) = \lambda^k f(w,z)$. The condition for $f_{12} > 0$ is $k > \rho$. The first order condition of maximization is:

$$(\delta w^\rho + (1-\delta)\gamma(w)^\rho)^{\frac{k}{\rho}-1}\gamma(w)^{\rho-1} = \frac{1}{k(1-\delta)(b+c)}.$$

Differentiating with respect to w we obtain and using the first order condition we get:

$$\gamma'(w) = \frac{(k-\rho)\delta w^{\rho-1}\gamma(w)}{(1-k)(1-\delta)(\gamma(w))^\rho + \delta(1-\rho)w^\rho}.$$

As $k > \rho$ by assumption the numerator is positive. Also, the denominator is positive as $1-k \geq 0$ and $\rho \in (0,1)$ and $\delta \in (0,1)$. Therefore $\gamma(.)$ is increasing. Observe that $\frac{w\gamma'(w)}{\gamma(w)} = \frac{(k-\rho)\delta w^\rho}{(1-k)(1-\delta)(\gamma(w))^\rho + \delta(1-\rho)w^\rho} \leq 1$ since the numerator is less than the second term in the denominator (which follows from $k \leq 1$). Differentiating the expression for $\gamma'(w)$, the sign of $\gamma''(w)$ turns out to be the same as that of the following expression:

$$(1-\rho)\left\{(1-k)(1-\delta)w^{\rho-2}\gamma(w)^{\rho+1} + \delta w^{2\rho-2}\gamma(w)\right\}\left\{\frac{w\gamma'(w)}{\gamma(w)} - 1\right\}.$$

This expression is non-negative under our assumptions, and the fact that $\frac{w\gamma'(w)}{\gamma(w)} \leq 1$. For $k = 1$, $\frac{w\gamma'(w)}{\gamma(w)} = 1$ and so the expression is equal to 0. Therefore $\gamma(w)$ is concave, and strictly so for $k < 1$. ■

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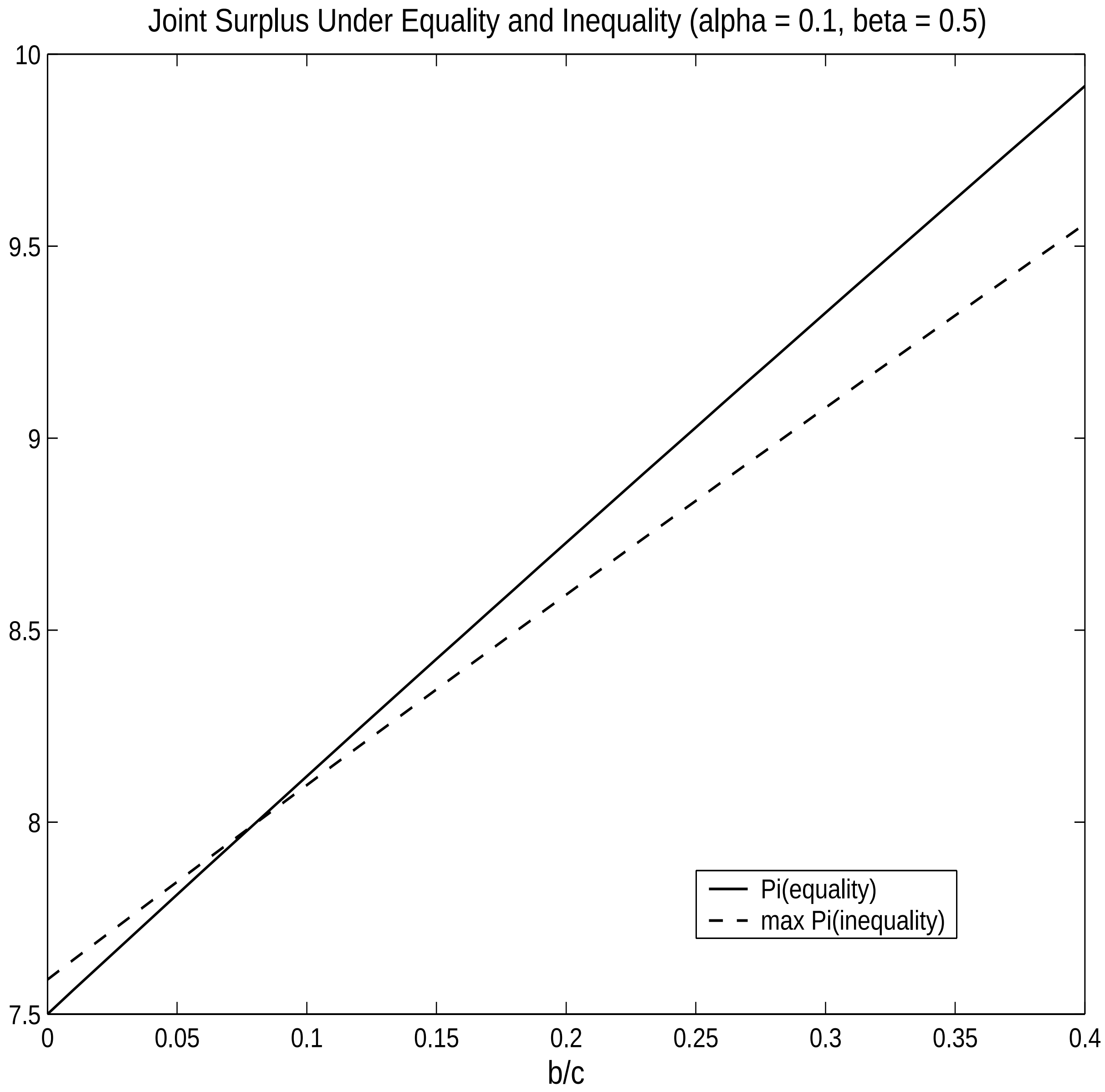
Joint Surplus Under Equality and Inequality (alpha = 0.1, beta = 0.5)
10
9.5
9
8.5
8
7.5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
b/c
Pi(equality)
max Pi(inequality)

Figure 1:

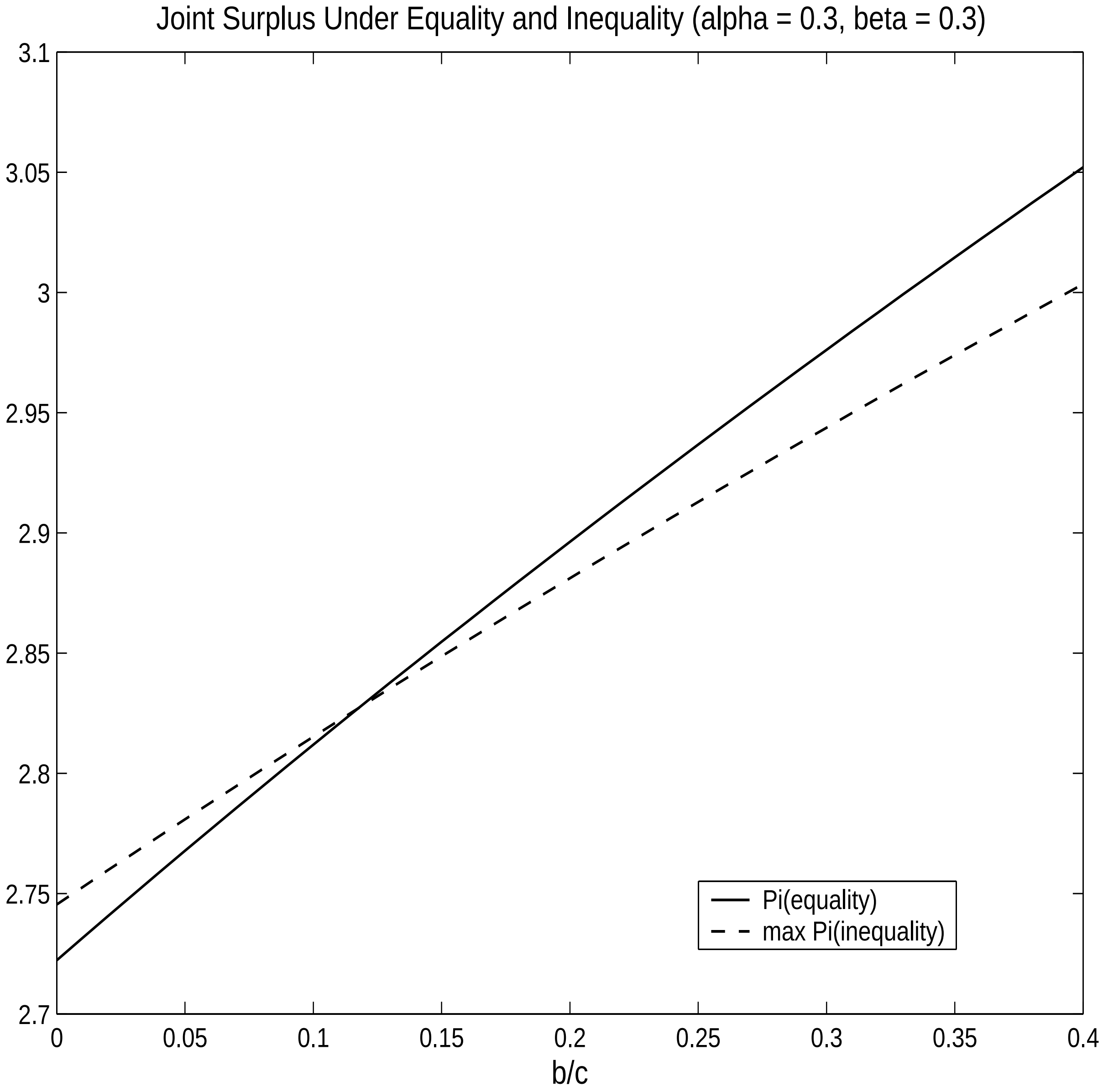
Joint Surplus Under Equality and Inequality (alpha = 0.3, beta = 0.3)
3.1
3.05
3
2.95
2.9
2.85
2.8
2.75
2.7
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
b/c
Pi(equality)
max Pi(inequality)

Figure 2:

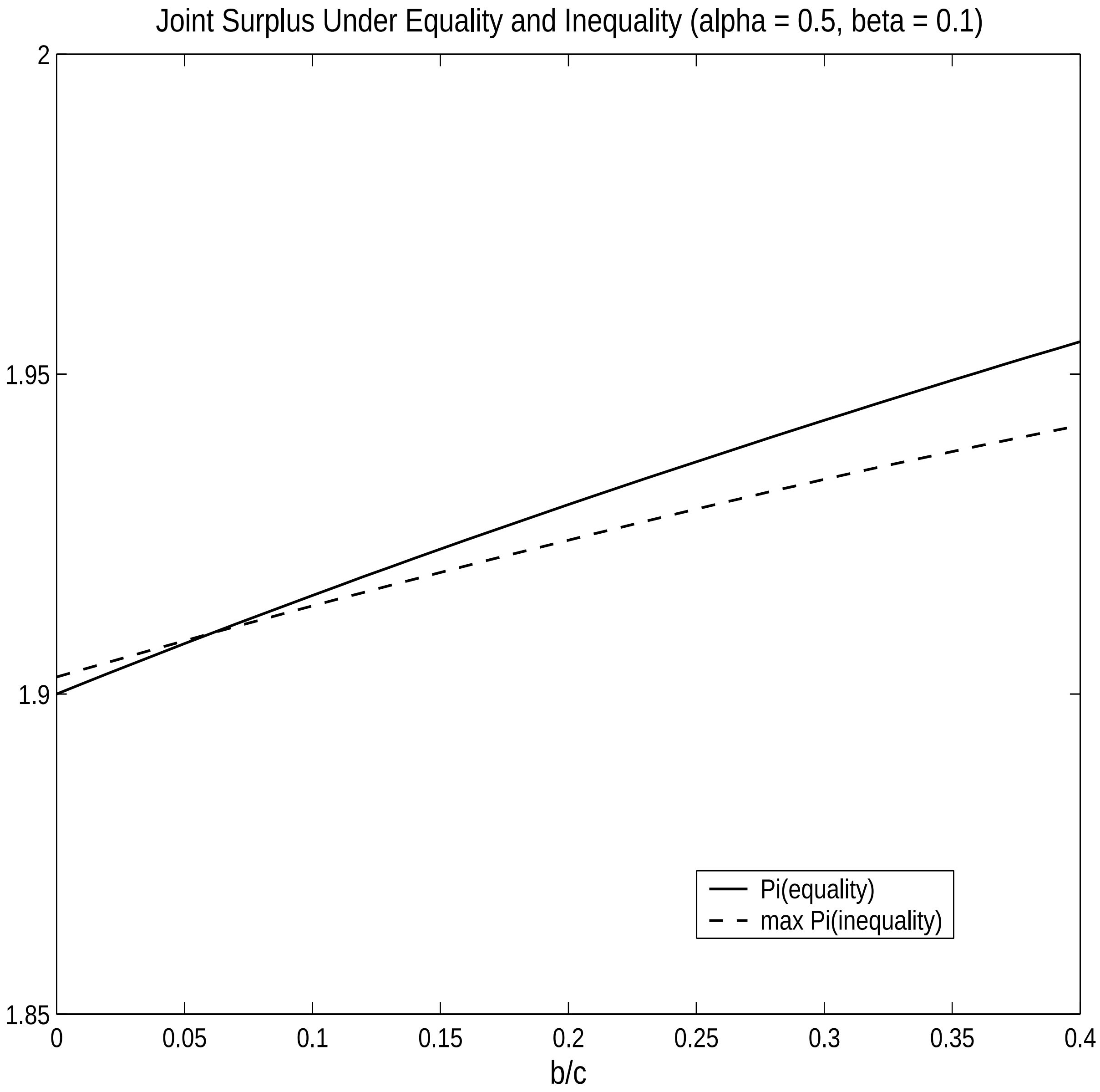
Joint Surplus Under Equality and Inequality (alpha = 0.5, beta = 0.1)
2
1.95
1.9
1.85
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
b/c
Pi(equality)
max Pi(inequality)

Figure 3: